

Computer Algebra Independent Integration Tests

Summer 2023 edition

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/167-6.2.3-e-x-^m-
a+b-cosh-c+d-xⁿ-^p

Nasser M. Abbasi

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CHAPTER 1

INTRODUCTION

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This report gives the result of running the computer algebra independent integration test. The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [68]. This is test number [167].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.3 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (68)	0.00 (0)
Mathematica	100.00 (68)	0.00 (0)
Maxima	91.18 (62)	8.82 (6)
Fricas	88.24 (60)	11.76 (8)
Maple	85.29 (58)	14.71 (10)
Giac	63.24 (43)	36.76 (25)
Sympy	35.29 (24)	64.71 (44)
Mupad	30.88 (21)	69.12 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons <ol style="list-style-type: none"> 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

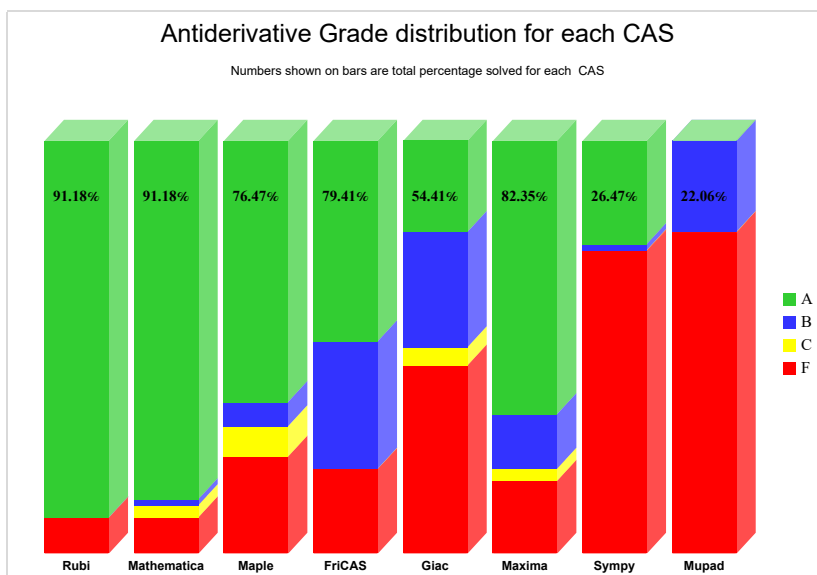
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

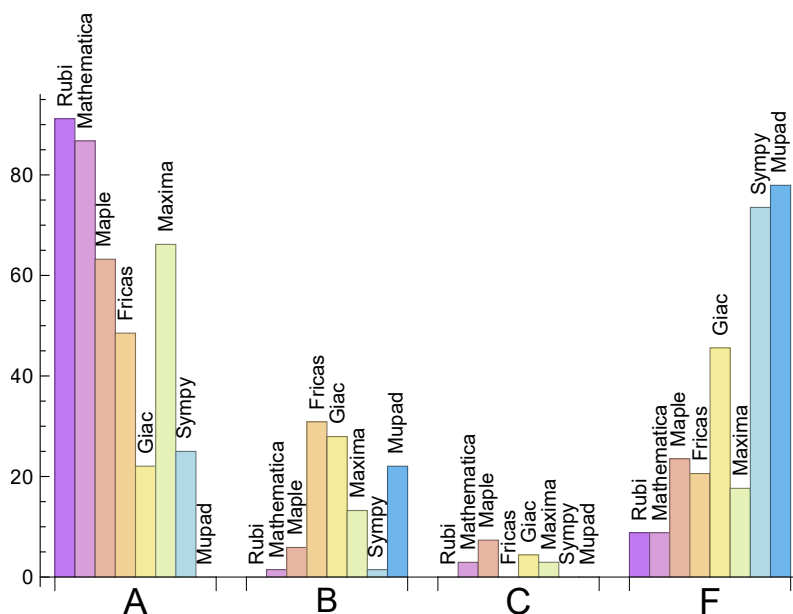
System	% A grade	% B grade	% C grade	% F grade
Rubi	91.176	0.000	0.000	8.824
Mathematica	86.765	1.471	2.941	8.824
Maxima	66.176	13.235	2.941	17.647
Maple	63.235	5.882	7.353	23.529
Fricas	48.529	30.882	0.000	20.588
Sympy	25.000	1.471	0.000	73.529
Giac	22.059	27.941	4.412	45.588
Mupad	0.000	22.059	0.000	77.941

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima

and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Maxima	6	100.00	0.00	0.00
Fricas	8	100.00	0.00	0.00
Maple	10	100.00	0.00	0.00
Giac	25	100.00	0.00	0.00
Sympy	44	100.00	0.00	0.00
Mupad	47	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Rubi	0.08
Maxima	0.25
Fricas	0.25
Maple	0.28
Mathematica	0.47
Giac	0.66
Mupad	1.29
Sympy	3.32

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	30.14	1.00	24.00	0.89
Sympy	52.67	1.16	25.50	1.08
Mathematica	74.84	0.94	53.00	0.95
Rubi	86.72	1.00	66.50	1.00
Maxima	113.13	1.55	61.00	1.01
Maple	116.10	1.18	57.00	1.07
Fricas	149.35	1.82	70.00	1.56
Giac	165.00	1.83	65.00	1.70

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

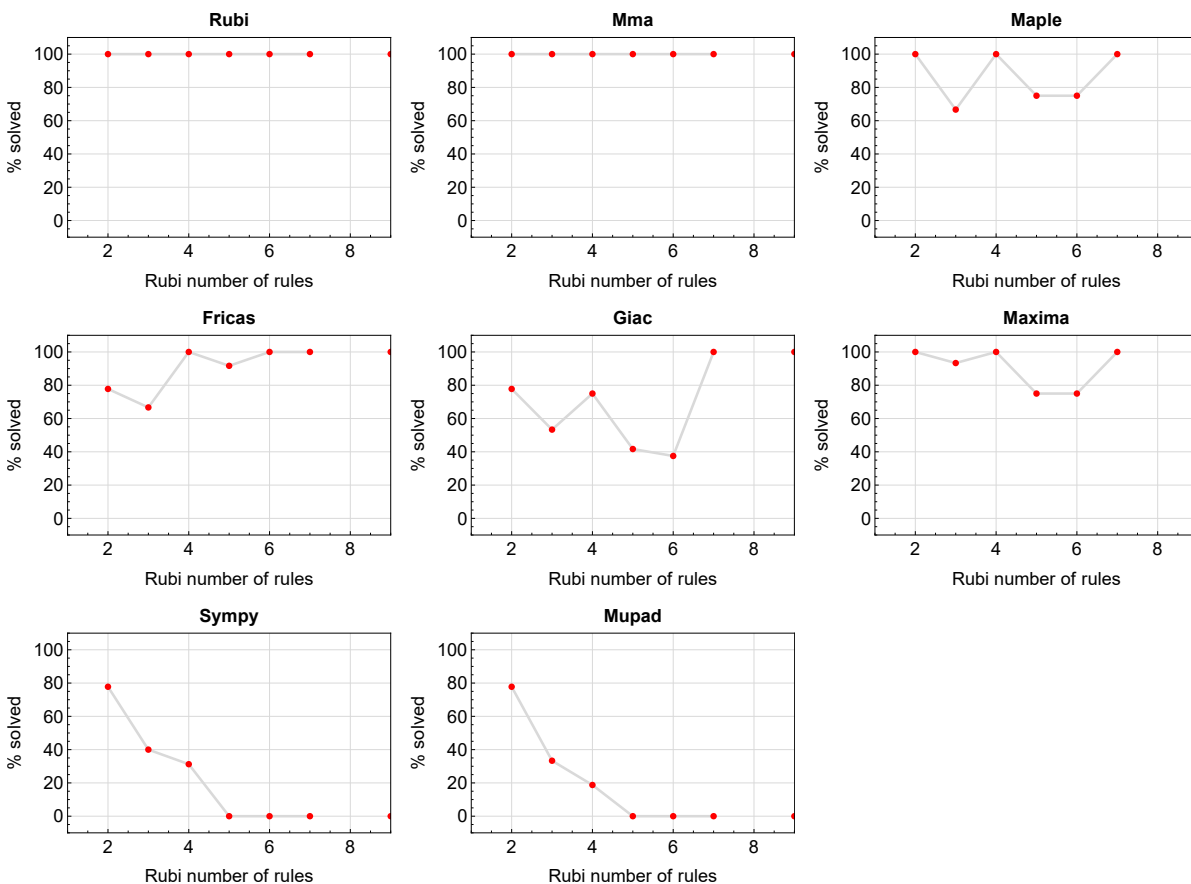


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

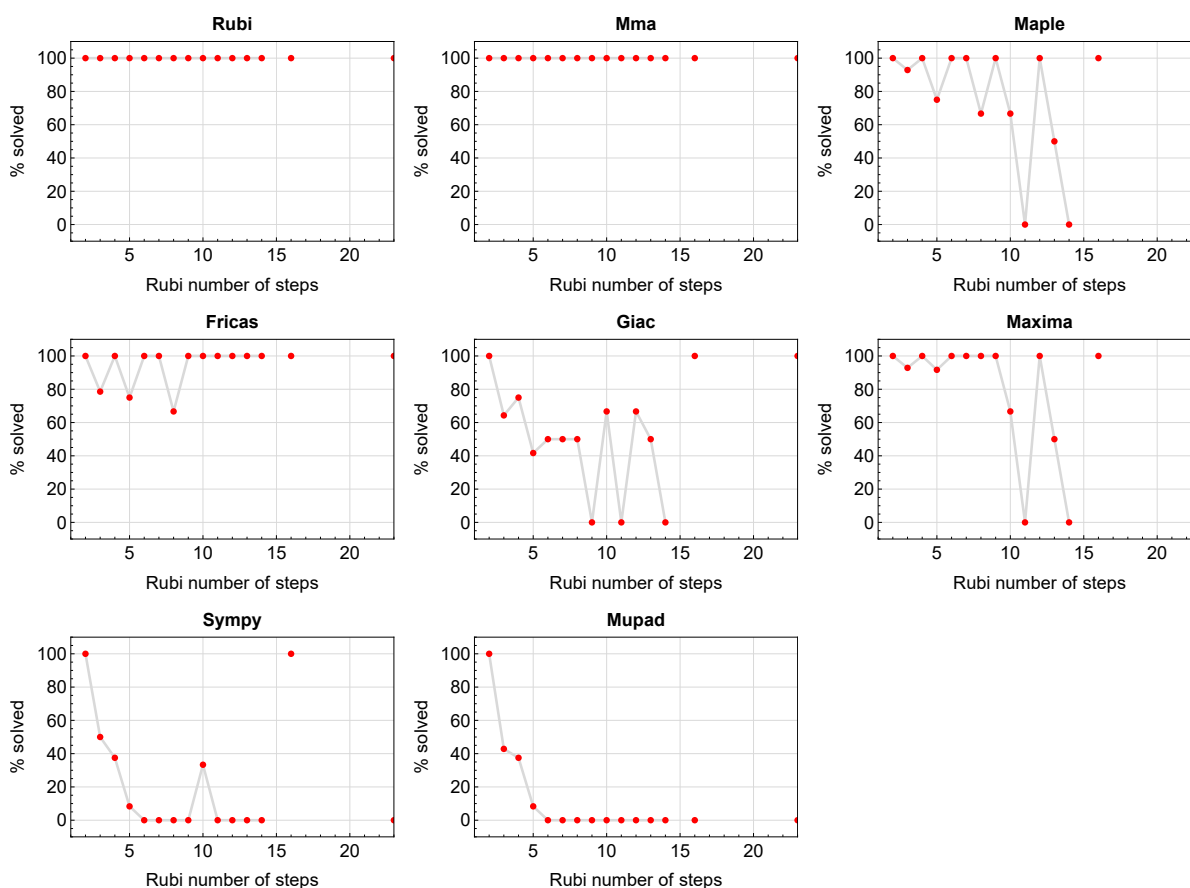


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the percentage of solved integrals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

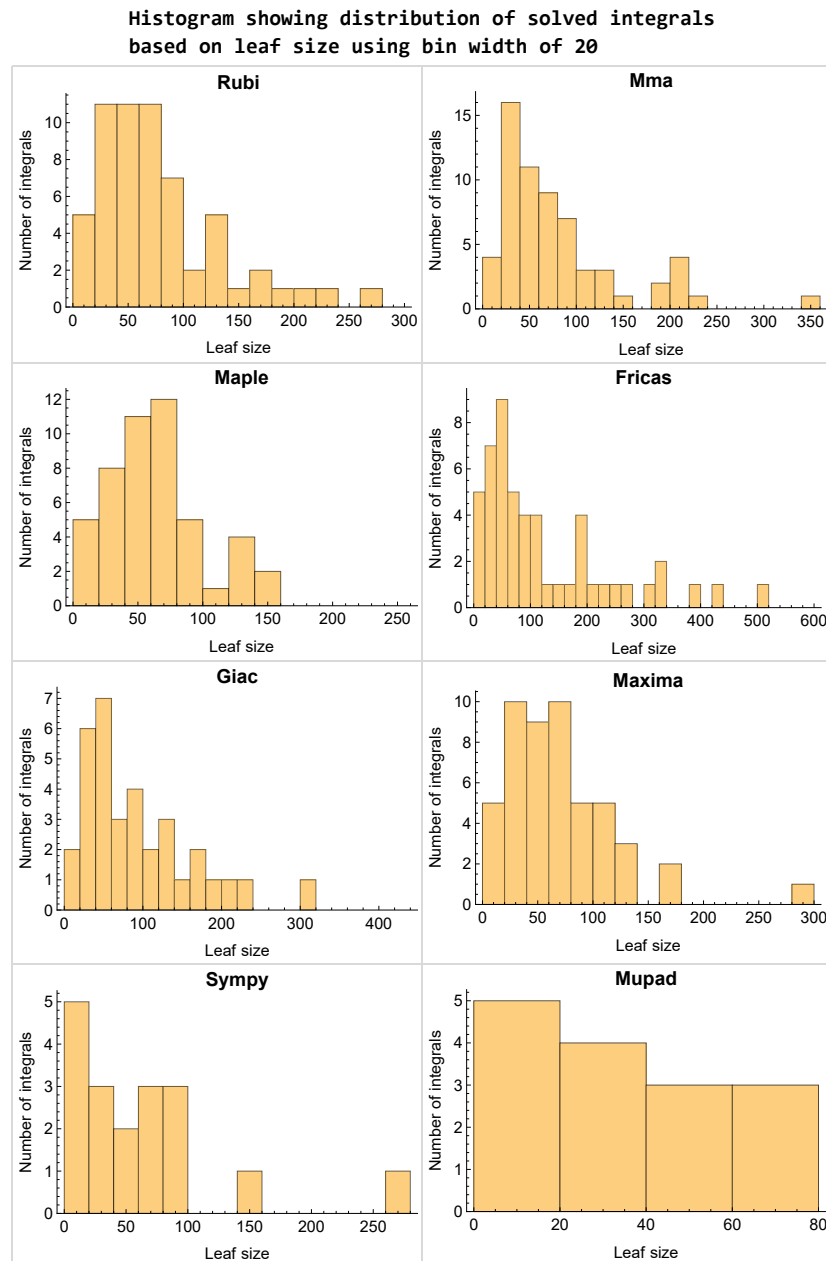


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

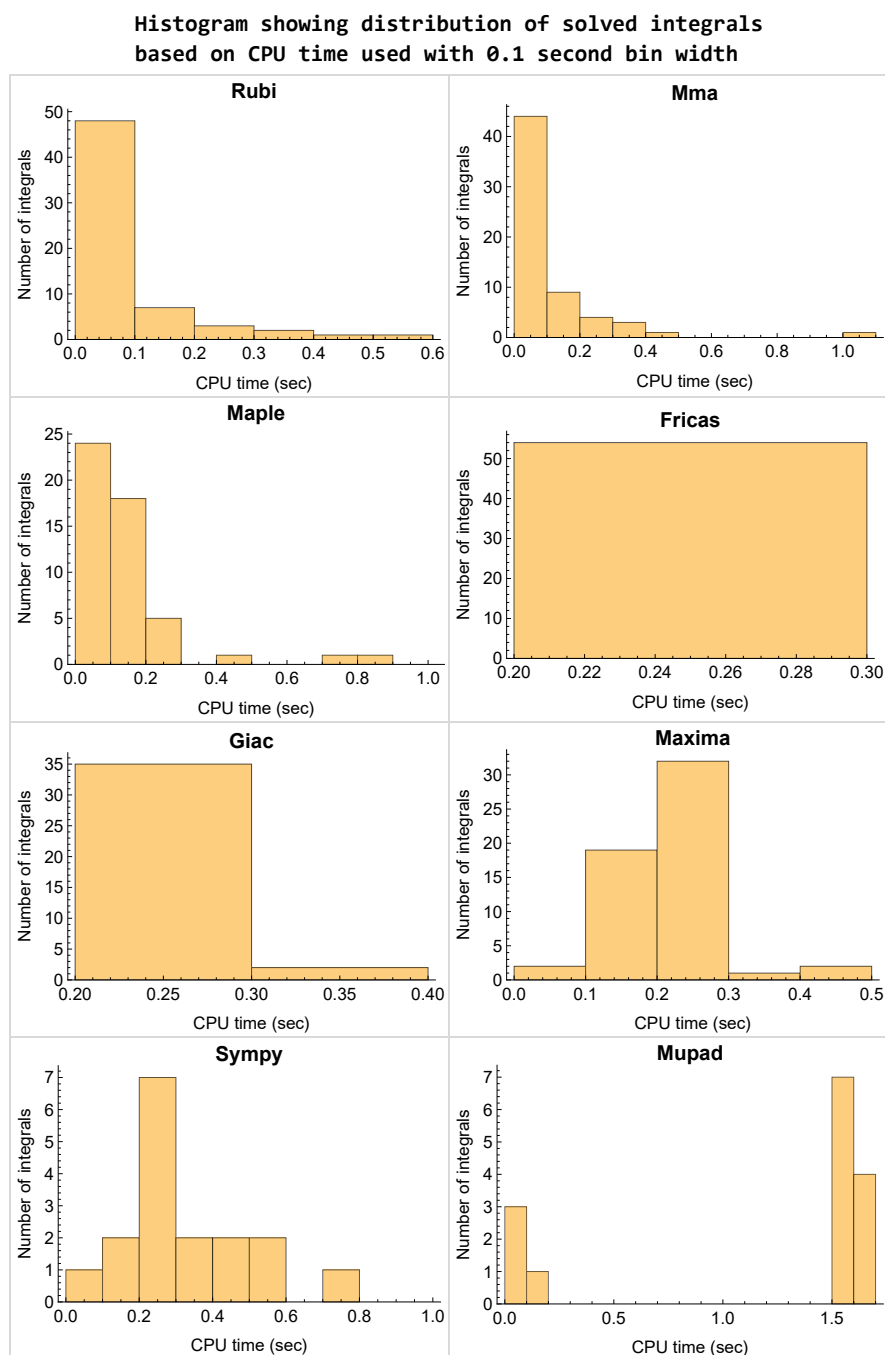


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fracas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

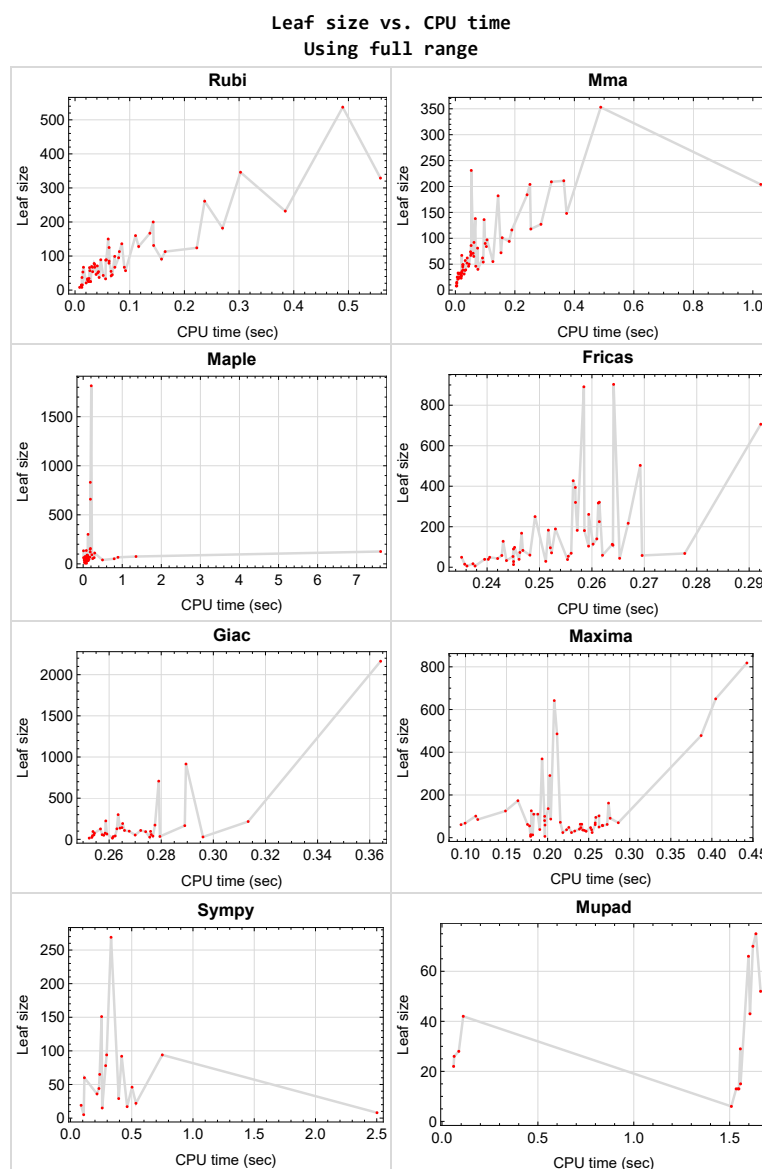


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{41, 42, 44, 46, 57, 58}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime')
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See <https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/> for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
```

```
x, aa = expr.operator(), expr.operands()
if x is None:
    return 1
else:
    return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
    # 1.7 is a fudge factor since it is low side from actual leaf count
    leafCount = round(1.7*count_ops(anti))

except Exception as ee:
    leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine, 'cos(x)*sin(x)')
the_variable = evalin(symengine, 'x')
anti = int(integrand, the_variable)
```

Which gives $\sin(x)^2/2$

1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer. the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

The following fields are present only in *Rubi Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,...}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntax

Nasser M. Abbasi
June 27, 2023
Design-vide

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1	List of integrals sorted by grade for each CAS	22
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2.3	Detailed conclusion table specific for Rubi results	39

2.1 List of integrals sorted by grade for each CAS

Rubi	22
Mma	22
Maple	23
Fricas	23
Maxima	23
Giac	24
Mupad	24
Sympy	24

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68 }

B grade { }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 61, 62, 63, 64, 65, 66 }

B grade { 3 }

C grade { 67, 68 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 38, 40, 50, 51, 52, 53, 61, 66 }

B grade { 59, 60, 64, 65 }

C grade { 35, 47, 54, 55, 56 }

F normal fail { 37, 39, 43, 45, 48, 49, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 3, 4, 5, 7, 8, 10, 11, 12, 14, 15, 17, 18, 19, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 38, 40, 53, 59, 60, 61, 64, 65, 66 }

B grade { 2, 6, 9, 13, 16, 20, 21, 22, 30, 34, 36, 50, 51, 52, 54, 55, 56, 62, 63, 67, 68 }

C grade { }

F normal fail { 35, 37, 39, 43, 45, 47, 48, 49 }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 3, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 23, 24, 25, 26, 27, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 47, 48, 49, 50, 51, 52, 53, 59, 60, 64, 65, 66 }

B grade { 1, 2, 4, 17, 22, 54, 55, 56, 61 }

C grade { 28, 29 }

F normal fail { 43, 45, 62, 63, 67, 68 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac**A grade** { 2, 4, 5, 9, 10, 11, 12, 16, 17, 18, 19, 22, 53, 61, 66 }**B grade** { 1, 3, 7, 8, 14, 15, 21, 23, 24, 25, 26, 27, 28, 29, 33, 59, 60, 64, 65 }**C grade** { 54, 55, 56 }**F normal fail** { 6, 13, 20, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 62, 63, 67, 68 }**F(-1) timeout fail** { }**F(-2) exception fail** { }**Mupad****A grade** { }**B grade** { 1, 3, 8, 10, 15, 17, 22, 23, 24, 27, 28, 29, 33, 61, 66 }**C grade** { }**F normal fail** { }**F(-1) timeout fail** { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 25, 26, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 59, 60, 62, 63, 64, 65, 67, 68 }**F(-2) exception fail** { }**Sympy****A grade** { 1, 3, 8, 15, 17, 22, 23, 24, 26, 27, 28, 29, 33, 59, 60, 61, 66 }**B grade** { 10 }**C grade** { }**F normal fail** { 2, 4, 5, 6, 7, 9, 11, 12, 13, 14, 16, 18, 19, 20, 21, 25, 30, 31, 32, 34, 35, 36, 37, 38, 39, 40, 43, 45, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 62, 63, 64, 65, 67, 68 }**F(-1) timeout fail** { }**F(-2) exception fail** { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column **N.S.** means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name **Mathematica** was abbreviated to **MMA**.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	34	34	31	41	80	29	36	75	28
N.S.	1	1.00	0.91	1.21	2.35	0.85	1.06	2.21	0.82
time (sec)	N/A	0.026	0.029	0.067	0.196	0.251	0.217	0.254	0.088

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	67	74	110	189	0	75	0
N.S.	1	1.00	0.97	1.07	1.59	2.74	0.00	1.09	0.00
time (sec)	N/A	0.030	0.052	0.049	0.183	0.253	0.000	0.258	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	B	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	31	14	13	13	19	27	13
N.S.	1	1.00	2.07	0.93	0.87	0.87	1.27	1.80	0.87
time (sec)	N/A	0.012	0.011	0.034	0.180	0.245	0.087	0.276	1.543

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	45	40	87	49	0	41	0
N.S.	1	1.00	0.85	0.75	1.64	0.92	0.00	0.77	0.00
time (sec)	N/A	0.014	0.025	0.028	0.204	0.235	0.000	0.262	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	24	39	0	24	0
N.S.	1	1.00	0.92	1.32	0.96	1.56	0.00	0.96	0.00
time (sec)	N/A	0.028	0.012	0.047	0.254	0.240	0.000	0.253	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	70	70	55	183	0	0	0
N.S.	1	1.00	1.06	1.06	0.83	2.77	0.00	0.00	0.00
time (sec)	N/A	0.027	0.053	0.050	0.178	0.252	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	38	58	40	71	0	108	0
N.S.	1	1.00	0.90	1.38	0.95	1.69	0.00	2.57	0.00
time (sec)	N/A	0.066	0.032	0.058	0.239	0.252	0.000	0.272	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	51	51	40	46	59	60	78	142	42
N.S.	1	1.00	0.78	0.90	1.16	1.18	1.53	2.78	0.82
time (sec)	N/A	0.042	0.076	0.148	0.197	0.248	0.287	0.265	0.111

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	101	90	95	427	0	97	0
N.S.	1	1.00	1.02	0.91	0.96	4.31	0.00	0.98	0.00
time (sec)	N/A	0.073	0.157	0.104	0.259	0.256	0.000	0.276	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	27	24	38	28	60	54	22
N.S.	1	1.00	0.87	0.77	1.23	0.90	1.94	1.74	0.71
time (sec)	N/A	0.024	0.020	0.071	0.190	0.245	0.114	0.254	0.061

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	86	51	56	73	0	58	0
N.S.	1	1.00	1.10	0.65	0.72	0.94	0.00	0.74	0.00
time (sec)	N/A	0.035	0.053	0.065	0.267	0.246	0.000	0.276	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	33	34	31	49	0	35	0
N.S.	1	1.00	0.89	0.92	0.84	1.32	0.00	0.95	0.00
time (sec)	N/A	0.044	0.016	0.104	0.233	0.240	0.000	0.279	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	94	86	61	394	0	0	0
N.S.	1	1.00	1.07	0.98	0.69	4.48	0.00	0.00	0.00
time (sec)	N/A	0.056	0.181	0.101	0.258	0.257	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	66	36	90	0	126	0
N.S.	1	1.00	0.81	1.16	0.63	1.58	0.00	2.21	0.00
time (sec)	N/A	0.092	0.069	0.110	0.243	0.245	0.000	0.257	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	55	93	100	96	92	192	70
N.S.	1	1.00	0.70	1.18	1.27	1.22	1.16	2.43	0.89
time (sec)	N/A	0.062	0.126	0.223	0.196	0.252	0.418	0.265	1.621

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	184	157	162	903	0	166	0
N.S.	1	1.00	1.15	0.98	1.01	5.64	0.00	1.04	0.00
time (sec)	N/A	0.111	0.241	0.186	0.274	0.264	0.000	0.289	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	28	62	38	44	56	26
N.S.	1	1.00	1.00	0.85	1.88	1.15	1.33	1.70	0.79
time (sec)	N/A	0.023	0.009	0.088	0.175	0.255	0.233	0.257	0.063

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	136	86	91	113	0	95	0
N.S.	1	1.00	1.09	0.69	0.73	0.90	0.00	0.76	0.00
time (sec)	N/A	0.062	0.097	0.104	0.276	0.264	0.000	0.254	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	49	69	50	83	0	50	0
N.S.	1	1.00	0.89	1.25	0.91	1.51	0.00	0.91	0.00
time (sec)	N/A	0.066	0.025	0.170	0.263	0.247	0.000	0.258	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	204	149	102	891	0	0	0
N.S.	1	1.00	1.50	1.10	0.75	6.55	0.00	0.00	0.00
time (sec)	N/A	0.086	0.251	0.178	0.263	0.258	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	92	121	58	168	0	224	0
N.S.	1	1.00	1.01	1.33	0.64	1.85	0.00	2.46	0.00
time (sec)	N/A	0.158	0.063	0.181	0.267	0.247	0.000	0.259	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	B	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	52	126	128	94	108	52
N.S.	1	1.00	1.00	0.78	1.88	1.91	1.40	1.61	0.78
time (sec)	N/A	0.032	0.022	0.793	0.180	0.243	0.750	0.266	1.661

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	5	15	6
N.S.	1	1.00	1.00	0.88	0.75	0.75	0.62	1.88	0.75
time (sec)	N/A	0.009	0.003	0.053	0.179	0.238	0.108	0.261	1.509

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	8	8	8	7	6	6	8	15	15
N.S.	1	1.00	1.00	0.88	0.75	0.75	1.00	1.88	1.88
time (sec)	N/A	0.012	0.005	0.086	0.197	0.236	2.502	0.252	1.557

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	56	37	58	0	173	0
N.S.	1	1.00	1.00	1.70	1.12	1.76	0.00	5.24	0.00
time (sec)	N/A	0.056	0.021	0.110	0.223	0.243	0.000	0.278	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	27	24	39	17	43	0
N.S.	1	1.00	1.00	1.29	1.14	1.86	0.81	2.05	0.00
time (sec)	N/A	0.020	0.008	0.075	0.218	0.240	0.462	0.254	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	15	15	29	13
N.S.	1	1.00	1.00	1.08	1.00	1.15	1.15	2.23	1.00
time (sec)	N/A	0.012	0.005	0.052	0.179	0.236	0.259	0.296	1.548

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	43	47	33	29	93	29
N.S.	1	1.00	1.00	1.48	1.62	1.14	1.00	3.21	1.00
time (sec)	N/A	0.023	0.020	0.088	0.252	0.244	0.394	0.274	1.556

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	C	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	39	65	48	43	46	216	66
N.S.	1	1.00	0.85	1.41	1.04	0.93	1.00	4.70	1.43
time (sec)	N/A	0.039	0.035	0.117	0.226	0.242	0.503	0.313	1.598

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	71	70	72	225	0	0	0
N.S.	1	1.00	1.06	1.04	1.07	3.36	0.00	0.00	0.00
time (sec)	N/A	0.036	0.060	0.079	0.215	0.261	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	33	24	39	0	0	0
N.S.	1	1.00	1.00	1.32	0.96	1.56	0.00	0.00	0.00
time (sec)	N/A	0.024	0.014	0.073	0.229	0.246	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	49	44	63	53	0	0	0
N.S.	1	1.00	0.86	0.77	1.11	0.93	0.00	0.00	0.00
time (sec)	N/A	0.027	0.026	0.078	0.240	0.245	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	15	15	15	14	13	17	22	33	13
N.S.	1	1.00	1.00	0.93	0.87	1.13	1.47	2.20	0.87
time (sec)	N/A	0.013	0.005	0.060	0.182	0.237	0.534	0.262	1.534

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	75	75	74	82	63	250	0	0	0
N.S.	1	1.00	0.99	1.09	0.84	3.33	0.00	0.00	0.00
time (sec)	N/A	0.036	0.051	0.112	0.242	0.249	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	62	74	61	0	0	0	0
N.S.	1	1.00	0.93	1.10	0.91	0.00	0.00	0.00	0.00
time (sec)	N/A	0.015	0.040	0.111	0.095	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	23	33	30	54	0	0	0
N.S.	1	1.00	0.92	1.32	1.20	2.16	0.00	0.00	0.00
time (sec)	N/A	0.028	0.020	0.130	0.247	0.255	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	89	89	81	0	68	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.76	0.00	0.00	0.00	0.00
time (sec)	N/A	0.057	0.075	0.000	0.100	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	36	40	37	69	0	0	0
N.S.	1	1.00	0.84	0.93	0.86	1.60	0.00	0.00	0.00
time (sec)	N/A	0.052	0.022	0.495	0.254	0.256	0.000	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	24	24	20	24	24
N.S.	1	1.00	1.09	1.00	1.09	1.09	0.91	1.09	1.09
time (sec)	N/A	0.042	4.414	0.053	0.420	0.256	7.942	1.322	1.547

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	131	131	148	0	0	0	0	0	0
N.S.	1	1.00	1.13	0.00	0.00	0.00	0.00	0.00	0.00
time (sec)	N/A	0.144	0.374	0.000	0.000	0.000	0.000	0.000	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	1.08
time (sec)	N/A	0.047	6.373	0.051	0.421	0.277	24.084	7.296	1.547

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	84	110	85	0	0	0	0
N.S.	1	1.00	0.94	1.24	0.96	0.00	0.00	0.00	0.00
time (sec)	N/A	0.047	0.104	0.295	0.115	0.000	0.000	0.000	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	116	0	101	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.79	0.00	0.00	0.00	0.00
time (sec)	N/A	0.116	0.190	0.000	0.112	0.000	0.000	0.000	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	200	200	182	0	173	0	0	0	0
N.S.	1	1.00	0.91	0.00	0.86	0.00	0.00	0.00	0.00
time (sec)	N/A	0.143	0.144	0.000	0.164	0.000	0.000	0.000	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	46	63	34	140	0	0	0
N.S.	1	1.00	1.02	1.40	0.76	3.11	0.00	0.00	0.00
time (sec)	N/A	0.068	0.044	0.281	0.245	0.261	0.000	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	54	75	47	182	0	0	0
N.S.	1	1.00	0.81	1.12	0.70	2.72	0.00	0.00	0.00
time (sec)	N/A	0.090	0.094	1.352	0.240	0.257	0.000	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	B	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	97	126	70	320	0	0	0
N.S.	1	1.00	0.86	1.12	0.62	2.83	0.00	0.00	0.00
time (sec)	N/A	0.165	0.107	7.612	0.286	0.257	0.000	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F	A	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	57	54	69	98	0	52	0
N.S.	1	1.00	0.80	0.76	0.97	1.38	0.00	0.73	0.00
time (sec)	N/A	0.041	0.033	0.244	0.258	0.245	0.000	0.270	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	62	136	818	321	0	137	0
N.S.	1	1.00	0.55	1.20	7.24	2.84	0.00	1.21	0.00
time (sec)	N/A	0.081	0.091	0.081	0.443	0.261	0.000	0.264	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	39	66	650	261	0	99	0
N.S.	1	1.00	0.72	1.22	12.04	4.83	0.00	1.83	0.00
time (sec)	N/A	0.043	0.025	0.050	0.405	0.259	0.000	0.268	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	C	B	B	F	C	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	25	36	478	59	0	39	0
N.S.	1	1.00	0.68	0.97	12.92	1.59	0.00	1.05	0.00
time (sec)	N/A	0.013	0.006	0.029	0.387	0.262	0.000	0.277	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	20	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.67	1.17	1.17
time (sec)	N/A	0.026	1.091	0.020	0.846	0.246	3.020	0.273	1.651

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	14	23	22	14	14
N.S.	1	1.00	1.17	1.00	1.17	1.92	1.83	1.17	1.17
time (sec)	N/A	0.029	3.360	0.020	0.610	0.251	3.036	0.271	1.572

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	346	346	209	831	486	104	269	915	0
N.S.	1	1.00	0.60	2.40	1.40	0.30	0.78	2.64	0.00
time (sec)	N/A	0.303	0.323	0.183	0.211	0.259	0.331	0.289	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	72	301	291	68	151	300	0
N.S.	1	1.00	0.43	1.80	1.74	0.41	0.90	1.80	0.00
time (sec)	N/A	0.137	0.153	0.125	0.203	0.278	0.254	0.263	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	50	63	110	44	65	65	43
N.S.	1	1.00	0.93	1.17	2.04	0.81	1.20	1.20	0.80
time (sec)	N/A	0.032	0.046	0.011	0.188	0.265	0.239	0.259	1.606

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	124	124	127	0	0	217	0	0	0
N.S.	1	1.00	1.02	0.00	0.00	1.75	0.00	0.00	0.00
time (sec)	N/A	0.223	0.288	0.000	0.000	0.267	0.000	0.000	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	182	182	204	0	0	317	0	0	0
N.S.	1	1.00	1.12	0.00	0.00	1.74	0.00	0.00	0.00
time (sec)	N/A	0.270	1.027	0.000	0.000	0.261	0.000	0.000	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	537	537	353	1815	642	181	0	2163	0
N.S.	1	1.00	0.66	3.38	1.20	0.34	0.00	4.03	0.00
time (sec)	N/A	0.490	0.489	0.207	0.208	0.259	0.000	0.364	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	B	A	A	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	118	659	369	109	0	707	0
N.S.	1	1.00	0.45	2.52	1.41	0.42	0.00	2.71	0.00
time (sec)	N/A	0.237	0.254	0.186	0.193	0.264	0.000	0.279	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	65	133	136	58	94	128	75
N.S.	1	1.00	0.76	1.56	1.60	0.68	1.11	1.51	0.88
time (sec)	N/A	0.062	0.062	0.009	0.201	0.270	0.296	0.263	1.637

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	232	232	231	0	0	503	0	0	0
N.S.	1	1.00	1.00	0.00	0.00	2.17	0.00	0.00	0.00
time (sec)	N/A	0.384	0.053	0.000	0.000	0.269	0.000	0.000	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	C	F	F	B	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	329	329	211	0	0	706	0	0	0
N.S.	1	1.00	0.64	0.00	0.00	2.15	0.00	0.00	0.00
time (sec)	N/A	0.559	0.364	0.000	0.000	0.292	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [54] had the largest ratio of [.7500000000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	3	1.00	12	0.250
2	A	4	4	1.00	12	0.333
3	A	2	2	1.00	10	0.200
4	A	3	3	1.00	8	0.375
5	A	3	3	1.00	12	0.250
6	A	4	4	1.00	12	0.333
7	A	5	5	1.00	12	0.417
8	A	3	3	1.00	14	0.214
9	A	6	5	1.00	14	0.357
10	A	3	3	1.00	12	0.250
11	A	5	4	1.00	10	0.400
12	A	5	4	1.00	14	0.286
13	A	6	6	1.00	14	0.429
14	A	7	6	1.00	14	0.429
15	A	4	4	1.00	14	0.286
16	A	10	5	1.00	14	0.357
17	A	3	2	1.00	12	0.167
18	A	8	4	1.00	10	0.400
19	A	8	4	1.00	14	0.286
20	A	9	5	1.00	14	0.357
21	A	12	6	1.00	14	0.429
22	A	3	2	1.00	12	0.167
23	A	2	2	1.00	8	0.250
24	A	2	2	1.00	8	0.250

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
25	A	5	5	1.00	8	0.625
26	A	3	3	1.00	12	0.250
27	A	2	2	1.00	12	0.167
28	A	3	3	1.00	12	0.250
29	A	4	3	1.00	12	0.250
30	A	5	5	1.00	8	0.625
31	A	3	3	1.00	12	0.250
32	A	4	4	1.00	12	0.333
33	A	2	2	1.00	12	0.167
34	A	5	5	1.00	12	0.417
35	A	3	2	1.00	8	0.250
36	A	3	3	1.00	12	0.250
37	A	5	3	1.00	10	0.300
38	A	5	4	1.00	14	0.286
39	A	8	3	1.00	10	0.300
40	A	8	4	1.00	14	0.286
41	N/A	0	0	1.00	18	0.000
42	N/A	0	0	1.00	20	0.000
43	A	3	3	1.00	20	0.150
44	N/A	0	0	1.00	22	0.000
45	A	5	5	1.00	22	0.227
46	N/A	0	0	1.00	24	0.000
47	A	3	2	1.00	12	0.167
48	A	5	3	1.00	14	0.214
49	A	8	3	1.00	14	0.214
50	A	5	5	1.00	16	0.312
51	A	7	6	1.00	18	0.333
52	A	12	6	1.00	18	0.333
53	A	4	4	1.00	18	0.222
54	A	12	9	1.00	12	0.750
55	A	8	7	1.00	10	0.700
56	A	4	4	1.00	8	0.500
57	N/A	0	0	1.00	12	0.000
58	N/A	0	0	1.00	12	0.000

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
59	A	16	4	1.00	18	0.222
60	A	10	4	1.00	16	0.250
61	A	4	4	1.00	14	0.286
62	A	10	5	1.00	18	0.278
63	A	11	6	1.00	18	0.333
64	A	23	6	1.00	18	0.333
65	A	13	5	1.00	16	0.312
66	A	5	4	1.00	14	0.286
67	A	13	5	1.00	18	0.278
68	A	14	6	1.00	18	0.333

CHAPTER 3

LISTING OF INTEGRALS

3.1	$\int x^3 \cosh(a + bx^2) dx$	46
3.2	$\int x^2 \cosh(a + bx^2) dx$	50
3.3	$\int x \cosh(a + bx^2) dx$	54
3.4	$\int \cosh(a + bx^2) dx$	58
3.5	$\int \frac{\cosh(a+bx^2)}{x} dx$	62
3.6	$\int \frac{\cosh(a+bx^2)}{x^2} dx$	66
3.7	$\int \frac{\cosh(a+bx^2)}{x^3} dx$	70
3.8	$\int x^3 \cosh^2(a + bx^2) dx$	75
3.9	$\int x^2 \cosh^2(a + bx^2) dx$	79
3.10	$\int x \cosh^2(a + bx^2) dx$	84
3.11	$\int \cosh^2(a + bx^2) dx$	88
3.12	$\int \frac{\cosh^2(a+bx^2)}{x} dx$	92
3.13	$\int \frac{\cosh^2(a+bx^2)}{x^2} dx$	96
3.14	$\int \frac{\cosh^2(a+bx^2)}{x^3} dx$	101
3.15	$\int x^3 \cosh^3(a + bx^2) dx$	106
3.16	$\int x^2 \cosh^3(a + bx^2) dx$	111
3.17	$\int x \cosh^3(a + bx^2) dx$	117
3.18	$\int \cosh^3(a + bx^2) dx$	121
3.19	$\int \frac{\cosh^3(a+bx^2)}{x} dx$	126
3.20	$\int \frac{\cosh^3(a+bx^2)}{x^2} dx$	130
3.21	$\int \frac{\cosh^3(a+bx^2)}{x^3} dx$	135
3.22	$\int x \cosh^7(a + bx^2) dx$	140
3.23	$\int x^2 \cosh(x^3) dx$	144
3.24	$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx$	148

3.25	$\int \cosh\left(a + \frac{b}{x}\right) dx$	152
3.26	$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx$	157
3.27	$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx$	161
3.28	$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx$	165
3.29	$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx$	169
3.30	$\int \cosh\left(a + \frac{b}{x^2}\right) dx$	173
3.31	$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx$	178
3.32	$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx$	182
3.33	$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx$	186
3.34	$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx$	190
3.35	$\int \cosh(a + bx^n) dx$	195
3.36	$\int \frac{x}{\cosh(a + bx^n)} dx$	199
3.37	$\int \cosh^2(a + bx^n) dx$	203
3.38	$\int \frac{\cosh^2(a + bx^n)}{x} dx$	207
3.39	$\int \cosh^3(a + bx^n) dx$	211
3.40	$\int \frac{\cosh^3(a + bx^n)}{x} dx$	215
3.41	$\int (ex)^m (b \cosh(c + dx^n))^p dx$	219
3.42	$\int (ex)^m (a + b \cosh(c + dx^n))^p dx$	222
3.43	$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx$	225
3.44	$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx$	229
3.45	$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$	232
3.46	$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx$	237
3.47	$\int x^m \cosh(a + bx^n) dx$	240
3.48	$\int x^m \cosh^2(a + bx^n) dx$	244
3.49	$\int x^m \cosh^3(a + bx^n) dx$	248
3.50	$\int x^{-1-n} \cosh(a + bx^n) dx$	253
3.51	$\int x^{-1-n} \cosh^2(a + bx^n) dx$	257
3.52	$\int x^{-1-n} \cosh^3(a + bx^n) dx$	262
3.53	$\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx$	267
3.54	$\int x^2 \cosh((a + bx)^2) dx$	271
3.55	$\int x \cosh((a + bx)^2) dx$	277
3.56	$\int \cosh((a + bx)^2) dx$	282
3.57	$\int \frac{\cosh((a + bx)^2)}{x} dx$	286
3.58	$\int \frac{\cosh((a + bx)^2)}{x^2} dx$	289
3.59	$\int x^2 \cosh(a + b\sqrt{c + dx}) dx$	292
3.60	$\int x \cosh(a + b\sqrt{c + dx}) dx$	301
3.61	$\int \cosh(a + b\sqrt{c + dx}) dx$	307
3.62	$\int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx$	311

3.63	$\int \frac{\cosh(a+b\sqrt{c+dx})}{x^2} dx$	316
3.64	$\int x^2 \cosh(a+b\sqrt[3]{c+dx}) dx$	322
3.65	$\int x \cosh(a+b\sqrt[3]{c+dx}) dx$	339
3.66	$\int \cosh(a+b\sqrt[3]{c+dx}) dx$	347
3.67	$\int \frac{\cosh(a+b\sqrt[3]{c+dx})}{x} dx$	352
3.68	$\int \frac{\cosh(a+b\sqrt[3]{c+dx})}{x^2} dx$	360

3.1 $\int x^3 \cosh(a + bx^2) dx$

Optimal result	46
Rubi [A] (verified)	46
Mathematica [A] (verified)	47
Maple [A] (verified)	47
Fricas [A] (verification not implemented)	48
Sympy [A] (verification not implemented)	48
Maxima [B] (verification not implemented)	48
Giac [B] (verification not implemented)	49
Mupad [B] (verification not implemented)	49

Optimal result

Integrand size = 12, antiderivative size = 34

$$\int x^3 \cosh(a + bx^2) dx = -\frac{\cosh(a + bx^2)}{2b^2} + \frac{x^2 \sinh(a + bx^2)}{2b}$$

[Out] $-1/2*\cosh(b*x^2+a)/b^2+1/2*x^2*\sinh(b*x^2+a)/b$

Rubi [A] (verified)

Time = 0.03 (sec), antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 3377, 2718}

$$\int x^3 \cosh(a + bx^2) dx = \frac{x^2 \sinh(a + bx^2)}{2b} - \frac{\cosh(a + bx^2)}{2b^2}$$

[In] $\text{Int}[x^3*\text{Cosh}[a + b*x^2], x]$

[Out] $-1/2*\text{Cosh}[a + b*x^2]/b^2 + (x^2*\text{Sinh}[a + b*x^2])/(2*b)$

Rule 2718

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/d, x] /; \text{FreeQ}[\{c, d\}, x]$

Rule 3377

$\text{Int}[(c_.) + (d_.)*(x_.)]^{(m_.)*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-(c + d*x)^m*(\text{Cos}[e + f*x]/f), x] + \text{Dist}[d*(m/f), \text{Int}[(c + d*x)^{(m-1)}*\text{Cos}[e + f*x], x], x] /; \text{FreeQ}[\{c, d, e, f\}, x] \&\& \text{GtQ}[m, 0]$

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \text{Subst} \left(\int x \cosh(a + bx) dx, x, x^2 \right) \\ &= \frac{x^2 \sinh(a + bx^2)}{2b} - \frac{\text{Subst}(\int \sinh(a + bx) dx, x, x^2)}{2b} \\ &= -\frac{\cosh(a + bx^2)}{2b^2} + \frac{x^2 \sinh(a + bx^2)}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.91

$$\int x^3 \cosh(a + bx^2) dx = \frac{-\cosh(a + bx^2) + bx^2 \sinh(a + bx^2)}{2b^2}$$

[In] Integrate[x^3*Cosh[a + b*x^2],x]

[Out] (-Cosh[a + b*x^2] + b*x^2*Sinh[a + b*x^2])/(2*b^2)

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.21

method	result	size
parallelrisc	$\frac{1 - \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)x^2b}{b^2 \left(\tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)^2 - 1\right)}$	41
risc	$\frac{(bx^2-1)e^{bx^2+a}}{4b^2} - \frac{(bx^2+1)e^{-bx^2-a}}{4b^2}$	45
meijerg	$-\frac{\cosh(a)\sqrt{\pi} \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cosh(bx^2)}{2\sqrt{\pi}} - \frac{bx^2 \sinh(bx^2)}{2\sqrt{\pi}}\right)}{b^2} + \frac{\sinh(a)(\cosh(bx^2)bx^2 - \sinh(bx^2))}{2b^2}$	71

[In] int(x^3*cosh(b*x^2+a),x,method=_RETURNVERBOSE)

[Out] (1-tanh(1/2*b*x^2+1/2*a)*x^2*b)/b^2/(tanh(1/2*b*x^2+1/2*a)^2-1)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int x^3 \cosh(a + bx^2) dx = \frac{bx^2 \sinh(bx^2 + a) - \cosh(bx^2 + a)}{2b^2}$$

[In] integrate(x^3*cosh(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*(b*x^2*sinh(b*x^2 + a) - cosh(b*x^2 + a))/b^2

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int x^3 \cosh(a + bx^2) dx = \begin{cases} \frac{x^2 \sinh(a+bx^2)}{2b} - \frac{\cosh(a+bx^2)}{2b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cosh(a)}{4} & \text{otherwise} \end{cases}$$

[In] integrate(x**3*cosh(b*x**2+a),x)

[Out] Piecewise((x**2*sinh(a + b*x**2)/(2*b) - cosh(a + b*x**2)/(2*b**2), Ne(b, 0)), (x**4*cosh(a)/4, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. 2(30) = 60.

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.35

$$\int x^3 \cosh(a + bx^2) dx = \frac{1}{4} x^4 \cosh(bx^2 + a) - \frac{1}{8} b \left(\frac{(b^2 x^4 e^a - 2bx^2 e^a + 2e^a)e^{(bx^2)}}{b^3} + \frac{(b^2 x^4 + 2bx^2 + 2)e^{(-bx^2 - a)}}{b^3} \right)$$

[In] integrate(x^3*cosh(b*x^2+a),x, algorithm="maxima")

[Out] 1/4*x^4*cosh(b*x^2 + a) - 1/8*b*((b^2*x^4*e^a - 2*b*x^2*e^a + 2*e^a)*e^(b*x^2)/b^3 + (b^2*x^4 + 2*b*x^2 + 2)*e^(-b*x^2 - a)/b^3)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(30) = 60$.

Time = 0.25 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.21

$$\int x^3 \cosh(a + bx^2) dx = \frac{(bx^2 + a - 1)e^{(bx^2+a)} - (bx^2 + a + 1)e^{(-bx^2-a)}}{4b^2} - \frac{ae^{(bx^2+a)} - ae^{(-bx^2-a)}}{4b^2}$$

[In] integrate(x^3*cosh(b*x^2+a),x, algorithm="giac")

[Out] $\frac{1}{4} * ((b * x^2 + a - 1) * e^{(b * x^2 + a)} - (b * x^2 + a + 1) * e^{(-b * x^2 - a)}) / b^2 - \frac{1}{4} * (a * e^{(b * x^2 + a)} - a * e^{(-b * x^2 - a)}) / b^2$

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.82

$$\int x^3 \cosh(a + bx^2) dx = -\frac{\cosh(bx^2 + a) - bx^2 \sinh(bx^2 + a)}{2b^2}$$

[In] int(x^3*cosh(a + b*x^2),x)

[Out] $-(\cosh(a + b * x^2) - b * x^2 * \sinh(a + b * x^2)) / (2 * b^2)$

3.2 $\int x^2 \cosh(a + bx^2) dx$

Optimal result	50
Rubi [A] (verified)	50
Mathematica [A] (verified)	51
Maple [A] (verified)	52
Fricas [B] (verification not implemented)	52
Sympy [F]	52
Maxima [B] (verification not implemented)	53
Giac [A] (verification not implemented)	53
Mupad [F(-1)]	53

Optimal result

Integrand size = 12, antiderivative size = 69

$$\int x^2 \cosh(a + bx^2) dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})}{8b^{3/2}} - \frac{e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx})}{8b^{3/2}} + \frac{x \sinh(a + bx^2)}{2b}$$

[Out] $1/2*x*\sinh(b*x^2+a)/b+1/8*\operatorname{erf}(x*b^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(3/2)}/\exp(a)-1/8*\exp(a)*\operatorname{erfi}(x*b^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(3/2)}$

Rubi [A] (verified)

Time = 0.03 (sec), antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5433, 5406, 2235, 2236}

$$\int x^2 \cosh(a + bx^2) dx = \frac{\sqrt{\pi}e^{-a}\operatorname{erf}(\sqrt{bx})}{8b^{3/2}} - \frac{\sqrt{\pi}e^a\operatorname{erfi}(\sqrt{bx})}{8b^{3/2}} + \frac{x \sinh(a + bx^2)}{2b}$$

[In] $\operatorname{Int}[x^2*\operatorname{Cosh}[a + b*x^2], x]$

[Out] $(\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*x])/ (8*b^{(3/2)}*E^a) - (E^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x])/ (8*b^{(3/2)}) + (x*\operatorname{Sinh}[a + b*x^2])/ (2*b)$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)n], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)n]*((e_.)*(x_)m), x_Symbol] := Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{x \sinh(a + bx^2)}{2b} - \frac{\int \sinh(a + bx^2) dx}{2b} \\
 &= \frac{x \sinh(a + bx^2)}{2b} + \frac{\int e^{-a-bx^2} dx}{4b} - \frac{\int e^{a+bx^2} dx}{4b} \\
 &= \frac{e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{b}x)}{8b^{3/2}} - \frac{e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{b}x)}{8b^{3/2}} + \frac{x \sinh(a + bx^2)}{2b}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.97

$$\begin{aligned}
 &\int x^2 \cosh(a + bx^2) dx \\
 &= \frac{\sqrt{\pi}\operatorname{erf}(\sqrt{b}x) (\cosh(a) - \sinh(a)) - \sqrt{\pi}\operatorname{erfi}(\sqrt{b}x) (\cosh(a) + \sinh(a)) + 4\sqrt{b}x \sinh(a + bx^2)}{8b^{3/2}}
 \end{aligned}$$

```
[In] Integrate[x^2*Cosh[a + b*x^2],x]
```

```
[Out] (Sqrt[Pi]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) - Sqrt[Pi]*Erfi[Sqrt[b]*x]*(Co
sh[a] + Sinh[a]) + 4*Sqrt[b]*x*Sinh[a + b*x^2])/(8*b^(3/2))
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07

method	result
risch	$-\frac{e^{-a} x e^{-b x^2}}{4b} + \frac{\operatorname{erf}(x\sqrt{b})\sqrt{\pi} e^{-a}}{8b^{\frac{3}{2}}} + \frac{e^a e^{b x^2} x}{4b} - \frac{e^a \sqrt{\pi} \operatorname{erf}(\sqrt{-b} x)}{8b\sqrt{-b}}$
meijerg	$-\frac{i \cosh(a)\sqrt{\pi}\sqrt{2} \left(\frac{x\sqrt{2}(ib)^{\frac{3}{2}} e^{b x^2}}{4\sqrt{\pi} b} - \frac{x\sqrt{2}(ib)^{\frac{3}{2}} e^{-b x^2}}{4\sqrt{\pi} b} + \frac{(ib)^{\frac{3}{2}} \sqrt{2} \operatorname{erf}(x\sqrt{b})}{8b^{\frac{3}{2}}} - \frac{(ib)^{\frac{3}{2}} \sqrt{2} \operatorname{erfi}(x\sqrt{b})}{8b^{\frac{3}{2}}} \right)}{2b\sqrt{ib}} - \frac{\sinh(a)\sqrt{\pi}\sqrt{2} \left(\frac{x\sqrt{2}(ib)^{\frac{5}{2}} e^{-b x^2}}{4\sqrt{\pi} b^2} + \dots \right)}{2b\sqrt{ib}}$

```
[In] int(x^2*cosh(b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/exp(a)/b*x*exp(-b*x^2)+1/8*erf(x*b^(1/2))*Pi^(1/2)/b^(3/2)/exp(a)+1/4*exp(a)*exp(b*x^2)*x/b-1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(49) = 98.

Time = 0.25 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.74

$$\int x^2 \cosh(a + bx^2) dx$$

$$= \frac{2bx \cosh(bx^2 + a)^2 + 4bx \cosh(bx^2 + a) \sinh(bx^2 + a) + 2bx \sinh(bx^2 + a)^2 + \sqrt{\pi}(\cosh(bx^2 + a) \cosh(a) + \sinh(bx^2 + a) \sinh(a)) \operatorname{erf}(\sqrt{-b}x) + \sqrt{\pi}(\cosh(bx^2 + a) \cosh(a) + (\cosh(a) - \sinh(a)) \sinh(bx^2 + a) - \cosh(bx^2 + a) \sinh(a)) \operatorname{erf}(\sqrt{b}x) - 2bx}{b^2 \cosh(bx^2 + a) + b^2 \sinh(bx^2 + a)}$$

```
[In] integrate(x^2*cosh(b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*(2*b*x*cosh(b*x^2 + a)^2 + 4*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + 2*b*x*sinh(b*x^2 + a)^2 + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) + sinh(a))*sinh(b*x^2 + a) + cosh(b*x^2 + a)*sinh(a))*sqrt(-b)*erf(sqrt(-b)*x) + sqrt(pi)*(cosh(b*x^2 + a)*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a) - cosh(b*x^2 + a)*sinh(a))*sqrt(b)*erf(sqrt(b)*x) - 2*b*x)/(b^2*cosh(b*x^2 + a) + b^2*sinh(b*x^2 + a))
```

Sympy [F]

$$\int x^2 \cosh(a + bx^2) dx = \int x^2 \cosh(a + bx^2) dx$$

```
[In] integrate(x**2*cosh(b*x**2+a),x)
```

```
[Out] Integral(x**2*cosh(a + b*x**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. 2(49) = 98.

Time = 0.18 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.59

$$\int x^2 \cosh(a + bx^2) dx = \frac{1}{3} x^3 \cosh(bx^2 + a) - \frac{1}{24} b \left(\frac{2(2bx^3e^a - 3xe^a)e^{(bx^2)}}{b^2} + \frac{2(2bx^3 + 3x)e^{(-bx^2-a)}}{b^2} - \frac{3\sqrt{\pi} \operatorname{erf}(\sqrt{bx}) e^{(-a)}}{b^{\frac{5}{2}}} + \frac{3\sqrt{\pi} \operatorname{erf}(\sqrt{-bx}) e^a}{\sqrt{-bb^2}} \right)$$

[In] integrate(x^2*cosh(b*x^2+a),x, algorithm="maxima")

[Out] 1/3*x^3*cosh(b*x^2 + a) - 1/24*b*(2*(2*b*x^3*e^a - 3*x*e^a)*e^(b*x^2)/b^2 + 2*(2*b*x^3 + 3*x)*e^(-b*x^2 - a)/b^2 - 3*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(5/2) + 3*sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b^2))

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.09

$$\int x^2 \cosh(a + bx^2) dx = \frac{xe^{(bx^2+a)}}{4b} - \frac{xe^{(-bx^2-a)}}{4b} - \frac{\sqrt{\pi} \operatorname{erf}(-\sqrt{bx}) e^{(-a)}}{8b^{\frac{3}{2}}} + \frac{\sqrt{\pi} \operatorname{erf}(-\sqrt{-bx}) e^a}{8\sqrt{-bb}}$$

[In] integrate(x^2*cosh(b*x^2+a),x, algorithm="giac")

[Out] 1/4*x*e^(b*x^2 + a)/b - 1/4*x*e^(-b*x^2 - a)/b - 1/8*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/b^(3/2) + 1/8*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/(sqrt(-b)*b)

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a + bx^2) dx = \int x^2 \cosh(bx^2 + a) dx$$

[In] int(x^2*cosh(a + b*x^2),x)

[Out] int(x^2*cosh(a + b*x^2), x)

3.3 $\int x \cosh(a + bx^2) dx$

Optimal result	54
Rubi [A] (verified)	54
Mathematica [B] (verified)	55
Maple [A] (verified)	55
Fricas [A] (verification not implemented)	56
Sympy [A] (verification not implemented)	56
Maxima [A] (verification not implemented)	56
Giac [B] (verification not implemented)	56
Mupad [B] (verification not implemented)	57

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int x \cosh(a + bx^2) dx = \frac{\sinh(a + bx^2)}{2b}$$

[Out] 1/2*sinh(b*x^2+a)/b

Rubi [A] (verified)

Time = 0.01 (sec), antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {5429, 2717}

$$\int x \cosh(a + bx^2) dx = \frac{\sinh(a + bx^2)}{2b}$$

[In] Int[x*Cosh[a + b*x^2],x]

[Out] Sinh[a + b*x^2]/(2*b)

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify

`[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \text{Subst} \left(\int \cosh(a + bx) dx, x, x^2 \right) \\ &= \frac{\sinh(a + bx^2)}{2b} \end{aligned}$$

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.

Time = 0.01 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.07

$$\int x \cosh(a + bx^2) dx = \frac{\cosh(bx^2) \sinh(a)}{2b} + \frac{\cosh(a) \sinh(bx^2)}{2b}$$

[In] `Integrate[x*Cosh[a + b*x^2],x]`

[Out] `(Cosh[b*x^2]*Sinh[a])/(2*b) + (Cosh[a]*Sinh[b*x^2])/(2*b)`

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativdivides	$\frac{\sinh(bx^2+a)}{2b}$	14
default	$\frac{\sinh(bx^2+a)}{2b}$	14
parallelsch	$\frac{\sinh(bx^2+a)}{2b}$	14
risch	$\frac{e^{bx^2+a}}{4b} - \frac{e^{-bx^2-a}}{4b}$	31
meijerg	$\frac{\cosh(a) \sinh(bx^2)}{2b} - \frac{\sinh(a) \sqrt{\pi} \left(\frac{1}{\sqrt{\pi}} - \frac{\cosh(bx^2)}{\sqrt{\pi}} \right)}{2b}$	40

[In] `int(x*cosh(b*x^2+a),x,method=_RETURNVERBOSE)`

[Out] `1/2*sinh(b*x^2+a)/b`

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \cosh(a + bx^2) dx = \frac{\sinh(bx^2 + a)}{2b}$$

[In] integrate(x*cosh(b*x^2+a),x, algorithm="fricas")

[Out] 1/2*sinh(b*x^2 + a)/b

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.27

$$\int x \cosh(a + bx^2) dx = \begin{cases} \frac{\sinh(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \cosh(a)}{2} & \text{otherwise} \end{cases}$$

[In] integrate(x*cosh(b*x**2+a),x)

[Out] Piecewise((sinh(a + b*x**2)/(2*b), Ne(b, 0)), (x**2*cosh(a)/2, True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \cosh(a + bx^2) dx = \frac{\sinh(bx^2 + a)}{2b}$$

[In] integrate(x*cosh(b*x^2+a),x, algorithm="maxima")

[Out] 1/2*sinh(b*x^2 + a)/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. 2(13) = 26.

Time = 0.28 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.80

$$\int x \cosh(a + bx^2) dx = \frac{e^{(bx^2+a)} - e^{(-bx^2-a)}}{4b}$$

[In] integrate(x*cosh(b*x^2+a),x, algorithm="giac")

[Out] 1/4*(e^(b*x^2 + a) - e^(-b*x^2 - a))/b

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int x \cosh(a + bx^2) dx = \frac{\sinh(bx^2 + a)}{2b}$$

[In] int(x*cosh(a + b*x^2),x)

[Out] sinh(a + b*x^2)/(2*b)

3.4 $\int \cosh(a + bx^2) dx$

Optimal result	58
Rubi [A] (verified)	58
Mathematica [A] (verified)	59
Maple [A] (verified)	59
Fricas [A] (verification not implemented)	60
Sympy [F]	60
Maxima [B] (verification not implemented)	60
Giac [A] (verification not implemented)	61
Mupad [F(-1)]	61

Optimal result

Integrand size = 8, antiderivative size = 53

$$\int \cosh(a + bx^2) dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})}{4\sqrt{b}} + \frac{e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx})}{4\sqrt{b}}$$

[Out] $1/4*\operatorname{erf}(x*b^{(1/2)})*\operatorname{Pi}^{(1/2)}/\exp(a)/b^{(1/2)}+1/4*\exp(a)*\operatorname{erfi}(x*b^{(1/2)})*\operatorname{Pi}^{(1/2)}/b^{(1/2)}$

Rubi [A] (verified)

Time = 0.01 (sec), antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {5407, 2235, 2236}

$$\int \cosh(a + bx^2) dx = \frac{\sqrt{\pi}e^{-a}\operatorname{erf}(\sqrt{bx})}{4\sqrt{b}} + \frac{\sqrt{\pi}e^a\operatorname{erfi}(\sqrt{bx})}{4\sqrt{b}}$$

[In] `Int[Cosh[a + b*x^2], x]`

[Out] $(\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*x])/(4*\operatorname{Sqrt}[b]*E^a) + (E^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x])/(4*\operatorname{Sqrt}[b])$

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \int e^{-a-bx^2} dx + \frac{1}{2} \int e^{a+bx^2} dx \\ &= \frac{e^{-a} \sqrt{\pi} \operatorname{erf}(\sqrt{b}x)}{4\sqrt{b}} + \frac{e^a \sqrt{\pi} \operatorname{erfi}(\sqrt{b}x)}{4\sqrt{b}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.85

$$\int \cosh(a+bx^2) dx = \frac{\sqrt{\pi} \left(\operatorname{erf}(\sqrt{b}x) (\cosh(a) - \sinh(a)) + \operatorname{erfi}(\sqrt{b}x) (\cosh(a) + \sinh(a)) \right)}{4\sqrt{b}}$$

```
[In] Integrate[Cosh[a + b*x^2], x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]*x]*(Cosh[a] +
Sinh[a])))/(4*Sqrt[b])
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
risch	$\frac{\operatorname{erf}(x\sqrt{b})\sqrt{\pi}e^{-a}}{4\sqrt{b}} + \frac{e^a\sqrt{\pi}\operatorname{erf}(\sqrt{-b}x)}{4\sqrt{-b}}$	40
meijerg	$\frac{\cosh(a)\sqrt{\pi}\sqrt{2} \left(\frac{\sqrt{ib}\sqrt{2}\operatorname{erf}(x\sqrt{b})}{2\sqrt{b}} + \frac{\sqrt{ib}\sqrt{2}\operatorname{erfi}(x\sqrt{b})}{2\sqrt{b}} \right)}{4\sqrt{ib}} - \frac{i\sinh(a)\sqrt{\pi}\sqrt{2} \left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\operatorname{erf}(x\sqrt{b})}{2b^{\frac{3}{2}}} + \frac{(ib)^{\frac{3}{2}}\sqrt{2}\operatorname{erfi}(x\sqrt{b})}{2b^{\frac{3}{2}}} \right)}{4\sqrt{ib}}$	117

```
[In] int(cosh(b*x^2+a), x, method=_RETURNVERBOSE)
```

```
[Out] 1/4*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*e
rf((-b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \cosh(a + bx^2) dx$$

$$= -\frac{\sqrt{\pi}\sqrt{-b}(\cosh(a) + \sinh(a))\operatorname{erf}(\sqrt{-b}x) - \sqrt{\pi}\sqrt{b}(\cosh(a) - \sinh(a))\operatorname{erf}(\sqrt{b}x)}{4b}$$

[In] integrate(cosh(b*x^2+a),x, algorithm="fricas")

[Out] -1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x) - sqrt(pi)*sqrt(b)*(cosh(a) - sinh(a))*erf(sqrt(b)*x))/b

Sympy [F]

$$\int \cosh(a + bx^2) dx = \int \cosh(a + bx^2) dx$$

[In] integrate(cosh(b*x**2+a),x)

[Out] Integral(cosh(a + b*x**2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. 2(35) = 70.

Time = 0.20 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.64

$$\int \cosh(a + bx^2) dx$$

$$= -\frac{1}{4}b \left(\frac{2xe^{(bx^2+a)}}{b} + \frac{2xe^{(-bx^2-a)}}{b} - \frac{\sqrt{\pi}\operatorname{erf}(\sqrt{b}x)e^{(-a)}}{b^{\frac{3}{2}}} - \frac{\sqrt{\pi}\operatorname{erf}(\sqrt{-b}x)e^a}{\sqrt{-bb}} \right)$$

$$+ x \cosh(bx^2 + a)$$

[In] integrate(cosh(b*x^2+a),x, algorithm="maxima")

[Out] -1/4*b*(2*x*e^(b*x^2 + a)/b + 2*x*e^(-b*x^2 - a)/b - sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/b^(3/2) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/(sqrt(-b)*b) + x*cosh(b*x^2 + a)

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \cosh(a + bx^2) dx = -\frac{\sqrt{\pi} \operatorname{erf}(-\sqrt{b}x) e^{(-a)}}{4\sqrt{b}} - \frac{\sqrt{\pi} \operatorname{erf}(-\sqrt{-b}x) e^a}{4\sqrt{-b}}$$

[In] integrate(cosh(b*x^2+a),x, algorithm="giac")

[Out] -1/4*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 1/4*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)

Mupad [F(-1)]

Timed out.

$$\int \cosh(a + bx^2) dx = \int \cosh(bx^2 + a) dx$$

[In] int(cosh(a + b*x^2),x)

[Out] int(cosh(a + b*x^2), x)

3.5 $\int \frac{\cosh(a+bx^2)}{x} dx$

Optimal result	62
Rubi [A] (verified)	62
Mathematica [A] (verified)	63
Maple [A] (verified)	63
Fricas [A] (verification not implemented)	64
Sympy [F]	64
Maxima [A] (verification not implemented)	64
Giac [A] (verification not implemented)	64
Mupad [F(-1)]	65

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cosh(a+bx^2)}{x} dx = \frac{1}{2} \cosh(a) \text{Chi}(bx^2) + \frac{1}{2} \sinh(a) \text{Shi}(bx^2)$$

[Out] 1/2*Chi(b*x^2)*cosh(a)+1/2*Shi(b*x^2)*sinh(a)

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5427, 5425, 5424}

$$\int \frac{\cosh(a+bx^2)}{x} dx = \frac{1}{2} \cosh(a) \text{Chi}(bx^2) + \frac{1}{2} \sinh(a) \text{Shi}(bx^2)$$

[In] Int[Cosh[a + b*x^2]/x,x]

[Out] (Cosh[a]*CoshIntegral[b*x^2])/2 + (Sinh[a]*SinhIntegral[b*x^2])/2

Rule 5424

Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinhIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5425

Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CoshIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a) \int \frac{\cosh(bx^2)}{x} dx + \sinh(a) \int \frac{\sinh(bx^2)}{x} dx \\ &= \frac{1}{2} \cosh(a) \text{Chi}(bx^2) + \frac{1}{2} \sinh(a) \text{Shi}(bx^2) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(a + bx^2)}{x} dx = \frac{1}{2} (\cosh(a) \text{Chi}(bx^2) + \sinh(a) \text{Shi}(bx^2))$$

```
[In] Integrate[Cosh[a + b*x^2]/x,x]
```

```
[Out] (Cosh[a]*CoshIntegral[b*x^2] + Sinh[a]*SinhIntegral[b*x^2])/2
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{e^{2a}e^{-a} \text{Ei}_1(-bx^2)}{4} - \frac{e^{-a} \text{Ei}_1(bx^2)}{4}$	33
meijerg	$\frac{\cosh(a)\sqrt{\pi} \left(\frac{2\gamma+4\ln(x)+2\ln(ib)}{\sqrt{\pi}} + \frac{2 \text{Chi}(bx^2)-2\ln(bx^2)-2\gamma}{\sqrt{\pi}} \right)}{4} + \frac{\text{Shi}(bx^2) \sinh(a)}{2}$	62

```
[In] int(cosh(b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*exp(2*a)*exp(-a)*Ei(1,-b*x^2)-1/4*exp(-a)*Ei(1,b*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\cosh(a + bx^2)}{x} dx = \frac{1}{4} (\text{Ei}(bx^2) + \text{Ei}(-bx^2)) \cosh(a) + \frac{1}{4} (\text{Ei}(bx^2) - \text{Ei}(-bx^2)) \sinh(a)$$

[In] integrate(cosh(b*x^2+a)/x,x, algorithm="fricas")

[Out] 1/4*(Ei(b*x^2) + Ei(-b*x^2))*cosh(a) + 1/4*(Ei(b*x^2) - Ei(-b*x^2))*sinh(a)

Sympy [F]

$$\int \frac{\cosh(a + bx^2)}{x} dx = \int \frac{\cosh(a + bx^2)}{x} dx$$

[In] integrate(cosh(b*x**2+a)/x,x)

[Out] Integral(cosh(a + b*x**2)/x, x)

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(a + bx^2)}{x} dx = \frac{1}{4} \text{Ei}(-bx^2) e^{(-a)} + \frac{1}{4} \text{Ei}(bx^2) e^a$$

[In] integrate(cosh(b*x^2+a)/x,x, algorithm="maxima")

[Out] 1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(a + bx^2)}{x} dx = \frac{1}{4} \text{Ei}(-bx^2) e^{(-a)} + \frac{1}{4} \text{Ei}(bx^2) e^a$$

[In] integrate(cosh(b*x^2+a)/x,x, algorithm="giac")

[Out] 1/4*Ei(-b*x^2)*e^(-a) + 1/4*Ei(b*x^2)*e^a

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a + bx^2)}{x} dx = \frac{\cosh(a) \operatorname{coshint}(bx^2)}{2} + \frac{\sinh(a) \operatorname{sinhint}(bx^2)}{2}$$

```
[In] int(cosh(a + b*x^2)/x,x)
```

```
[Out] (cosh(a)*coshint(b*x^2))/2 + (sinh(a)*sinhint(b*x^2))/2
```

3.6 $\int \frac{\cosh(a+bx^2)}{x^2} dx$

Optimal result	66
Rubi [A] (verified)	66
Mathematica [A] (verified)	67
Maple [A] (verified)	68
Fricas [B] (verification not implemented)	68
Sympy [F]	68
Maxima [A] (verification not implemented)	69
Giac [F]	69
Mupad [F(-1)]	69

Optimal result

Integrand size = 12, antiderivative size = 66

$$\int \frac{\cosh(a+bx^2)}{x^2} dx = -\frac{\cosh(a+bx^2)}{x} - \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{b}x) + \frac{1}{2}\sqrt{b}e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{b}x)$$

[Out] $-\cosh(b*x^2+a)/x-1/2*\operatorname{erf}(x*b^(1/2))*b^(1/2)*\operatorname{Pi}^(1/2)/\exp(a)+1/2*\exp(a)*\operatorname{erfi}(x*b^(1/2))*b^(1/2)*\operatorname{Pi}^(1/2)$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5435, 5406, 2235, 2236}

$$\int \frac{\cosh(a+bx^2)}{x^2} dx = -\frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\operatorname{erf}(\sqrt{b}x) + \frac{1}{2}\sqrt{\pi}e^a\sqrt{b}\operatorname{erfi}(\sqrt{b}x) - \frac{\cosh(a+bx^2)}{x}$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b*x^2]/x^2, x]$

[Out] $-(\operatorname{Cosh}[a + b*x^2]/x) - (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*x])/(2*\operatorname{E}^a) + (\operatorname{Sqrt}[b]*\operatorname{E}^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x])/2$

Rule 2235

$\operatorname{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]))], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5435

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_.)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
&& LtQ[m, -1]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cosh(a + bx^2)}{x} + (2b) \int \sinh(a + bx^2) dx \\ &= -\frac{\cosh(a + bx^2)}{x} - b \int e^{-a - bx^2} dx + b \int e^{a + bx^2} dx \\ &= -\frac{\cosh(a + bx^2)}{x} - \frac{1}{2} \sqrt{b} e^{-a} \sqrt{\pi} \operatorname{erf}(\sqrt{bx}) + \frac{1}{2} \sqrt{b} e^a \sqrt{\pi} \operatorname{erfi}(\sqrt{bx}) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a + bx^2)}{x^2} dx = \frac{-2 \cosh(a + bx^2) + \sqrt{b} \sqrt{\pi} x \operatorname{erf}(\sqrt{bx}) (-\cosh(a) + \sinh(a)) + \sqrt{b} \sqrt{\pi} x \operatorname{erfi}(\sqrt{bx}) (\cosh(a) + \sinh(a))}{2x}$$

```
[In] Integrate[Cosh[a + b*x^2]/x^2,x]
```

```
[Out] (-2*Cosh[a + b*x^2] + Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]
) + Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*(Cosh[a] + Sinh[a]))/(2*x)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{e^{-a}e^{-bx^2}}{2x} - \frac{\operatorname{erf}(x\sqrt{b})\sqrt{b}\sqrt{\pi}e^{-a}}{2} - \frac{e^ae^{bx^2}}{2x} + \frac{e^ab\sqrt{\pi}\operatorname{erf}(\sqrt{-b}x)}{2\sqrt{-b}}$
meijerg	$\frac{i \cosh(a)\sqrt{\pi}b\sqrt{2} \left(-\frac{2\sqrt{2}e^{bx^2}}{\sqrt{\pi}x\sqrt{ib}} - \frac{2\sqrt{2}e^{-bx^2}}{\sqrt{\pi}x\sqrt{ib}} - \frac{2\sqrt{2}\sqrt{b}\operatorname{erf}(x\sqrt{b})}{\sqrt{ib}} + \frac{2\sqrt{2}\sqrt{b}\operatorname{erfi}(x\sqrt{b})}{\sqrt{ib}} \right)}{8\sqrt{ib}} + \frac{\sinh(a)\sqrt{\pi}b\sqrt{2} \left(\frac{2\sqrt{2}\sqrt{ib}e^{-bx^2}}{\sqrt{\pi}xb} - \frac{2\sqrt{2}\sqrt{ib}e^{bx^2}}{\sqrt{\pi}xb} \right)}{8\sqrt{ib}}$

[In] `int(cosh(b*x^2+a)/x^2,x,method=_RETURNVERBOSE)`

[Out]
$$-1/2/\exp(a)/x*\exp(-b*x^2)-1/2*\operatorname{erf}(x*b^{(1/2)})*b^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(a)-1/2*\exp(a)*\exp(b*x^2)/x+1/2*\exp(a)*b*\operatorname{Pi}^{(1/2)}/(-b)^{(1/2)}*\operatorname{erf}((-b)^{(1/2)}*x)$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. 2(48) = 96.

Time = 0.25 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.77

$$\int \frac{\cosh(a + bx^2)}{x^2} dx = \frac{\sqrt{\pi}(x \cosh(bx^2 + a) \cosh(a) + x \cosh(bx^2 + a) \sinh(a) + (x \cosh(a) + x \sinh(a)) \sinh(bx^2 + a))\sqrt{-b}e^{-a} - \sqrt{\pi}(x \cosh(bx^2 + a) \cosh(a) - x \cosh(bx^2 + a) \sinh(a) + (x \cosh(a) - x \sinh(a)) \sinh(bx^2 + a))\sqrt{b}e^a}{8\sqrt{ib}}$$

[In] `integrate(cosh(b*x^2+a)/x^2,x, algorithm="fricas")`

[Out]
$$-1/2*(\operatorname{sqrt}(\operatorname{pi})*(x*\cosh(b*x^2 + a)*\cosh(a) + x*\cosh(b*x^2 + a)*\sinh(a) + (x*\cosh(a) + x*\sinh(a))*\sinh(b*x^2 + a))*\operatorname{sqrt}(-b)*\operatorname{erf}(\operatorname{sqrt}(-b)*x) + \operatorname{sqrt}(\operatorname{pi})*(x*\cosh(b*x^2 + a)*\cosh(a) - x*\cosh(b*x^2 + a)*\sinh(a) + (x*\cosh(a) - x*\sinh(a))*\sinh(b*x^2 + a))*\operatorname{sqrt}(b)*\operatorname{erf}(\operatorname{sqrt}(b)*x) + \cosh(b*x^2 + a)^2 + 2*\cosh(b*x^2 + a)*\sinh(b*x^2 + a) + \sinh(b*x^2 + a)^2 + 1)/(x*\cosh(b*x^2 + a) + x*\sinh(b*x^2 + a))$$

Sympy [F]

$$\int \frac{\cosh(a + bx^2)}{x^2} dx = \int \frac{\cosh(a + bx^2)}{x^2} dx$$

[In] `integrate(cosh(b*x**2+a)/x**2,x)`

[Out] `Integral(cosh(a + b*x**2)/x**2, x)`

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(a + bx^2)}{x^2} dx = -\frac{1}{2} \left(\frac{\sqrt{\pi} \operatorname{erf}(\sqrt{b}x) e^{-a}}{\sqrt{b}} - \frac{\sqrt{\pi} \operatorname{erf}(\sqrt{-b}x) e^a}{\sqrt{-b}} \right) b - \frac{\cosh(bx^2 + a)}{x}$$

[In] integrate(cosh(b*x^2+a)/x^2,x, algorithm="maxima")

[Out] -1/2*(sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) - sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b))*b - cosh(b*x^2 + a)/x

Giac [F]

$$\int \frac{\cosh(a + bx^2)}{x^2} dx = \int \frac{\cosh(bx^2 + a)}{x^2} dx$$

[In] integrate(cosh(b*x^2+a)/x^2,x, algorithm="giac")

[Out] integrate(cosh(b*x^2 + a)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a + bx^2)}{x^2} dx = \int \frac{\cosh(bx^2 + a)}{x^2} dx$$

[In] int(cosh(a + b*x^2)/x^2,x)

[Out] int(cosh(a + b*x^2)/x^2, x)

3.7 $\int \frac{\cosh(a+bx^2)}{x^3} dx$

Optimal result	70
Rubi [A] (verified)	70
Mathematica [A] (verified)	72
Maple [A] (verified)	72
Fricas [A] (verification not implemented)	72
Sympy [F]	73
Maxima [A] (verification not implemented)	73
Giac [B] (verification not implemented)	73
Mupad [F(-1)]	74

Optimal result

Integrand size = 12, antiderivative size = 42

$$\int \frac{\cosh(a+bx^2)}{x^3} dx = -\frac{\cosh(a+bx^2)}{2x^2} + \frac{1}{2}b\text{Chi}(bx^2)\sinh(a) + \frac{1}{2}b\cosh(a)\text{Shi}(bx^2)$$

[Out] $-1/2*\cosh(b*x^2+a)/x^2+1/2*b*\cosh(a)*\text{Shi}(b*x^2)+1/2*b*\text{Chi}(b*x^2)*\sinh(a)$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5429, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh(a+bx^2)}{x^3} dx = \frac{1}{2}b\sinh(a)\text{Chi}(bx^2) + \frac{1}{2}b\cosh(a)\text{Shi}(bx^2) - \frac{\cosh(a+bx^2)}{2x^2}$$

[In] `Int[Cosh[a + b*x^2]/x^3, x]`

[Out] $-1/2*\text{Cosh}[a + b*x^2]/x^2 + (b*\text{CoshIntegral}[b*x^2]*\text{Sinh}[a])/2 + (b*\text{Cosh}[a]*\text{ShiIntegral}[b*x^2])/2$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d],
Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
&& (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2} \text{Subst} \left(\int \frac{\cosh(a + bx)}{x^2} dx, x, x^2 \right) \\
&= -\frac{\cosh(a + bx^2)}{2x^2} + \frac{1}{2} b \text{Subst} \left(\int \frac{\sinh(a + bx)}{x} dx, x, x^2 \right) \\
&= -\frac{\cosh(a + bx^2)}{2x^2} + \frac{1}{2} (b \cosh(a)) \text{Subst} \left(\int \frac{\sinh(bx)}{x} dx, x, x^2 \right) \\
&\quad + \frac{1}{2} (b \sinh(a)) \text{Subst} \left(\int \frac{\cosh(bx)}{x} dx, x, x^2 \right) \\
&= -\frac{\cosh(a + bx^2)}{2x^2} + \frac{1}{2} b \text{Chi}(bx^2) \sinh(a) + \frac{1}{2} b \cosh(a) \text{Shi}(bx^2)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.90

$$\int \frac{\cosh(a + bx^2)}{x^3} dx = \frac{1}{2} \left(-\frac{\cosh(a + bx^2)}{x^2} + b \operatorname{Chi}(bx^2) \sinh(a) + b \cosh(a) \operatorname{Shi}(bx^2) \right)$$

[In] Integrate[Cosh[a + b*x^2]/x^3,x]

[Out] $(-\operatorname{Cosh}[a + b*x^2]/x^2) + b*\operatorname{CoshIntegral}[b*x^2]*\operatorname{Sinh}[a] + b*\operatorname{Cosh}[a]*\operatorname{SinhIntegral}[b*x^2])/2$

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

method	result
risch	$-\frac{-e^{-a} \operatorname{Ei}_1(bx^2)bx^2 + \operatorname{Ei}_1(-bx^2)e^a bx^2 + e^{-bx^2-a} + e^{bx^2+a}}{4x^2}$
meijerg	$\frac{i \cosh(a)\sqrt{\pi} b \left(\frac{4i \cosh(bx^2)}{bx^2\sqrt{\pi}} - \frac{4i \operatorname{Shi}(bx^2)}{\sqrt{\pi}} \right)}{8} + \frac{\sinh(a)\sqrt{\pi} b \left(\frac{4\gamma - 4 + 8 \ln(x) + 4 \ln(ib)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4 \sinh(bx^2)}{\sqrt{\pi} bx^2} + \frac{4 \operatorname{Chi}(bx^2) - 4 \ln(bx^2) - 4\gamma}{\sqrt{\pi}} \right)}{8}$

[In] int(cosh(b*x^2+a)/x^3,x,method=_RETURNVERBOSE)

[Out] $-1/4*(-\exp(-a)*\operatorname{Ei}(1,b*x^2)*b*x^2 + \operatorname{Ei}(1,-b*x^2)*\exp(a)*b*x^2 + \exp(-b*x^2-a) + \exp(b*x^2+a))/x^2$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\cosh(a + bx^2)}{x^3} dx = \frac{(bx^2 \operatorname{Ei}(bx^2) - bx^2 \operatorname{Ei}(-bx^2)) \cosh(a) + (bx^2 \operatorname{Ei}(bx^2) + bx^2 \operatorname{Ei}(-bx^2)) \sinh(a) - 2 \cosh(bx^2 + a)}{4x^2}$$

[In] integrate(cosh(b*x^2+a)/x^3,x, algorithm="fricas")

[Out] $1/4*((b*x^2*\operatorname{Ei}(b*x^2) - b*x^2*\operatorname{Ei}(-b*x^2))*\operatorname{cosh}(a) + (b*x^2*\operatorname{Ei}(b*x^2) + b*x^2*\operatorname{Ei}(-b*x^2))*\operatorname{sinh}(a) - 2*\operatorname{cosh}(b*x^2 + a))/x^2$

Sympy [F]

$$\int \frac{\cosh(a + bx^2)}{x^3} dx = \int \frac{\cosh(a + bx^2)}{x^3} dx$$

[In] integrate(cosh(b*x**2+a)/x**3,x)

[Out] Integral(cosh(a + b*x**2)/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.95

$$\int \frac{\cosh(a + bx^2)}{x^3} dx = -\frac{1}{4} (\text{Ei}(-bx^2) e^{-a} - \text{Ei}(bx^2) e^a) b - \frac{\cosh(bx^2 + a)}{2x^2}$$

[In] integrate(cosh(b*x^2+a)/x^3,x, algorithm="maxima")

[Out] -1/4*(Ei(-b*x^2)*e^(-a) - Ei(b*x^2)*e^a)*b - 1/2*cosh(b*x^2 + a)/x^2

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. 2(36) = 72.

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 2.57

$$\int \frac{\cosh(a + bx^2)}{x^3} dx = \frac{(bx^2 + a)b^2\text{Ei}(-bx^2) e^{-a} - ab^2\text{Ei}(-bx^2) e^{-a} - (bx^2 + a)b^2\text{Ei}(bx^2) e^a + ab^2\text{Ei}(bx^2) e^a + b^2e^{(bx^2+a)} + b^2e^{-(bx^2+a)}}{4b^2x^2}$$

[In] integrate(cosh(b*x^2+a)/x^3,x, algorithm="giac")

[Out] -1/4*((b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) - a*b^2*Ei(-b*x^2)*e^(-a) - (b*x^2 + a)*b^2*Ei(b*x^2)*e^a + a*b^2*Ei(b*x^2)*e^a + b^2*e^(b*x^2 + a) + b^2*e^(-b*x^2 - a))/(b^2*x^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a + bx^2)}{x^3} dx = \int \frac{\cosh(bx^2 + a)}{x^3} dx$$

```
[In] int(cosh(a + b*x^2)/x^3,x)
```

```
[Out] int(cosh(a + b*x^2)/x^3, x)
```

3.8 $\int x^3 \cosh^2(a + bx^2) dx$

Optimal result	75
Rubi [A] (verified)	75
Mathematica [A] (verified)	76
Maple [A] (verified)	76
Fricas [A] (verification not implemented)	77
Sympy [A] (verification not implemented)	77
Maxima [A] (verification not implemented)	77
Giac [B] (verification not implemented)	78
Mupad [B] (verification not implemented)	78

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int x^3 \cosh^2(a + bx^2) dx = \frac{x^4}{8} - \frac{\cosh^2(a + bx^2)}{8b^2} + \frac{x^2 \cosh(a + bx^2) \sinh(a + bx^2)}{4b}$$

[Out] 1/8*x^4-1/8*cosh(b*x^2+a)^2/b^2+1/4*x^2*cosh(b*x^2+a)*sinh(b*x^2+a)/b

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5429, 3391, 30}

$$\int x^3 \cosh^2(a + bx^2) dx = -\frac{\cosh^2(a + bx^2)}{8b^2} + \frac{x^2 \sinh(a + bx^2) \cosh(a + bx^2)}{4b} + \frac{x^4}{8}$$

[In] Int[x^3*Cosh[a + b*x^2]^2,x]

[Out] x^4/8 - Cosh[a + b*x^2]^2/(8*b^2) + (x^2*Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)

Rule 30

Int[(x_)^(m_), x_Symbol] := Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N eQ[m, -1]

Rule 3391

Int[((c_) + (d_)*(x_))*((b_)*sin[(e_) + (f_)*(x_)])^(n_), x_Symbol] := Simp[d*((b*Sine[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c + d*x)*(b*Sine[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b

```
*Sin[e + f*x])^(n - 1)/(f*n)), x] /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \text{Subst} \left(\int x \cosh^2(a + bx) dx, x, x^2 \right) \\ &= -\frac{\cosh^2(a + bx^2)}{8b^2} + \frac{x^2 \cosh(a + bx^2) \sinh(a + bx^2)}{4b} + \frac{1}{4} \text{Subst} \left(\int x dx, x, x^2 \right) \\ &= \frac{x^4}{8} - \frac{\cosh^2(a + bx^2)}{8b^2} + \frac{x^2 \cosh(a + bx^2) \sinh(a + bx^2)}{4b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int x^3 \cosh^2(a + bx^2) dx = -\frac{\cosh(2(a + bx^2)) - 2bx^2(bx^2 + \sinh(2(a + bx^2)))}{16b^2}$$

```
[In] Integrate[x^3*Cosh[a + b*x^2]^2,x]
```

```
[Out] -1/16*(Cosh[2*(a + b*x^2)] - 2*b*x^2*(b*x^2 + Sinh[2*(a + b*x^2)]))/b^2
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90

method	result	size
parallelrisch	$\frac{2b^2x^4 + 2bx^2 \sinh(2bx^2 + 2a) - \cosh(2bx^2 + 2a) + 1}{16b^2}$	46
risch	$\frac{x^4}{8} + \frac{(2bx^2 - 1)e^{2bx^2 + 2a}}{32b^2} - \frac{(2bx^2 + 1)e^{-2bx^2 - 2a}}{32b^2}$	55

```
[In] int(x^3*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(2*b^2*x^4+2*b*x^2*sinh(2*b*x^2+2*a)-cosh(2*b*x^2+2*a)+1)/b^2
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.18

$$\int x^3 \cosh^2(a + bx^2) dx = \frac{2b^2x^4 + 4bx^2 \cosh(bx^2 + a) \sinh(bx^2 + a) - \cosh(bx^2 + a)^2 - \sinh(bx^2 + a)^2}{16b^2}$$

[In] integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/16*(2*b^2*x^4 + 4*b*x^2*cosh(b*x^2 + a)*sinh(b*x^2 + a) - cosh(b*x^2 + a)^2 - sinh(b*x^2 + a)^2)/b^2

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.53

$$\int x^3 \cosh^2(a + bx^2) dx = \begin{cases} -\frac{x^4 \sinh^2(a+bx^2)}{8} + \frac{x^4 \cosh^2(a+bx^2)}{8} + \frac{x^2 \sinh(a+bx^2) \cosh(a+bx^2)}{4b} - \frac{\cosh^2(a+bx^2)}{8b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cosh^2(a)}{4} & \text{otherwise} \end{cases}$$

[In] integrate(x**3*cosh(b*x**2+a)**2,x)

[Out] Piecewise((-x**4*sinh(a + b*x**2)**2/8 + x**4*cosh(a + b*x**2)**2/8 + x**2*sinh(a + b*x**2)*cosh(a + b*x**2)/(4*b) - cosh(a + b*x**2)**2/(8*b**2), Ne(b, 0)), (x**4*cosh(a)**2/4, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16

$$\int x^3 \cosh^2(a + bx^2) dx = \frac{1}{8}x^4 + \frac{(2bx^2e^{(2a)} - e^{(2a)})e^{(2bx^2)}}{32b^2} - \frac{(2bx^2 + 1)e^{(-2bx^2-2a)}}{32b^2}$$

[In] integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/8*x^4 + 1/32*(2*b*x^2*e^(2*a) - e^(2*a))*e^(2*b*x^2)/b^2 - 1/32*(2*b*x^2 + 1)*e^(-2*b*x^2 - 2*a)/b^2

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.78

$$\int x^3 \cosh^2(a + bx^2) dx$$

$$= \frac{4(bx^2 + a)^2 + 2(bx^2 + a)e^{(2bx^2+2a)} - 2(bx^2 + a)e^{(-2bx^2-2a)} - e^{(2bx^2+2a)} - e^{(-2bx^2-2a)}}{32b^2}$$

$$- \frac{4(bx^2 + a)a + ae^{(2bx^2+2a)} - (2ae^{(2bx^2+2a)} + a)e^{(-2bx^2-2a)}}{16b^2}$$

[In] integrate(x^3*cosh(b*x^2+a)^2,x, algorithm="giac")

[Out] $\frac{1}{32} * (4 * (b * x^2 + a)^2 + 2 * (b * x^2 + a) * e^{(2 * b * x^2 + 2 * a)} - 2 * (b * x^2 + a) * e^{(-2 * b * x^2 - 2 * a)} - e^{(2 * b * x^2 + 2 * a)} - e^{(-2 * b * x^2 - 2 * a)}) / b^2 - \frac{1}{16} * (4 * (b * x^2 + a) * a + a * e^{(2 * b * x^2 + 2 * a)} - (2 * a * e^{(2 * b * x^2 + 2 * a)} + a) * e^{(-2 * b * x^2 - 2 * a)}) / b^2$

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int x^3 \cosh^2(a + bx^2) dx = \frac{x^4}{8} - \frac{\frac{\cosh(2bx^2+2a)}{16} - \frac{bx^2 \sinh(2bx^2+2a)}{8}}{b^2}$$

[In] int(x^3*cosh(a + b*x^2)^2,x)

[Out] $x^4/8 - (\cosh(2*a + 2*b*x^2)/16 - (b*x^2*\sinh(2*a + 2*b*x^2))/8)/b^2$

3.9 $\int x^2 \cosh^2(a + bx^2) dx$

Optimal result	79
Rubi [A] (verified)	79
Mathematica [A] (verified)	81
Maple [A] (verified)	81
Fricas [B] (verification not implemented)	81
Sympy [F]	82
Maxima [A] (verification not implemented)	82
Giac [A] (verification not implemented)	83
Mupad [F(-1)]	83

Optimal result

Integrand size = 14, antiderivative size = 99

$$\int x^2 \cosh^2(a + bx^2) dx = \frac{x^3}{6} + \frac{e^{-2a} \sqrt{\frac{\pi}{2}} \operatorname{erf}(\sqrt{2} \sqrt{bx})}{32b^{3/2}} - \frac{e^{2a} \sqrt{\frac{\pi}{2}} \operatorname{erfi}(\sqrt{2} \sqrt{bx})}{32b^{3/2}} + \frac{x \sinh(2a + 2bx^2)}{8b}$$

[Out] $1/6*x^3+1/8*x*\sinh(2*b*x^2+2*a)/b+1/64*\operatorname{erf}(x*2^{(1/2)}*b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(3/2)}/\exp(2*a)-1/64*\exp(2*a)*\operatorname{erfi}(x*2^{(1/2)}*b^{(1/2)})*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/b^{(3/2)}$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5449, 5433, 5406, 2235, 2236}

$$\int x^2 \cosh^2(a + bx^2) dx = \frac{\sqrt{\frac{\pi}{2}} e^{-2a} \operatorname{erf}(\sqrt{2} \sqrt{bx})}{32b^{3/2}} - \frac{\sqrt{\frac{\pi}{2}} e^{2a} \operatorname{erfi}(\sqrt{2} \sqrt{bx})}{32b^{3/2}} + \frac{x \sinh(2a + 2bx^2)}{8b} + \frac{x^3}{6}$$

[In] $\operatorname{Int}[x^2*\operatorname{Cosh}[a + b*x^2]^2,x]$

[Out] $x^3/6 + (\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*x])/(32*b^{(3/2)}*E^{(2*a)}) - (E^{(2*a)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*x])/(32*b^{(3/2)}) + (x*\operatorname{Sinh}[2*a + 2*b*x^2])/(8*b)$

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_)*((e_.)*(x_)^(m_.),
x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^2}{2} + \frac{1}{2}x^2 \cosh(2a + 2bx^2) \right) dx \\
&= \frac{x^3}{6} + \frac{1}{2} \int x^2 \cosh(2a + 2bx^2) dx \\
&= \frac{x^3}{6} + \frac{x \sinh(2a + 2bx^2)}{8b} - \frac{\int \sinh(2a + 2bx^2) dx}{8b} \\
&= \frac{x^3}{6} + \frac{x \sinh(2a + 2bx^2)}{8b} + \frac{\int e^{-2a-2bx^2} dx}{16b} - \frac{\int e^{2a+2bx^2} dx}{16b} \\
&= \frac{x^3}{6} + \frac{e^{-2a} \sqrt{\frac{\pi}{2}} \operatorname{erf}(\sqrt{2}\sqrt{bx})}{32b^{3/2}} - \frac{e^{2a} \sqrt{\frac{\pi}{2}} \operatorname{erfi}(\sqrt{2}\sqrt{bx})}{32b^{3/2}} + \frac{x \sinh(2a + 2bx^2)}{8b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02

$$\int x^2 \cosh^2(a + bx^2) dx$$

$$= \frac{3\sqrt{2\pi}\operatorname{erf}\left(\sqrt{2}\sqrt{bx}\right) (\cosh(2a) - \sinh(2a)) - 3\sqrt{2\pi}\operatorname{erfi}\left(\sqrt{2}\sqrt{bx}\right) (\cosh(2a) + \sinh(2a)) + 8\sqrt{bx}(4bx^2 + 3\sinh(2(a + bx^2)))}{192b^{3/2}}$$

[In] Integrate[x^2*Cosh[a + b*x^2]^2,x]

[Out] (3*Sqrt[2*Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*a]) - 3*Sqrt[2*Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]) + 8*Sqrt[b]*x*(4*b*x^2 + 3*Sinh[2*(a + b*x^2)]))/(192*b^(3/2))

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{x^3}{6} - \frac{e^{-2a}x e^{-2bx^2}}{16b} + \frac{e^{-2a}\sqrt{\pi}\sqrt{2}\operatorname{erf}(x\sqrt{2}\sqrt{b})}{64b^{3/2}} + \frac{e^{2a}x e^{2bx^2}}{16b} - \frac{e^{2a}\sqrt{\pi}\operatorname{erf}(\sqrt{-2b}x)}{32b\sqrt{-2b}}$	90

[In] int(x^2*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/6*x^3-1/16*exp(-2*a)/b*x*exp(-2*b*x^2)+1/64*exp(-2*a)/b^(3/2)*Pi^(1/2)*2^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/16*exp(2*a)/b*x*exp(2*b*x^2)-1/32*exp(2*a)/b*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. 2(71) = 142.

Time = 0.26 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.31

$$\int x^2 \cosh^2(a + bx^2) dx$$

$$= \frac{32b^2x^3 \cosh(bx^2 + a)^2 + 12bx \cosh(bx^2 + a)^4 + 48bx \cosh(bx^2 + a) \sinh(bx^2 + a)^3 + 12bx \sinh(bx^2 + a)^5}{192b^{3/2}}$$

[In] integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/192*(32*b^2*x^3*cosh(b*x^2 + a)^2 + 12*b*x*cosh(b*x^2 + a)^4 + 48*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^3 + 12*b*x*sinh(b*x^2 + a)^4 + 3*sqrt(2)*sqrt(pi)*(cosh(b*x^2 + a)^2*cosh(2*a) + (cosh(2*a) + sinh(2*a))*sinh(b*x^2 + a)^2)

$$2 + \cosh(bx^2 + a)^2 \sinh(2a) + 2(\cosh(bx^2 + a) \cosh(2a) + \cosh(bx^2 + a) \sinh(2a)) \sinh(bx^2 + a) \sqrt{-b} \operatorname{erf}(\sqrt{2} \sqrt{-b} x) + 3 \sqrt{2} \sqrt{\pi} (\cosh(bx^2 + a)^2 \cosh(2a) + (\cosh(2a) - \sinh(2a)) \sinh(bx^2 + a)^2 - \cosh(bx^2 + a)^2 \sinh(2a) + 2(\cosh(bx^2 + a) \cosh(2a) - \cosh(bx^2 + a) \sinh(2a)) \sinh(bx^2 + a)) \sqrt{b} \operatorname{erf}(\sqrt{2} \sqrt{b} x) + 8(4b^2 x^3 + 9bx \cosh(bx^2 + a)^2) \sinh(bx^2 + a)^2 - 12bx + 16(4b^2 x^3 \cosh(bx^2 + a) + 3bx \cosh(bx^2 + a)^3) \sinh(bx^2 + a) / (b^2 \cosh(bx^2 + a)^2 + 2b^2 \cosh(bx^2 + a) \sinh(bx^2 + a) + b^2 \sinh(bx^2 + a)^2)$$

Sympy [F]

$$\int x^2 \cosh^2(a + bx^2) dx = \int x^2 \cosh^2(a + bx^2) dx$$

[In] integrate(x**2*cosh(b*x**2+a)**2,x)

[Out] Integral(x**2*cosh(a + b*x**2)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.96

$$\int x^2 \cosh^2(a + bx^2) dx = \frac{1}{6} x^3 - \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}(\sqrt{2} \sqrt{-b} x) e^{(2a)}}{64 \sqrt{-b} b} + \frac{\sqrt{2} \sqrt{\pi} \operatorname{erf}(\sqrt{2} \sqrt{b} x) e^{(-2a)}}{64 b^{\frac{3}{2}}} + \frac{x e^{(2bx^2+2a)}}{16b} - \frac{x e^{(-2bx^2-2a)}}{16b}$$

[In] integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/6*x^3 - 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*b) + 1/64*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.98

$$\int x^2 \cosh^2(a + bx^2) dx = \frac{1}{6} x^3 + \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}(-\sqrt{2}\sqrt{-b}x) e^{(2a)}}{64\sqrt{-bb}} - \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}(-\sqrt{2}\sqrt{b}x) e^{(-2a)}}{64b^{\frac{3}{2}}} + \frac{x e^{(2bx^2+2a)}}{16b} - \frac{x e^{(-2bx^2-2a)}}{16b}$$

[In] integrate(x^2*cosh(b*x^2+a)^2,x, algorithm="giac")

[Out] 1/6*x^3 + 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/(sqrt(-b)*b) - 1/64*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/b^(3/2) + 1/16*x*e^(2*b*x^2 + 2*a)/b - 1/16*x*e^(-2*b*x^2 - 2*a)/b

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^2(a + bx^2) dx = \int x^2 \cosh(bx^2 + a)^2 dx$$

[In] int(x^2*cosh(a + b*x^2)^2,x)

[Out] int(x^2*cosh(a + b*x^2)^2, x)

3.10 $\int x \cosh^2(a + bx^2) dx$

Optimal result	84
Rubi [A] (verified)	84
Mathematica [A] (verified)	85
Maple [A] (verified)	85
Fricas [A] (verification not implemented)	86
Sympy [B] (verification not implemented)	86
Maxima [A] (verification not implemented)	86
Giac [A] (verification not implemented)	87
Mupad [B] (verification not implemented)	87

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int x \cosh^2(a + bx^2) dx = \frac{x^2}{4} + \frac{\cosh(a + bx^2) \sinh(a + bx^2)}{4b}$$

[Out] 1/4*x^2+1/4*cosh(b*x^2+a)*sinh(b*x^2+a)/b

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 2715, 8}

$$\int x \cosh^2(a + bx^2) dx = \frac{\sinh(a + bx^2) \cosh(a + bx^2)}{4b} + \frac{x^2}{4}$$

[In] Int[x*Cosh[a + b*x^2]^2,x]

[Out] x^2/4 + (Cosh[a + b*x^2]*Sinh[a + b*x^2])/(4*b)

Rule 8

Int[a_, x_Symbol] := Simp[a*x, x] /; FreeQ[a, x]

Rule 2715

Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[(-b)*Cos[c + d*x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n]

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol]
:=> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \text{Subst} \left(\int \cosh^2(a + bx) dx, x, x^2 \right) \\ &= \frac{\cosh(a + bx^2) \sinh(a + bx^2)}{4b} + \frac{1}{4} \text{Subst} \left(\int 1 dx, x, x^2 \right) \\ &= \frac{x^2}{4} + \frac{\cosh(a + bx^2) \sinh(a + bx^2)}{4b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int x \cosh^2(a + bx^2) dx = \frac{2(a + bx^2) + \sinh(2(a + bx^2))}{8b}$$

```
[In] Integrate[x*Cosh[a + b*x^2]^2,x]
```

```
[Out] (2*(a + b*x^2) + Sinh[2*(a + b*x^2)])/(8*b)
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{2bx^2 + \sinh(2bx^2 + 2a)}{8b}$	24
derivativedivides	$\frac{\cosh(bx^2 + a) \sinh(bx^2 + a)}{2} + \frac{bx^2}{2} + \frac{a}{2}$	34
default	$\frac{\cosh(bx^2 + a) \sinh(bx^2 + a)}{2} + \frac{bx^2}{2} + \frac{a}{2}$	34
risch	$\frac{x^2}{4} + \frac{e^{2bx^2 + 2a}}{16b} - \frac{e^{-2bx^2 - 2a}}{16b}$	39

```
[In] int(x*cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*(2*b*x^2+sinh(2*b*x^2+2*a))/b
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int x \cosh^2(a + bx^2) dx = \frac{bx^2 + \cosh(bx^2 + a) \sinh(bx^2 + a)}{4b}$$

`[In] integrate(x*cosh(b*x^2+a)^2,x, algorithm="fricas")``[Out] 1/4*(b*x^2 + cosh(b*x^2 + a)*sinh(b*x^2 + a))/b`**Sympy [B] (verification not implemented)**

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. 2(24) = 48.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int x \cosh^2(a + bx^2) dx = \begin{cases} -\frac{x^2 \sinh^2(a+bx^2)}{4} + \frac{x^2 \cosh^2(a+bx^2)}{4} + \frac{\sinh(a+bx^2) \cosh(a+bx^2)}{4b} & \text{for } b \neq 0 \\ \frac{x^2 \cosh^2(a)}{2} & \text{otherwise} \end{cases}$$

`[In] integrate(x*cosh(b*x**2+a)**2,x)``[Out] Piecewise((-x**2*sinh(a + b*x**2)**2/4 + x**2*cosh(a + b*x**2)**2/4 + sinh(a + b*x**2)*cosh(a + b*x**2)/(4*b), Ne(b, 0)), (x**2*cosh(a)**2/2, True))`**Maxima [A] (verification not implemented)**

none

Time = 0.19 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.23

$$\int x \cosh^2(a + bx^2) dx = \frac{1}{4} x^2 + \frac{e^{(2bx^2+2a)}}{16b} - \frac{e^{(-2bx^2-2a)}}{16b}$$

`[In] integrate(x*cosh(b*x^2+a)^2,x, algorithm="maxima")``[Out] 1/4*x^2 + 1/16*e^(2*b*x^2 + 2*a)/b - 1/16*e^(-2*b*x^2 - 2*a)/b`

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int x \cosh^2(a + bx^2) dx = \frac{4bx^2 - \left(2e^{(2bx^2+2a)} + 1\right)e^{(-2bx^2-2a)} + 4a + e^{(2bx^2+2a)}}{16b}$$

[In] integrate(x*cosh(b*x^2+a)^2,x, algorithm="giac")

[Out] 1/16*(4*b*x^2 - (2*e^(2*b*x^2 + 2*a) + 1)*e^(-2*b*x^2 - 2*a) + 4*a + e^(2*b*x^2 + 2*a))/b

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.71

$$\int x \cosh^2(a + bx^2) dx = \frac{\sinh(2bx^2 + 2a)}{8b} + \frac{x^2}{4}$$

[In] int(x*cosh(a + b*x^2)^2,x)

[Out] sinh(2*a + 2*b*x^2)/(8*b) + x^2/4

3.11 $\int \cosh^2(a + bx^2) dx$

Optimal result	88
Rubi [A] (verified)	88
Mathematica [A] (verified)	89
Maple [A] (verified)	90
Fricas [A] (verification not implemented)	90
Sympy [F]	90
Maxima [A] (verification not implemented)	91
Giac [A] (verification not implemented)	91
Mupad [F(-1)]	91

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \cosh^2(a + bx^2) dx = \frac{x}{2} + \frac{e^{-2a} \sqrt{\frac{\pi}{2}} \operatorname{erf}(\sqrt{2}\sqrt{bx})}{8\sqrt{b}} + \frac{e^{2a} \sqrt{\frac{\pi}{2}} \operatorname{erfi}(\sqrt{2}\sqrt{bx})}{8\sqrt{b}}$$

[Out] 1/2*x+1/16*erf(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/exp(2*a)/b^(1/2)+1/16*exp(2*a)*erfi(x*2^(1/2)*b^(1/2))*2^(1/2)*Pi^(1/2)/b^(1/2)

Rubi [A] (verified)

Time = 0.03 (sec), antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5409, 5407, 2235, 2236}

$$\int \cosh^2(a + bx^2) dx = \frac{\sqrt{\frac{\pi}{2}} e^{-2a} \operatorname{erf}(\sqrt{2}\sqrt{bx})}{8\sqrt{b}} + \frac{\sqrt{\frac{\pi}{2}} e^{2a} \operatorname{erfi}(\sqrt{2}\sqrt{bx})}{8\sqrt{b}} + \frac{x}{2}$$

[In] Int[Cosh[a + b*x^2]^2,x]

[Out] x/2 + (Sqrt[Pi/2]*Erf[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b]*E^(2*a)) + (E^(2*a)*Sqrt[Pi/2]*Erfi[Sqrt[2]*Sqrt[b]*x])/(8*Sqrt[b])

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236


```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5409

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] := Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{1}{2} + \frac{1}{2} \cosh(2a + 2bx^2) \right) dx \\
 &= \frac{x}{2} + \frac{1}{2} \int \cosh(2a + 2bx^2) dx \\
 &= \frac{x}{2} + \frac{1}{4} \int e^{-2a-2bx^2} dx + \frac{1}{4} \int e^{2a+2bx^2} dx \\
 &= \frac{x}{2} + \frac{e^{-2a} \sqrt{\frac{\pi}{2}} \operatorname{erf}(\sqrt{2}\sqrt{b}x)}{8\sqrt{b}} + \frac{e^{2a} \sqrt{\frac{\pi}{2}} \operatorname{erfi}(\sqrt{2}\sqrt{b}x)}{8\sqrt{b}}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.10

$$\int \cosh^2(a + bx^2) dx = \frac{4\sqrt{2}\sqrt{b}x + \sqrt{\pi}\operatorname{erf}(\sqrt{2}\sqrt{b}x) (\cosh(2a) - \sinh(2a)) + \sqrt{\pi}\operatorname{erfi}(\sqrt{2}\sqrt{b}x) (\cosh(2a) + \sinh(2a))}{8\sqrt{2}\sqrt{b}}$$

```
[In] Integrate[Cosh[a + b*x^2]^2,x]
```

```
[Out] (4*Sqrt[2]*Sqrt[b]*x + Sqrt[Pi]*Erf[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] - Sinh[2*
a]) + Sqrt[Pi]*Erfi[Sqrt[2]*Sqrt[b]*x]*(Cosh[2*a] + Sinh[2*a]))/(8*Sqrt[2]*
Sqrt[b])
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.65

method	result	size
risch	$\frac{x}{2} + \frac{e^{-2a}\sqrt{\pi}\sqrt{2}\operatorname{erf}(x\sqrt{2}\sqrt{b})}{16\sqrt{b}} + \frac{e^{2a}\sqrt{\pi}\operatorname{erf}(\sqrt{-2b}x)}{8\sqrt{-2b}}$	51

[In] int(cosh(b*x^2+a)^2,x,method=_RETURNVERBOSE)

[Out] 1/2*x+1/16*exp(-2*a)*Pi^(1/2)*2^(1/2)/b^(1/2)*erf(x*2^(1/2)*b^(1/2))+1/8*exp(2*a)*Pi^(1/2)/(-2*b)^(1/2)*erf((-2*b)^(1/2)*x)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.94

$$\int \cosh^2(a + bx^2) dx = \frac{\sqrt{2}\sqrt{\pi}\sqrt{-b}(\cosh(2a) + \sinh(2a))\operatorname{erf}(\sqrt{2}\sqrt{-b}x) - \sqrt{2}\sqrt{\pi}\sqrt{b}(\cosh(2a) - \sinh(2a))\operatorname{erf}(\sqrt{2}\sqrt{b}x) - 8bx}{16b}$$

[In] integrate(cosh(b*x^2+a)^2,x, algorithm="fricas")

[Out] -1/16*(sqrt(2)*sqrt(pi)*sqrt(-b)*(cosh(2*a) + sinh(2*a))*erf(sqrt(2)*sqrt(-b)*x) - sqrt(2)*sqrt(pi)*sqrt(b)*(cosh(2*a) - sinh(2*a))*erf(sqrt(2)*sqrt(b)*x) - 8*b*x)/b

Sympy [F]

$$\int \cosh^2(a + bx^2) dx = \int \cosh^2(a + bx^2) dx$$

[In] integrate(cosh(b*x**2+a)**2,x)

[Out] Integral(cosh(a + b*x**2)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.72

$$\int \cosh^2(a + bx^2) dx = \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}(\sqrt{2}\sqrt{-b}x) e^{(2a)}}{16\sqrt{-b}} + \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}(\sqrt{2}\sqrt{b}x) e^{(-2a)}}{16\sqrt{b}} + \frac{1}{2}x$$

[In] integrate(cosh(b*x^2+a)^2,x, algorithm="maxima")

[Out] 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) + 1/16*sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) + 1/2*x

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.74

$$\int \cosh^2(a + bx^2) dx = -\frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}(-\sqrt{2}\sqrt{-b}x) e^{(2a)}}{16\sqrt{-b}} - \frac{\sqrt{2}\sqrt{\pi} \operatorname{erf}(-\sqrt{2}\sqrt{b}x) e^{(-2a)}}{16\sqrt{b}} + \frac{1}{2}x$$

[In] integrate(cosh(b*x^2+a)^2,x, algorithm="giac")

[Out] -1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(-b)*x)*e^(2*a)/sqrt(-b) - 1/16*sqrt(2)*sqrt(pi)*erf(-sqrt(2)*sqrt(b)*x)*e^(-2*a)/sqrt(b) + 1/2*x

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(a + bx^2) dx = \int \cosh(bx^2 + a)^2 dx$$

[In] int(cosh(a + b*x^2)^2,x)

[Out] int(cosh(a + b*x^2)^2, x)

3.12 $\int \frac{\cosh^2(a+bx^2)}{x} dx$

Optimal result	92
Rubi [A] (verified)	92
Mathematica [A] (verified)	93
Maple [A] (verified)	93
Fricas [A] (verification not implemented)	94
Sympy [F]	94
Maxima [A] (verification not implemented)	94
Giac [A] (verification not implemented)	95
Mupad [F(-1)]	95

Optimal result

Integrand size = 14, antiderivative size = 37

$$\int \frac{\cosh^2(a+bx^2)}{x} dx = \frac{1}{4} \cosh(2a) \operatorname{Chi}(2bx^2) + \frac{\log(x)}{2} + \frac{1}{4} \sinh(2a) \operatorname{Shi}(2bx^2)$$

[Out] 1/4*Chi(2*b*x^2)*cosh(2*a)+1/2*ln(x)+1/4*Shi(2*b*x^2)*sinh(2*a)

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5449, 5427, 5425, 5424}

$$\int \frac{\cosh^2(a+bx^2)}{x} dx = \frac{1}{4} \cosh(2a) \operatorname{Chi}(2bx^2) + \frac{1}{4} \sinh(2a) \operatorname{Shi}(2bx^2) + \frac{\log(x)}{2}$$

[In] Int[Cosh[a + b*x^2]^2/x, x]

[Out] (Cosh[2*a]*CoshIntegral[2*b*x^2])/4 + Log[x]/2 + (Sinh[2*a]*SinhIntegral[2*b*x^2])/4

Rule 5424

Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinhIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5425

Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CoshIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5427

`Int[Cosh[(c_) + (d_)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d, n}, x]`

Rule 5449

`Int[((a_) + Cosh[(c_) + (d_)*(x_)^(n_)]*(b_))^(p_)*((e_)*(x_))^(m_), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{1}{2x} + \frac{\cosh(2a + 2bx^2)}{2x} \right) dx \\
 &= \frac{\log(x)}{2} + \frac{1}{2} \int \frac{\cosh(2a + 2bx^2)}{x} dx \\
 &= \frac{\log(x)}{2} + \frac{1}{2} \cosh(2a) \int \frac{\cosh(2bx^2)}{x} dx + \frac{1}{2} \sinh(2a) \int \frac{\sinh(2bx^2)}{x} dx \\
 &= \frac{1}{4} \cosh(2a) \text{Chi}(2bx^2) + \frac{\log(x)}{2} + \frac{1}{4} \sinh(2a) \text{Shi}(2bx^2)
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^2(a + bx^2)}{x} dx = \frac{1}{4} (\cosh(2a) \text{Chi}(2bx^2) + 2 \log(x) + \sinh(2a) \text{Shi}(2bx^2))$$

[In] `Integrate[Cosh[a + b*x^2]^2/x,x]`

[Out] `(Cosh[2*a]*CoshIntegral[2*b*x^2] + 2*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^2])/4`

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.92

method	result	size
risch	$\frac{\ln(x)}{2} - \frac{e^{-2a} \text{Ei}_1(2bx^2)}{8} - \frac{e^{2a} \text{Ei}_1(-2bx^2)}{8}$	34

```
[In] int(cosh(b*x^2+a)^2/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x)-1/8*exp(-2*a)*Ei(1,2*b*x^2)-1/8*exp(2*a)*Ei(1,-2*b*x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.32

$$\int \frac{\cosh^2(a + bx^2)}{x} dx = \frac{1}{8} (\text{Ei}(2bx^2) + \text{Ei}(-2bx^2)) \cosh(2a) + \frac{1}{8} (\text{Ei}(2bx^2) - \text{Ei}(-2bx^2)) \sinh(2a) + \frac{1}{2} \log(x)$$

```
[In] integrate(cosh(b*x^2+a)^2/x,x, algorithm="fricas")
```

```
[Out] 1/8*(Ei(2*b*x^2) + Ei(-2*b*x^2))*cosh(2*a) + 1/8*(Ei(2*b*x^2) - Ei(-2*b*x^2))*sinh(2*a) + 1/2*log(x)
```

Sympy [F]

$$\int \frac{\cosh^2(a + bx^2)}{x} dx = \int \frac{\cosh^2(a + bx^2)}{x} dx$$

```
[In] integrate(cosh(b*x**2+a)**2/x,x)
```

```
[Out] Integral(cosh(a + b*x**2)**2/x, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(a + bx^2)}{x} dx = \frac{1}{8} \text{Ei}(2bx^2) e^{(2a)} + \frac{1}{8} \text{Ei}(-2bx^2) e^{(-2a)} + \frac{1}{2} \log(x)$$

```
[In] integrate(cosh(b*x^2+a)^2/x,x, algorithm="maxima")
```

```
[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) + 1/2*log(x)
```

Giac [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^2(a + bx^2)}{x} dx = \frac{1}{8} \operatorname{Ei}(2bx^2) e^{(2a)} + \frac{1}{8} \operatorname{Ei}(-2bx^2) e^{(-2a)} + \frac{1}{4} \log(bx^2)$$

[In] integrate(cosh(b*x^2+a)^2/x,x, algorithm="giac")

[Out] 1/8*Ei(2*b*x^2)*e^(2*a) + 1/8*Ei(-2*b*x^2)*e^(-2*a) + 1/4*log(b*x^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a + bx^2)}{x} dx = \int \frac{\cosh(bx^2 + a)^2}{x} dx$$

[In] int(cosh(a + b*x^2)^2/x,x)

[Out] int(cosh(a + b*x^2)^2/x, x)

3.13 $\int \frac{\cosh^2(a+bx^2)}{x^2} dx$

Optimal result	96
Rubi [A] (verified)	96
Mathematica [A] (verified)	98
Maple [A] (verified)	98
Fricas [B] (verification not implemented)	98
Sympy [F]	99
Maxima [A] (verification not implemented)	99
Giac [F]	99
Mupad [F(-1)]	100

Optimal result

Integrand size = 14, antiderivative size = 88

$$\int \frac{\cosh^2(a+bx^2)}{x^2} dx = -\frac{\cosh^2(a+bx^2)}{x} - \frac{1}{2}\sqrt{b}e^{-2a}\sqrt{\frac{\pi}{2}}\operatorname{erf}(\sqrt{2}\sqrt{b}x) + \frac{1}{2}\sqrt{b}e^{2a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}(\sqrt{2}\sqrt{b}x)$$

[Out] $-\cosh(b*x^2+a)^2/x-1/4*\operatorname{erf}(x*2^{(1/2)}*b^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(2*a)+1/4*\exp(2*a)*\operatorname{erfi}(x*2^{(1/2)}*b^{(1/2)})*b^{(1/2)}*2^{(1/2)}*\operatorname{Pi}^{(1/2)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5439, 5736, 5422, 5406, 2235, 2236}

$$\int \frac{\cosh^2(a+bx^2)}{x^2} dx = -\frac{1}{2}\sqrt{\frac{\pi}{2}}e^{-2a}\sqrt{b}\operatorname{erf}(\sqrt{2}\sqrt{b}x) + \frac{1}{2}\sqrt{\frac{\pi}{2}}e^{2a}\sqrt{b}\operatorname{erfi}(\sqrt{2}\sqrt{b}x) - \frac{\cosh^2(a+bx^2)}{x}$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b*x^2]^2/x^2, x]$

[Out] $-(\operatorname{Cosh}[a + b*x^2]^2/x) - (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*x])/(2*\operatorname{E}^{(2*a)}) + (\operatorname{Sqrt}[b]*\operatorname{E}^{(2*a)}*\operatorname{Sqrt}[\operatorname{Pi}/2]*\operatorname{ErFi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*x])/2$

Rule 2235

$\operatorname{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2), x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{ErFi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{$

F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 5406

Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ[n, 1]

Rule 5422

Int[((a_.) + (b_.)*Sinh[u_])^(p_.), x_Symbol] := Int[(a + b*Sinh[ExpandToSum[u, x]])^p, x] /; FreeQ[{a, b, p}, x] && BinomialQ[u, x] && !BinomialMatchQ[u, x]

Rule 5439

Int[Cosh[(a_.) + (b_.)*(x_)^(n_)]^(p_)*(x_)^(m_), x_Symbol] := Simp[-Cosh[a + b*x^n]^p/((n - 1)*x^(n - 1)), x] + Dist[b*n*(p/(n - 1)), Int[Cosh[a + b*x^n]^(p - 1)*Sinh[a + b*x^n], x], x] /; FreeQ[{a, b}, x] && IntegersQ[n, p] && EqQ[m + n, 0] && GtQ[p, 1] && NeQ[n, 1]

Rule 5736

Int[Cosh[w_]^(p_)*(u_.)*Sinh[v_]^(p_), x_Symbol] := Dist[1/2^p, Int[u*Sinh[2*v]^p, x], x] /; EqQ[w, v] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\frac{\cosh^2(a + bx^2)}{x} + (4b) \int \cosh(a + bx^2) \sinh(a + bx^2) dx \\
 &= -\frac{\cosh^2(a + bx^2)}{x} + (2b) \int \sinh(2(a + bx^2)) dx \\
 &= -\frac{\cosh^2(a + bx^2)}{x} + (2b) \int \sinh(2a + 2bx^2) dx \\
 &= -\frac{\cosh^2(a + bx^2)}{x} - b \int e^{-2a-2bx^2} dx + b \int e^{2a+2bx^2} dx \\
 &= -\frac{\cosh^2(a + bx^2)}{x} - \frac{1}{2} \sqrt{b} e^{-2a} \sqrt{\frac{\pi}{2}} \operatorname{erf}(\sqrt{2}\sqrt{b}x) + \frac{1}{2} \sqrt{b} e^{2a} \sqrt{\frac{\pi}{2}} \operatorname{erfi}(\sqrt{2}\sqrt{b}x)
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(a + bx^2)}{x^2} dx = \frac{-4 \cosh^2(a + bx^2) + \sqrt{b}\sqrt{2\pi}x \operatorname{erf}(\sqrt{2}\sqrt{bx}) (-\cosh(2a) + \sinh(2a)) + \sqrt{b}\sqrt{2\pi}x \operatorname{erfi}(\sqrt{2}\sqrt{bx}) (\cosh(2a) + \sinh(2a))}{4x}$$

[In] Integrate[Cosh[a + b*x^2]^2/x^2,x]

[Out] $(-4*\operatorname{Cosh}[a + b*x^2]^2 + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[2*\operatorname{Pi}]*x*\operatorname{Erf}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*x]*(-\operatorname{Cosh}[2*a] + \operatorname{Sinh}[2*a]) + \operatorname{Sqrt}[b]*\operatorname{Sqrt}[2*\operatorname{Pi}]*x*\operatorname{Erfi}[\operatorname{Sqrt}[2]*\operatorname{Sqrt}[b]*x]*(\operatorname{Cosh}[2*a] + \operatorname{Sinh}[2*a]))/(4*x)$

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

method	result	size
risch	$-\frac{1}{2x} - \frac{e^{-2a}e^{-2bx^2}}{4x} - \frac{e^{-2a}\sqrt{b}\sqrt{\pi}\sqrt{2}\operatorname{erf}(x\sqrt{2}\sqrt{b})}{4} - \frac{e^{2a}e^{2bx^2}}{4x} + \frac{e^{2a}b\sqrt{\pi}\operatorname{erf}(\sqrt{-2b}x)}{2\sqrt{-2b}}$	86

[In] int(cosh(b*x^2+a)^2/x^2,x,method=_RETURNVERBOSE)

[Out] $-1/2/x - 1/4*\exp(-2*a)/x*\exp(-2*b*x^2) - 1/4*\exp(-2*a)*b^{(1/2)}*\operatorname{Pi}^{(1/2)}*2^{(1/2)}*\operatorname{erf}(x*2^{(1/2)}*b^{(1/2)}) - 1/4*\exp(2*a)/x*\exp(2*b*x^2) + 1/2*\exp(2*a)*b*\operatorname{Pi}^{(1/2)}/(-2*b)^{(1/2)}*\operatorname{erf}((-2*b)^{(1/2)}*x)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(64) = 128.

Time = 0.26 (sec) , antiderivative size = 394, normalized size of antiderivative = 4.48

$$\int \frac{\cosh^2(a + bx^2)}{x^2} dx = \frac{\cosh(bx^2 + a)^4 + 4 \cosh(bx^2 + a) \sinh(bx^2 + a)^3 + \sinh(bx^2 + a)^4 + \sqrt{2}\sqrt{\pi} \left(x \cosh(bx^2 + a)^2 \cosh(2a) + x \sinh(bx^2 + a)^2 \sinh(2a) \right)}{4x^2}$$

[In] integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="fricas")

[Out] $-1/4*(\cosh(b*x^2 + a)^4 + 4*\cosh(b*x^2 + a)*\sinh(b*x^2 + a)^3 + \sinh(b*x^2 + a)^4 + \operatorname{sqr}(2)*\operatorname{sqr}(pi)*(x*\cosh(b*x^2 + a)^2*\cosh(2*a) + x*\cosh(b*x^2 + a)^2*\sinh(2*a)))/4x^2$

)^2*sinh(2*a) + (x*cosh(2*a) + x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) + x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(2)*sqrt(-b)*x) + sqrt(2)*sqrt(pi)*(x*cosh(b*x^2 + a)^2*cosh(2*a) - x*cosh(b*x^2 + a)^2*sinh(2*a) + (x*cosh(2*a) - x*sinh(2*a))*sinh(b*x^2 + a)^2 + 2*(x*cosh(b*x^2 + a)*cosh(2*a) - x*cosh(b*x^2 + a)*sinh(2*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(2)*sqrt(b)*x) + 2*(3*cosh(b*x^2 + a)^2 + 1)*sinh(b*x^2 + a)^2 + 2*cosh(b*x^2 + a)^2 + 4*(cosh(b*x^2 + a)^3 + cosh(b*x^2 + a))*sinh(b*x^2 + a) + 1)/(x*cosh(b*x^2 + a)^2 + 2*x*cosh(b*x^2 + a)*sinh(b*x^2 + a) + x*sinh(b*x^2 + a)^2)

Sympy [F]

$$\int \frac{\cosh^2(a + bx^2)}{x^2} dx = \int \frac{\cosh^2(a + bx^2)}{x^2} dx$$

[In] integrate(cosh(b*x**2+a)**2/x**2,x)

[Out] Integral(cosh(a + b*x**2)**2/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.69

$$\int \frac{\cosh^2(a + bx^2)}{x^2} dx = -\frac{\sqrt{2}\sqrt{bx^2}e^{(-2a)}\Gamma(-\frac{1}{2}, 2bx^2)}{8x} - \frac{\sqrt{2}\sqrt{-bx^2}e^{(2a)}\Gamma(-\frac{1}{2}, -2bx^2)}{8x} - \frac{1}{2x}$$

[In] integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="maxima")

[Out] -1/8*sqrt(2)*sqrt(b*x^2)*e^(-2*a)*gamma(-1/2, 2*b*x^2)/x - 1/8*sqrt(2)*sqrt(-b*x^2)*e^(2*a)*gamma(-1/2, -2*b*x^2)/x - 1/2/x

Giac [F]

$$\int \frac{\cosh^2(a + bx^2)}{x^2} dx = \int \frac{\cosh(bx^2 + a)^2}{x^2} dx$$

[In] integrate(cosh(b*x^2+a)^2/x^2,x, algorithm="giac")

[Out] integrate(cosh(b*x^2 + a)^2/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a + bx^2)}{x^2} dx = \int \frac{\cosh(bx^2 + a)^2}{x^2} dx$$

```
[In] int(cosh(a + b*x^2)^2/x^2, x)
```

```
[Out] int(cosh(a + b*x^2)^2/x^2, x)
```

3.14 $\int \frac{\cosh^2(a+bx^2)}{x^3} dx$

Optimal result	101
Rubi [A] (verified)	101
Mathematica [A] (verified)	103
Maple [A] (verified)	103
Fricas [A] (verification not implemented)	104
Sympy [F]	104
Maxima [A] (verification not implemented)	104
Giac [B] (verification not implemented)	105
Mupad [F(-1)]	105

Optimal result

Integrand size = 14, antiderivative size = 57

$$\int \frac{\cosh^2(a+bx^2)}{x^3} dx = -\frac{1}{4x^2} - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{2}b\text{Chi}(2bx^2)\sinh(2a) + \frac{1}{2}b\cosh(2a)\text{Shi}(2bx^2)$$

[Out] -1/4/x^2-1/4*cosh(2*b*x^2+2*a)/x^2+1/2*b*cosh(2*a)*Shi(2*b*x^2)+1/2*b*Chi(2*b*x^2)*sinh(2*a)

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5449, 5429, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^2(a+bx^2)}{x^3} dx = \frac{1}{2}b\sinh(2a)\text{Chi}(2bx^2) + \frac{1}{2}b\cosh(2a)\text{Shi}(2bx^2) - \frac{\cosh(2(a+bx^2))}{4x^2} - \frac{1}{4x^2}$$

[In] Int[Cosh[a + b*x^2]^2/x^3,x]

[Out] -1/4*1/x^2 - Cosh[2*(a + b*x^2)]/(4*x^2) + (b*CoshIntegral[2*b*x^2]*Sinh[2*a])/2 + (b*Cosh[2*a]*SinhIntegral[2*b*x^2])/2

Rule 3378

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c

```
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d],
Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x]
/; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x_Symbol]
:> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x]
&& IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{1}{2x^3} + \frac{\cosh(2a + 2bx^2)}{2x^3} \right) dx \\ &= -\frac{1}{4x^2} + \frac{1}{2} \int \frac{\cosh(2a + 2bx^2)}{x^3} dx \\ &= -\frac{1}{4x^2} + \frac{1}{4} \text{Subst} \left(\int \frac{\cosh(2a + 2bx)}{x^2} dx, x, x^2 \right) \end{aligned}$$

$$\begin{aligned}
&= -\frac{1}{4x^2} - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{2}b \text{Subst}\left(\int \frac{\sinh(2a+2bx)}{x} dx, x, x^2\right) \\
&= -\frac{1}{4x^2} - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{2}(b \cosh(2a)) \text{Subst}\left(\int \frac{\sinh(2bx)}{x} dx, x, x^2\right) \\
&\quad + \frac{1}{2}(b \sinh(2a)) \text{Subst}\left(\int \frac{\cosh(2bx)}{x} dx, x, x^2\right) \\
&= -\frac{1}{4x^2} - \frac{\cosh(2(a+bx^2))}{4x^2} + \frac{1}{2}b \text{Chi}(2bx^2) \sinh(2a) + \frac{1}{2}b \cosh(2a) \text{Shi}(2bx^2)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\cosh^2(a+bx^2)}{x^3} dx = \frac{1}{2} \left(-\frac{\cosh^2(a+bx^2)}{x^2} + b \text{Chi}(2bx^2) \sinh(2a) + b \cosh(2a) \text{Shi}(2bx^2) \right)$$

[In] Integrate[Cosh[a + b*x^2]^2/x^3,x]

[Out] $(-\text{Cosh}[a + b*x^2]^2/x^2) + b*\text{CoshIntegral}[2*b*x^2]*\text{Sinh}[2*a] + b*\text{Cosh}[2*a]*\text{SinhIntegral}[2*b*x^2])/2$

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.16

method	result	size
risch	$-\frac{2e^{-2a} \text{Ei}_1(2bx^2)bx^2 + 2e^{2a} \text{Ei}_1(-2bx^2)bx^2 + e^{-2bx^2-2a} + e^{2bx^2+2a} + 2}{8x^2}$	66

[In] int(cosh(b*x^2+a)^2/x^3,x,method=_RETURNVERBOSE)

[Out] $-1/8*(-2*\exp(-2*a)*\text{Ei}(1,2*b*x^2)*b*x^2 + 2*\exp(2*a)*\text{Ei}(1,-2*b*x^2)*b*x^2 + \exp(-2*b*x^2-2*a) + \exp(2*b*x^2+2*a))/x^2$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\cosh^2(a + bx^2)}{x^3} dx = \frac{\cosh(bx^2 + a)^2 - (bx^2 \operatorname{Ei}(2bx^2) - bx^2 \operatorname{Ei}(-2bx^2)) \cosh(2a) + \sinh(bx^2 + a)^2 - (bx^2 \operatorname{Ei}(2bx^2) + bx^2 \operatorname{Ei}(-2bx^2)) \sinh(2a)}{4x^2}$$

[In] integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="fricas")

[Out] -1/4*(cosh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) - b*x^2*Ei(-2*b*x^2))*cosh(2*a) + sinh(b*x^2 + a)^2 - (b*x^2*Ei(2*b*x^2) + b*x^2*Ei(-2*b*x^2))*sinh(2*a) + 1)/x^2

Sympy [F]

$$\int \frac{\cosh^2(a + bx^2)}{x^3} dx = \int \frac{\cosh^2(a + bx^2)}{x^3} dx$$

[In] integrate(cosh(b*x**2+a)**2/x**3,x)

[Out] Integral(cosh(a + b*x**2)**2/x**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.63

$$\int \frac{\cosh^2(a + bx^2)}{x^3} dx = -\frac{1}{4} b e^{(-2a)} \Gamma(-1, 2bx^2) + \frac{1}{4} b e^{(2a)} \Gamma(-1, -2bx^2) - \frac{1}{4x^2}$$

[In] integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="maxima")

[Out] -1/4*b*e^(-2*a)*gamma(-1, 2*b*x^2) + 1/4*b*e^(2*a)*gamma(-1, -2*b*x^2) - 1/4/x^2

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(50) = 100.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.21

$$\int \frac{\cosh^2(a + bx^2)}{x^3} dx$$

$$= \frac{2(bx^2 + a)b^2\text{Ei}(2bx^2)e^{(2a)} - 2ab^2\text{Ei}(2bx^2)e^{(2a)} - 2(bx^2 + a)b^2\text{Ei}(-2bx^2)e^{(-2a)} + 2ab^2\text{Ei}(-2bx^2)e^{(-2a)}}{8b^2x^2}$$

[In] integrate(cosh(b*x^2+a)^2/x^3,x, algorithm="giac")

[Out] 1/8*(2*(b*x^2 + a)*b^2*Ei(2*b*x^2)*e^(2*a) - 2*a*b^2*Ei(2*b*x^2)*e^(2*a) - 2*(b*x^2 + a)*b^2*Ei(-2*b*x^2)*e^(-2*a) + 2*a*b^2*Ei(-2*b*x^2)*e^(-2*a) - b^2*e^(2*b*x^2 + 2*a) - b^2*e^(-2*b*x^2 - 2*a) - 2*b^2)/(b^2*x^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a + bx^2)}{x^3} dx = \int \frac{\cosh(bx^2 + a)^2}{x^3} dx$$

[In] int(cosh(a + b*x^2)^2/x^3,x)

[Out] int(cosh(a + b*x^2)^2/x^3, x)

3.15 $\int x^3 \cosh^3(a + bx^2) dx$

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Optimal result

Integrand size = 14, antiderivative size = 79

$$\int x^3 \cosh^3(a + bx^2) dx = -\frac{\cosh(a + bx^2)}{3b^2} - \frac{\cosh^3(a + bx^2)}{18b^2} + \frac{x^2 \sinh(a + bx^2)}{3b} + \frac{x^2 \cosh^2(a + bx^2) \sinh(a + bx^2)}{6b}$$

[Out] $-1/3*\cosh(b*x^2+a)/b^2-1/18*\cosh(b*x^2+a)^3/b^2+1/3*x^2*\sinh(b*x^2+a)/b+1/6*x^2*\cosh(b*x^2+a)^2*\sinh(b*x^2+a)/b$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5429, 3391, 3377, 2718}

$$\int x^3 \cosh^3(a + bx^2) dx = -\frac{\cosh^3(a + bx^2)}{18b^2} - \frac{\cosh(a + bx^2)}{3b^2} + \frac{x^2 \sinh(a + bx^2)}{3b} + \frac{x^2 \sinh(a + bx^2) \cosh^2(a + bx^2)}{6b}$$

[In] $\text{Int}[x^3*\text{Cosh}[a + b*x^2]^3, x]$

[Out] $-1/3*\text{Cosh}[a + b*x^2]/b^2 - \text{Cosh}[a + b*x^2]^3/(18*b^2) + (x^2*\text{Sinh}[a + b*x^2])/ (3*b) + (x^2*\text{Cosh}[a + b*x^2]^2*\text{Sinh}[a + b*x^2])/(6*b)$

Rule 2718

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \text{ :> Simp}[-\text{Cos}[c + d*x]/d, x] \text{ /; FreeQ}[\{c, d\}, x]$

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-
(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 3391

```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :=>
Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (Dist[b^2*((n - 1)/n), Int[(c
+ d*x)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[b*(c + d*x)*Cos[e + f*x]*((b
*Sin[e + f*x])^(n - 1)/(f*n)), x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1
]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] :=> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2} \text{Subst} \left(\int x \cosh^3(a + bx) dx, x, x^2 \right) \\
&= -\frac{\cosh^3(a + bx^2)}{18b^2} + \frac{x^2 \cosh^2(a + bx^2) \sinh(a + bx^2)}{6b} + \frac{1}{3} \text{Subst} \left(\int x \cosh(a + bx) dx, x, x^2 \right) \\
&= -\frac{\cosh^3(a + bx^2)}{18b^2} + \frac{x^2 \sinh(a + bx^2)}{3b} \\
&\quad + \frac{x^2 \cosh^2(a + bx^2) \sinh(a + bx^2)}{6b} - \frac{\text{Subst}(\int \sinh(a + bx) dx, x, x^2)}{3b} \\
&= -\frac{\cosh(a + bx^2)}{3b^2} - \frac{\cosh^3(a + bx^2)}{18b^2} + \frac{x^2 \sinh(a + bx^2)}{3b} + \frac{x^2 \cosh^2(a + bx^2) \sinh(a + bx^2)}{6b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.70

$$\int x^3 \cosh^3(a + bx^2) dx$$

$$= -\frac{27 \cosh(a + bx^2) + \cosh(3(a + bx^2)) - 3bx^2(9 \sinh(a + bx^2) + \sinh(3(a + bx^2)))}{72b^2}$$

[In] Integrate[x^3*Cosh[a + b*x^2]^3,x]

[Out] -1/72*(27*Cosh[a + b*x^2] + Cosh[3*(a + b*x^2)] - 3*b*x^2*(9*Sinh[a + b*x^2] + Sinh[3*(a + b*x^2)]))/b^2

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.18

method	result	size
risch	$\frac{(3bx^2-1)e^{3bx^2+3a}}{144b^2} + \frac{3(bx^2-1)e^{bx^2+a}}{16b^2} - \frac{3(bx^2+1)e^{-bx^2-a}}{16b^2} - \frac{(3bx^2+1)e^{-3bx^2-3a}}{144b^2}$	93
parallelrisc	$\frac{-9 \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)^5 x^2 b + 6 \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)^3 x^2 b + 9 \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)^4 - 9 \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right) x^2 b - 12 \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)^2 + 7}{9b^2 \left(1 + \tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right)\right)^3 \left(\tanh\left(\frac{bx^2}{2} + \frac{a}{2}\right) - 1\right)^3}$	123

[In] int(x^3*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/144*(3*b*x^2-1)/b^2*exp(3*b*x^2+3*a)+3/16*(b*x^2-1)/b^2*exp(b*x^2+a)-3/16*(b*x^2+1)/b^2*exp(-b*x^2-a)-1/144*(3*b*x^2+1)/b^2*exp(-3*b*x^2-3*a)

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.22

$$\int x^3 \cosh^3(a + bx^2) dx$$

$$= \frac{3bx^2 \sinh(bx^2 + a)^3 - \cosh(bx^2 + a)^3 - 3 \cosh(bx^2 + a) \sinh(bx^2 + a)^2 + 9 \left(bx^2 \cosh(bx^2 + a)^2 + 3bx^2\right)}{72b^2}$$

[In] integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/72*(3*b*x^2*sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3 - 3*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 + 9*(b*x^2*cosh(b*x^2 + a)^2 + 3*b*x^2)*sinh(b*x^2 + a) - 27*cosh(b*x^2 + a))/b^2

Sympy [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.16

$$\int x^3 \cosh^3(a + bx^2) dx = \begin{cases} -\frac{x^2 \sinh^3(a+bx^2)}{3b} + \frac{x^2 \sinh(a+bx^2) \cosh^2(a+bx^2)}{2b} + \frac{\sinh^2(a+bx^2) \cosh(a+bx^2)}{3b^2} - \frac{7 \cosh^3(a+bx^2)}{18b^2} & \text{for } b \neq 0 \\ \frac{x^4 \cosh^3(a)}{4} & \text{otherwise} \end{cases}$$

[In] integrate(x**3*cosh(b*x**2+a)**3,x)

[Out] Piecewise((-x**2*sinh(a + b*x**2)**3/(3*b) + x**2*sinh(a + b*x**2)*cosh(a + b*x**2)**2/(2*b) + sinh(a + b*x**2)**2*cosh(a + b*x**2)/(3*b**2) - 7*cosh(a + b*x**2)**3/(18*b**2), Ne(b, 0)), (x**4*cosh(a)**3/4, True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.27

$$\int x^3 \cosh^3(a + bx^2) dx = \frac{(3bx^2e^{(3a)} - e^{(3a)})e^{(3bx^2)}}{144b^2} + \frac{3(bx^2e^a - e^a)e^{(bx^2)}}{16b^2} - \frac{3(bx^2 + 1)e^{(-bx^2 - a)}}{16b^2} - \frac{(3bx^2 + 1)e^{(-3bx^2 - 3a)}}{144b^2}$$

[In] integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/144*(3*b*x^2*e^(3*a) - e^(3*a))*e^(3*b*x^2)/b^2 + 3/16*(b*x^2*e^a - e^a)*e^(b*x^2)/b^2 - 3/16*(b*x^2 + 1)*e^(-b*x^2 - a)/b^2 - 1/144*(3*b*x^2 + 1)*e^(-3*b*x^2 - 3*a)/b^2

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(71) = 142.

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.43

$$\int x^3 \cosh^3(a + bx^2) dx = \frac{3(bx^2 + a)e^{(3bx^2+3a)} + 27(bx^2 + a)e^{(bx^2+a)} - 27(bx^2 + a)e^{(-bx^2-a)} - 3(bx^2 + a)e^{(-3bx^2-3a)} - e^{(3bx^2+3a)}}{144b^2} - \frac{ae^{(3bx^2+3a)} + 9ae^{(bx^2+a)} - (9ae^{(2bx^2+2a)} + a)e^{(-3bx^2-3a)}}{48b^2}$$

[In] integrate(x^3*cosh(b*x^2+a)^3,x, algorithm="giac")

[Out] $\frac{1}{144} \cdot (3 \cdot (b \cdot x^2 + a) \cdot e^{(3 \cdot b \cdot x^2 + 3 \cdot a)} + 27 \cdot (b \cdot x^2 + a) \cdot e^{(b \cdot x^2 + a)} - 27 \cdot (b \cdot x^2 + a) \cdot e^{(-b \cdot x^2 - a)} - 3 \cdot (b \cdot x^2 + a) \cdot e^{(-3 \cdot b \cdot x^2 - 3 \cdot a)} - e^{(3 \cdot b \cdot x^2 + 3 \cdot a)} - 27 \cdot e^{(b \cdot x^2 + a)} - 27 \cdot e^{(-b \cdot x^2 - a)} - e^{(-3 \cdot b \cdot x^2 - 3 \cdot a)}) / b^2 - 1 / 48 \cdot (a \cdot e^{(3 \cdot b \cdot x^2 + 3 \cdot a)} + 9 \cdot a \cdot e^{(b \cdot x^2 + a)} - (9 \cdot a \cdot e^{(2 \cdot b \cdot x^2 + 2 \cdot a)} + a) \cdot e^{(-3 \cdot b \cdot x^2 - 3 \cdot a)}) / b^2$

Mupad [B] (verification not implemented)

Time = 1.62 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x^3 \cosh^3(a + bx^2) dx = \frac{\frac{x^2 \sinh(bx^2+a)}{3} + \frac{x^2 \cosh(bx^2+a)^2 \sinh(bx^2+a)}{6}}{b} - \frac{\cosh(bx^2+a)^3}{18b^2} - \frac{\cosh(bx^2+a)}{3b^2}$$

[In] int(x^3*cosh(a + b*x^2)^3,x)

[Out] $((x^2 \cdot \sinh(a + b \cdot x^2)) / 3 + (x^2 \cdot \cosh(a + b \cdot x^2)^2 \cdot \sinh(a + b \cdot x^2)) / 6) / b - \cosh(a + b \cdot x^2)^3 / (18 \cdot b^2) - \cosh(a + b \cdot x^2) / (3 \cdot b^2)$

3.16 $\int x^2 \cosh^3(a + bx^2) dx$

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Optimal result

Integrand size = 14, antiderivative size = 160

$$\int x^2 \cosh^3(a + bx^2) dx = \frac{3e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})}{32b^{3/2}} + \frac{e^{-3a}\sqrt{\frac{\pi}{3}}\operatorname{erf}(\sqrt{3}\sqrt{bx})}{96b^{3/2}} - \frac{3e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx})}{32b^{3/2}} \\ - \frac{e^{3a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{96b^{3/2}} + \frac{3x \sinh(a + bx^2)}{8b} + \frac{x \sinh(3a + 3bx^2)}{24b}$$

[Out] $\frac{3}{8}x\sinh(bx^2+a)/b+1/24x\sinh(3bx^2+3a)/b+1/288\operatorname{erf}(x^{3/2}b^{1/2})\pi^{1/2}/b^{3/2}/\exp(3a)-1/288\exp(3a)\operatorname{erfi}(x^{3/2}b^{1/2})\pi^{1/2}/b^{3/2}+3/32\operatorname{erf}(xb^{1/2})\pi^{1/2}/b^{3/2}/\exp(a)-3/32\exp(a)\operatorname{erfi}(xb^{1/2})\pi^{1/2}/b^{3/2}$

Rubi [A] (verified)

Time = 0.11 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5449, 5433, 5406, 2235, 2236}

$$\int x^2 \cosh^3(a + bx^2) dx = \frac{3\sqrt{\pi}e^{-a}\operatorname{erf}(\sqrt{bx})}{32b^{3/2}} + \frac{\sqrt{\frac{\pi}{3}}e^{-3a}\operatorname{erf}(\sqrt{3}\sqrt{bx})}{96b^{3/2}} - \frac{3\sqrt{\pi}e^a\operatorname{erfi}(\sqrt{bx})}{32b^{3/2}} \\ - \frac{\sqrt{\frac{\pi}{3}}e^{3a}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{96b^{3/2}} + \frac{3x \sinh(a + bx^2)}{8b} + \frac{x \sinh(3a + 3bx^2)}{24b}$$

[In] $\operatorname{Int}[x^2\operatorname{Cosh}[a + bx^2]^3, x]$

[Out] $(3\sqrt{\pi}\operatorname{Erf}[\sqrt{b}x])/(32b^{3/2}E^a) + (\sqrt{\pi/3}\operatorname{Erf}[\sqrt{3}\sqrt{b}x])/(96b^{3/2}E^{3a}) - (3E^a\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}x])/(32b^{3/2}) - (E^{3a}\sqrt{\pi/3}\operatorname{Erfi}[\sqrt{3}\sqrt{b}x])/(96b^{3/2}) + (3x\sinh(a+bx^2))/(8b) + (x\sinh(3a+3bx^2))/(24b)$

) - (E^(3*a)*Sqrt[Pi/3]*Erfi[Sqrt[3]*Sqrt[b]*x])/(96*b^(3/2)) + (3*x*Sinh[a + b*x^2])/(8*b) + (x*Sinh[3*a + 3*b*x^2])/(24*b)

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2)), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 5406

Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ[n, 1]

Rule 5433

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.)), x_Symbol] := Simp[e^(n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[0, n, m + 1]

Rule 5449

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{3}{4}x^2 \cosh(a + bx^2) + \frac{1}{4}x^2 \cosh(3a + 3bx^2) \right) dx \\
 &= \frac{1}{4} \int x^2 \cosh(3a + 3bx^2) dx + \frac{3}{4} \int x^2 \cosh(a + bx^2) dx \\
 &= \frac{3x \sinh(a + bx^2)}{8b} + \frac{x \sinh(3a + 3bx^2)}{24b} - \frac{\int \sinh(3a + 3bx^2) dx}{24b} - \frac{3 \int \sinh(a + bx^2) dx}{8b} \\
 &= \frac{3x \sinh(a + bx^2)}{8b} + \frac{x \sinh(3a + 3bx^2)}{24b} + \frac{\int e^{-3a-3bx^2} dx}{48b} \\
 &\quad - \frac{\int e^{3a+3bx^2} dx}{48b} + \frac{3 \int e^{-a-bx^2} dx}{16b} - \frac{3 \int e^{a+bx^2} dx}{16b}
 \end{aligned}$$

$$= \frac{3e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})}{32b^{3/2}} + \frac{e^{-3a}\sqrt{\frac{\pi}{3}}\operatorname{erf}(\sqrt{3}\sqrt{bx})}{96b^{3/2}} - \frac{3e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx})}{32b^{3/2}} - \frac{e^{3a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{96b^{3/2}} + \frac{3x\sinh(a+bx^2)}{8b} + \frac{x\sinh(3a+3bx^2)}{24b}$$

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.15

$$\int x^2 \cosh^3(a+bx^2) dx = \frac{-27\sqrt{\pi}\cosh(a)\operatorname{erfi}(\sqrt{bx}) - \sqrt{3\pi}\cosh(3a)\operatorname{erfi}(\sqrt{3}\sqrt{bx}) + 27\sqrt{\pi}\operatorname{erf}(\sqrt{bx})(\cosh(a) - \sinh(a)) - 27\sqrt{\pi}e^a\operatorname{erfi}(\sqrt{bx}) + 27\sqrt{\pi}e^{-a}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{288b^{3/2}}$$

[In] Integrate[x^2*Cosh[a + b*x^2]^3,x]

[Out] $(-27\sqrt{\pi}\operatorname{Cosh}[a]\operatorname{Erfi}[\sqrt{b}x] - \sqrt{3\pi}\operatorname{Cosh}[3a]\operatorname{Erfi}[\sqrt{3}\sqrt{b}x] + 27\sqrt{\pi}\operatorname{Erf}[\sqrt{b}x](\operatorname{Cosh}[a] - \operatorname{Sinh}[a]) - 27\sqrt{\pi}\operatorname{Erfi}[\sqrt{b}x]\operatorname{Sinh}[a] + \sqrt{3\pi}\operatorname{Erf}[\sqrt{3}\sqrt{b}x](\operatorname{Cosh}[3a] - \operatorname{Sinh}[3a]) - \sqrt{3\pi}\operatorname{Erfi}[\sqrt{3}\sqrt{b}x]\operatorname{Sinh}[3a] + 108\sqrt{b}x\operatorname{Sinh}[a + b*x^2] + 12\sqrt{b}x\operatorname{Sinh}[3(a + b*x^2)])/(288*b^{3/2})$

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 157, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{e^{-3a}x e^{-3bx^2}}{48b} + \frac{e^{-3a}\sqrt{\pi}\sqrt{3}\operatorname{erf}(x\sqrt{3}\sqrt{b})}{288b^{3/2}} - \frac{3e^{-a}x e^{-bx^2}}{16b} + \frac{3\operatorname{erf}(x\sqrt{b})\sqrt{\pi}e^{-a}}{32b^{3/2}} + \frac{e^{3a}x e^{3bx^2}}{48b} - \frac{e^{3a}\sqrt{\pi}\operatorname{erf}(\sqrt{-3b}x)}{96b\sqrt{-3b}}$

[In] int(x^2*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] $-1/48/\exp(a)^3/b*x*\exp(-3*b*x^2)+1/288/\exp(a)^3/b^{3/2}*Pi^{1/2}*3^{1/2}*erf(x*3^{1/2}*b^{1/2})-3/16/\exp(a)/b*x*\exp(-b*x^2)+3/32*erf(x*b^{1/2})*Pi^{1/2}/b^{3/2}/\exp(a)+1/48*\exp(a)^3/b*x*\exp(3*b*x^2)-1/96*\exp(a)^3/b*Pi^{1/2}/(-3*b)^{1/2}*erf((-3*b)^{1/2}*x)+3/16*\exp(a)*\exp(b*x^2)*x/b-3/32*\exp(a)/b*Pi^{1/2}/(-b)^{1/2}*erf((-b)^{1/2}*x)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 903 vs. 2(114) = 228.

Time = 0.26 (sec) , antiderivative size = 903, normalized size of antiderivative = 5.64

$$\int x^2 \cosh^3(a + bx^2) dx = \text{Too large to display}$$

[In] integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/288*(6*b*x*cosh(b*x^2 + a)^6 + 36*b*x*cosh(b*x^2 + a)*sinh(b*x^2 + a)^5 + 6*b*x*sinh(b*x^2 + a)^6 + 54*b*x*cosh(b*x^2 + a)^4 + 18*(5*b*x*cosh(b*x^2 + a)^2 + 3*b*x)*sinh(b*x^2 + a)^4 - 54*b*x*cosh(b*x^2 + a)^2 + 24*(5*b*x*cosh(b*x^2 + a)^3 + 9*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a)^3 + sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) + sinh(3*a))*sinh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) + cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) + cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(3)*sqrt(-b)*x) + sqrt(3)*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(3*a) + (cosh(3*a) - sinh(3*a))*sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(3*a) + 3*(cosh(b*x^2 + a)*cosh(3*a) - cosh(b*x^2 + a)*sinh(3*a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(3*a) - cosh(b*x^2 + a)^2*sinh(3*a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(3)*sqrt(b)*x) + 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) + sinh(a))*sinh(b*x^2 + a)^3 + cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) + cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) + cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(-b)*erf(sqrt(-b)*x) + 27*sqrt(pi)*(cosh(b*x^2 + a)^3*cosh(a) + (cosh(a) - sinh(a))*sinh(b*x^2 + a)^3 - cosh(b*x^2 + a)^3*sinh(a) + 3*(cosh(b*x^2 + a)*cosh(a) - cosh(b*x^2 + a)*sinh(a))*sinh(b*x^2 + a)^2 + 3*(cosh(b*x^2 + a)^2*cosh(a) - cosh(b*x^2 + a)^2*sinh(a))*sinh(b*x^2 + a))*sqrt(b)*erf(sqrt(b)*x) + 18*(5*b*x*cosh(b*x^2 + a)^4 + 18*b*x*cosh(b*x^2 + a)^2 - 3*b*x)*sinh(b*x^2 + a)^2 - 6*b*x + 36*(b*x*cosh(b*x^2 + a)^5 + 6*b*x*cosh(b*x^2 + a)^3 - 3*b*x*cosh(b*x^2 + a))*sinh(b*x^2 + a))/(b^2*cosh(b*x^2 + a)^3 + 3*b^2*cosh(b*x^2 + a)^2*sinh(b*x^2 + a) + 3*b^2*cosh(b*x^2 + a)*sinh(b*x^2 + a)^2 + b^2*sinh(b*x^2 + a)^3)

Sympy [F]

$$\int x^2 \cosh^3(a + bx^2) dx = \int x^2 \cosh^3(a + bx^2) dx$$

[In] integrate(x**2*cosh(b*x**2+a)**3,x)

[Out] Integral(x**2*cosh(a + b*x**2)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.01

$$\int x^2 \cosh^3(a + bx^2) dx = -\frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(\sqrt{3}\sqrt{-bx}) e^{(3a)}}{288\sqrt{-bb}} + \frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(\sqrt{3}\sqrt{bx}) e^{(-3a)}}{288b^{\frac{3}{2}}} + \frac{xe^{(3bx^2+3a)}}{48b} + \frac{3xe^{(bx^2+a)}}{16b} - \frac{3xe^{(-bx^2-a)}}{16b} - \frac{xe^{(-3bx^2-3a)}}{48b} + \frac{3\sqrt{\pi} \operatorname{erf}(\sqrt{bx}) e^{(-a)}}{32b^{\frac{3}{2}}} - \frac{3\sqrt{\pi} \operatorname{erf}(\sqrt{-bx}) e^a}{32\sqrt{-bb}}$$

[In] integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="maxima")

[Out] $-1/288*\sqrt{3}*\sqrt{\pi}*\operatorname{erf}(\sqrt{3}*\sqrt{-b}*x)*e^{(3*a)}/(\sqrt{-b}*b) + 1/288*\sqrt{3}*\sqrt{\pi}*\operatorname{erf}(\sqrt{3}*\sqrt{b}*x)*e^{(-3*a)}/b^{(3/2)} + 1/48*x*e^{(3*b*x^2 + 3*a)}/b + 3/16*x*e^{(b*x^2 + a)}/b - 3/16*x*e^{(-b*x^2 - a)}/b - 1/48*x*e^{(-3*b*x^2 - 3*a)}/b + 3/32*\sqrt{\pi}*\operatorname{erf}(\sqrt{b}*x)*e^{(-a)}/b^{(3/2)} - 3/32*\sqrt{\pi}*\operatorname{erf}(\sqrt{-b}*x)*e^a/(\sqrt{-b}*b)$

Giac [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.04

$$\int x^2 \cosh^3(a + bx^2) dx = \frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(-\sqrt{3}\sqrt{-bx}) e^{(3a)}}{288\sqrt{-bb}} - \frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(-\sqrt{3}\sqrt{bx}) e^{(-3a)}}{288b^{\frac{3}{2}}} + \frac{xe^{(3bx^2+3a)}}{48b} + \frac{3xe^{(bx^2+a)}}{16b} - \frac{3xe^{(-bx^2-a)}}{16b} - \frac{xe^{(-3bx^2-3a)}}{48b} - \frac{3\sqrt{\pi} \operatorname{erf}(-\sqrt{bx}) e^{(-a)}}{32b^{\frac{3}{2}}} + \frac{3\sqrt{\pi} \operatorname{erf}(-\sqrt{-bx}) e^a}{32\sqrt{-bb}}$$

[In] integrate(x^2*cosh(b*x^2+a)^3,x, algorithm="giac")

[Out] $1/288*\sqrt{3}*\sqrt{\pi}*\operatorname{erf}(-\sqrt{3}*\sqrt{-b}*x)*e^{(3*a)}/(\sqrt{-b}*b) - 1/288*\sqrt{3}*\sqrt{\pi}*\operatorname{erf}(-\sqrt{3}*\sqrt{b}*x)*e^{(-3*a)}/b^{(3/2)} + 1/48*x*e^{(3*b*x^2 + 3*a)}/b + 3/16*x*e^{(b*x^2 + a)}/b - 3/16*x*e^{(-b*x^2 - a)}/b - 1/48*x*e^{(-3*b*x^2 - 3*a)}/b - 3/32*\sqrt{\pi}*\operatorname{erf}(-\sqrt{b}*x)*e^{(-a)}/b^{(3/2)} + 3/32*\sqrt{\pi}*\operatorname{erf}(-\sqrt{-b}*x)*e^a/(\sqrt{-b}*b)$

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh^3(a + bx^2) dx = \int x^2 \cosh(bx^2 + a)^3 dx$$

```
[In] int(x^2*cosh(a + b*x^2)^3,x)
```

```
[Out] int(x^2*cosh(a + b*x^2)^3, x)
```

3.17 $\int x \cosh^3(a + bx^2) dx$

Optimal result	117
Rubi [A] (verified)	117
Mathematica [A] (verified)	118
Maple [A] (verified)	118
Fricas [A] (verification not implemented)	119
Sympy [A] (verification not implemented)	119
Maxima [B] (verification not implemented)	119
Giac [A] (verification not implemented)	120
Mupad [B] (verification not implemented)	120

Optimal result

Integrand size = 12, antiderivative size = 33

$$\int x \cosh^3(a + bx^2) dx = \frac{\sinh(a + bx^2)}{2b} + \frac{\sinh^3(a + bx^2)}{6b}$$

[Out] 1/2*sinh(b*x^2+a)/b+1/6*sinh(b*x^2+a)^3/b

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2713}

$$\int x \cosh^3(a + bx^2) dx = \frac{\sinh^3(a + bx^2)}{6b} + \frac{\sinh(a + bx^2)}{2b}$$

[In] Int[x*Cosh[a + b*x^2]^3,x]

[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(6*b)

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)^(n_.)], x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify

$[(m + 1)/n], 0))$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \text{Subst} \left(\int \cosh^3(a + bx) dx, x, x^2 \right) \\ &= \frac{i \text{Subst} \left(\int (1 - x^2) dx, x, -i \sinh(a + bx^2) \right)}{2b} \\ &= \frac{\sinh(a + bx^2)}{2b} + \frac{\sinh^3(a + bx^2)}{6b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int x \cosh^3(a + bx^2) dx = \frac{\sinh(a + bx^2)}{2b} + \frac{\sinh^3(a + bx^2)}{6b}$$

[In] Integrate[x*Cosh[a + b*x^2]^3,x]

[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(6*b)

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3} + \frac{\cosh(bx^2+a)^2}{3}\right) \sinh(bx^2+a)}{2b}$	28
default	$\frac{\left(\frac{2}{3} + \frac{\cosh(bx^2+a)^2}{3}\right) \sinh(bx^2+a)}{2b}$	28
parallelrisch	$\frac{\sinh(3bx^2+3a)+9\sinh(bx^2+a)}{24b}$	28
risch	$\frac{e^{3bx^2+3a}}{48b} + \frac{3e^{bx^2+a}}{16b} - \frac{3e^{-bx^2-a}}{16b} - \frac{e^{-3bx^2-3a}}{48b}$	63

[In] int(x*cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] 1/2/b*(2/3+1/3*cosh(b*x^2+a)^2)*sinh(b*x^2+a)

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.15

$$\int x \cosh^3(a + bx^2) dx = \frac{\sinh(bx^2 + a)^3 + 3(\cosh(bx^2 + a)^2 + 3)\sinh(bx^2 + a)}{24b}$$

[In] integrate(x*cosh(b*x^2+a)^3,x, algorithm="fricas")

[Out] 1/24*(sinh(b*x^2 + a)^3 + 3*(cosh(b*x^2 + a)^2 + 3)*sinh(b*x^2 + a))/b

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.33

$$\int x \cosh^3(a + bx^2) dx = \begin{cases} -\frac{\sinh^3(a+bx^2)}{3b} + \frac{\sinh(a+bx^2)\cosh^2(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \cosh^3(a)}{2} & \text{otherwise} \end{cases}$$

[In] integrate(x*cosh(b*x**2+a)**3,x)

[Out] Piecewise((-sinh(a + b*x**2)**3/(3*b) + sinh(a + b*x**2)*cosh(a + b*x**2)**2/(2*b), Ne(b, 0)), (x**2*cosh(a)**3/2, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 62 vs. 2(29) = 58.

Time = 0.18 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.88

$$\int x \cosh^3(a + bx^2) dx = \frac{e^{(3bx^2+3a)}}{48b} + \frac{3e^{(bx^2+a)}}{16b} - \frac{3e^{(-bx^2-a)}}{16b} - \frac{e^{(-3bx^2-3a)}}{48b}$$

[In] integrate(x*cosh(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/48*e^(3*b*x^2 + 3*a)/b + 3/16*e^(b*x^2 + a)/b - 3/16*e^(-b*x^2 - a)/b - 1/48*e^(-3*b*x^2 - 3*a)/b

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

$$\int x \cosh^3(a + bx^2) dx = -\frac{(9e^{(2bx^2+2a)} + 1)e^{(-3bx^2-3a)} - e^{(3bx^2+3a)} - 9e^{(bx^2+a)}}{48b}$$

[In] integrate(x*cosh(b*x^2+a)^3,x, algorithm="giac")

[Out] -1/48*((9*e^(2*b*x^2 + 2*a) + 1)*e^(-3*b*x^2 - 3*a) - e^(3*b*x^2 + 3*a) - 9*e^(b*x^2 + a))/b

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

$$\int x \cosh^3(a + bx^2) dx = \frac{\sinh(bx^2 + a)^3 + 3 \sinh(bx^2 + a)}{6b}$$

[In] int(x*cosh(a + b*x^2)^3,x)

[Out] (3*sinh(a + b*x^2) + sinh(a + b*x^2)^3)/(6*b)

3.18 $\int \cosh^3(a + bx^2) dx$

Optimal result	121
Rubi [A] (verified)	121
Mathematica [A] (verified)	123
Maple [A] (verified)	123
Fricas [A] (verification not implemented)	123
Sympy [F]	124
Maxima [A] (verification not implemented)	124
Giac [A] (verification not implemented)	124
Mupad [F(-1)]	125

Optimal result

Integrand size = 10, antiderivative size = 125

$$\int \cosh^3(a + bx^2) dx = \frac{3e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})}{16\sqrt{b}} + \frac{e^{-3a}\sqrt{\frac{\pi}{3}}\operatorname{erf}(\sqrt{3}\sqrt{bx})}{16\sqrt{b}} \\ + \frac{3e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx})}{16\sqrt{b}} + \frac{e^{3a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{16\sqrt{b}}$$

[Out] 1/48*erf(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/exp(3*a)/b^(1/2)+1/48*exp(3*a)*erfi(x*3^(1/2)*b^(1/2))*3^(1/2)*Pi^(1/2)/b^(1/2)+3/16*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+3/16*exp(a)*erfi(x*b^(1/2))*Pi^(1/2)/b^(1/2)

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {5409, 5407, 2235, 2236}

$$\int \cosh^3(a + bx^2) dx = \frac{3\sqrt{\pi}e^{-a}\operatorname{erf}(\sqrt{bx})}{16\sqrt{b}} + \frac{\sqrt{\frac{\pi}{3}}e^{-3a}\operatorname{erf}(\sqrt{3}\sqrt{bx})}{16\sqrt{b}} \\ + \frac{3\sqrt{\pi}e^a\operatorname{erfi}(\sqrt{bx})}{16\sqrt{b}} + \frac{\sqrt{\frac{\pi}{3}}e^{3a}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{16\sqrt{b}}$$

[In] Int[Cosh[a + b*x^2]^3,x]

[Out] (3*sqrt(Pi)*Erf[sqrt[b]*x])/(16*sqrt[b]*E^a) + (sqrt(Pi/3)*Erf[sqrt[3]*sqrt[b]*x])/(16*sqrt[b]*E^(3*a)) + (3*E^a*sqrt(Pi)*Erfi[sqrt[b]*x])/(16*sqrt[b]) + (E^(3*a)*sqrt(Pi/3)*Erfi[sqrt[3]*sqrt[b]*x])/(16*sqrt[b])

Rule 2235

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5409

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_), x_Symbol] := Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[n, 1] && IGtQ[p, 1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4} \cosh(a + bx^2) + \frac{1}{4} \cosh(3a + 3bx^2) \right) dx \\
&= \frac{1}{4} \int \cosh(3a + 3bx^2) dx + \frac{3}{4} \int \cosh(a + bx^2) dx \\
&= \frac{1}{8} \int e^{-3a-3bx^2} dx + \frac{1}{8} \int e^{3a+3bx^2} dx + \frac{3}{8} \int e^{-a-bx^2} dx + \frac{3}{8} \int e^{a+bx^2} dx \\
&= \frac{3e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})}{16\sqrt{b}} + \frac{e^{-3a}\sqrt{\frac{\pi}{3}}\operatorname{erf}(\sqrt{3}\sqrt{bx})}{16\sqrt{b}} + \frac{3e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx})}{16\sqrt{b}} + \frac{e^{3a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}(\sqrt{3}\sqrt{bx})}{16\sqrt{b}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.09

$$\int \cosh^3(a + bx^2) dx = \frac{\sqrt{\frac{\pi}{3}} \left(3\sqrt{3} \cosh(a) \operatorname{erfi}(\sqrt{bx}) + \cosh(3a) \operatorname{erfi}(\sqrt{3}\sqrt{bx}) + 3\sqrt{3} \operatorname{erf}(\sqrt{bx}) (\cosh(a) - \sinh(a)) + 3\sqrt{3} \operatorname{erfi}(\sqrt{bx}) \right)}{16\sqrt{b}}$$

`[In] Integrate[Cosh[a + b*x^2]^3,x]`

```
[Out] (Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[a]*Erfi[Sqrt[b]*x] + Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] + 3*Sqrt[3]*Erf[Sqrt[b]*x]*(Cosh[a] - Sinh[a]) + 3*Sqrt[3]*Erfi[Sqrt[b]*x]*Sinh[a] + Erf[Sqrt[3]*Sqrt[b]*x]*(Cosh[3*a] - Sinh[3*a]) + Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a]))/(16*Sqrt[b])
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.69

method	result	size
risch	$\frac{e^{-3a} \sqrt{\pi} \sqrt{3} \operatorname{erf}(x\sqrt{3}\sqrt{b})}{48\sqrt{b}} + \frac{3 \operatorname{erf}(x\sqrt{b}) \sqrt{\pi} e^{-a}}{16\sqrt{b}} + \frac{e^{3a} \sqrt{\pi} \operatorname{erf}(\sqrt{-3b}x)}{16\sqrt{-3b}} + \frac{3e^a \sqrt{\pi} \operatorname{erf}(\sqrt{-b}x)}{16\sqrt{-b}}$	86

`[In] int(cosh(b*x^2+a)^3,x,method=_RETURNVERBOSE)`

```
[Out] 1/48/exp(a)^3*Pi^(1/2)*3^(1/2)/b^(1/2)*erf(x*3^(1/2)*b^(1/2))+3/16*erf(x*b^(1/2))*Pi^(1/2)/exp(a)/b^(1/2)+1/16*exp(a)^3*Pi^(1/2)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)+3/16*exp(a)*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)
```

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.90

$$\int \cosh^3(a + bx^2) dx = \frac{\sqrt{3}\sqrt{\pi}\sqrt{-b}(\cosh(3a) + \sinh(3a)) \operatorname{erf}(\sqrt{3}\sqrt{-bx}) - \sqrt{3}\sqrt{\pi}\sqrt{b}(\cosh(3a) - \sinh(3a)) \operatorname{erf}(\sqrt{3}\sqrt{bx})}{48b}$$

`[In] integrate(cosh(b*x^2+a)^3,x, algorithm="fricas")`

```
[Out] -1/48*(sqrt(3)*sqrt(pi)*sqrt(-b)*(cosh(3*a) + sinh(3*a))*erf(sqrt(3)*sqrt(-b)*x) - sqrt(3)*sqrt(pi)*sqrt(b)*(cosh(3*a) - sinh(3*a))*erf(sqrt(3)*sqrt(b
```

) * x) + 9 * sqrt(pi) * sqrt(-b) * (cosh(a) + sinh(a)) * erf(sqrt(-b) * x) - 9 * sqrt(pi) * sqrt(b) * (cosh(a) - sinh(a)) * erf(sqrt(b) * x)) / b

Sympy [F]

$$\int \cosh^3(a + bx^2) dx = \int \cosh^3(a + bx^2) dx$$

[In] integrate(cosh(b*x**2+a)**3,x)

[Out] Integral(cosh(a + b*x**2)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.73

$$\int \cosh^3(a + bx^2) dx = \frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(\sqrt{3}\sqrt{-bx}) e^{3a}}{48\sqrt{-b}} + \frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(\sqrt{3}\sqrt{bx}) e^{-3a}}{48\sqrt{b}} + \frac{3\sqrt{\pi} \operatorname{erf}(\sqrt{bx}) e^{-a}}{16\sqrt{b}} + \frac{3\sqrt{\pi} \operatorname{erf}(\sqrt{-bx}) e^a}{16\sqrt{-b}}$$

[In] integrate(cosh(b*x^2+a)^3,x, algorithm="maxima")

[Out] 1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) + 1/48*sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqrt(b)*x)*e^(-a)/sqrt(b) + 3/16*sqrt(pi)*erf(sqrt(-b)*x)*e^a/sqrt(-b)

Giac [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.76

$$\int \cosh^3(a + bx^2) dx = -\frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(-\sqrt{3}\sqrt{-bx}) e^{3a}}{48\sqrt{-b}} - \frac{\sqrt{3}\sqrt{\pi} \operatorname{erf}(-\sqrt{3}\sqrt{bx}) e^{-3a}}{48\sqrt{b}} - \frac{3\sqrt{\pi} \operatorname{erf}(-\sqrt{bx}) e^{-a}}{16\sqrt{b}} - \frac{3\sqrt{\pi} \operatorname{erf}(-\sqrt{-bx}) e^a}{16\sqrt{-b}}$$

[In] integrate(cosh(b*x^2+a)^3,x, algorithm="giac")

[Out] -1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(-b)*x)*e^(3*a)/sqrt(-b) - 1/48*sqrt(3)*sqrt(pi)*erf(-sqrt(3)*sqrt(b)*x)*e^(-3*a)/sqrt(b) - 3/16*sqrt(pi)*erf(-sqrt(b)*x)*e^(-a)/sqrt(b) - 3/16*sqrt(pi)*erf(-sqrt(-b)*x)*e^a/sqrt(-b)

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(a + bx^2) dx = \int \cosh(bx^2 + a)^3 dx$$

```
[In] int(cosh(a + b*x^2)^3,x)
```

```
[Out] int(cosh(a + b*x^2)^3, x)
```

3.19 $\int \frac{\cosh^3(a+bx^2)}{x} dx$

Optimal result	126
Rubi [A] (verified)	126
Mathematica [A] (verified)	127
Maple [A] (verified)	128
Fricas [A] (verification not implemented)	128
Sympy [F]	128
Maxima [A] (verification not implemented)	129
Giac [A] (verification not implemented)	129
Mupad [F(-1)]	129

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{\cosh^3(a+bx^2)}{x} dx = \frac{3}{8} \cosh(a) \operatorname{Chi}(bx^2) + \frac{1}{8} \cosh(3a) \operatorname{Chi}(3bx^2) \\ + \frac{3}{8} \sinh(a) \operatorname{Shi}(bx^2) + \frac{1}{8} \sinh(3a) \operatorname{Shi}(3bx^2)$$

[Out] 3/8*Chi(b*x^2)*cosh(a)+1/8*Chi(3*b*x^2)*cosh(3*a)+3/8*Shi(b*x^2)*sinh(a)+1/8*Shi(3*b*x^2)*sinh(3*a)

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5449, 5427, 5425, 5424}

$$\int \frac{\cosh^3(a+bx^2)}{x} dx = \frac{3}{8} \cosh(a) \operatorname{Chi}(bx^2) + \frac{1}{8} \cosh(3a) \operatorname{Chi}(3bx^2) \\ + \frac{3}{8} \sinh(a) \operatorname{Shi}(bx^2) + \frac{1}{8} \sinh(3a) \operatorname{Shi}(3bx^2)$$

[In] Int[Cosh[a + b*x^2]^3/x,x]

[Out] (3*Cosh[a]*CoshIntegral[b*x^2])/8 + (Cosh[3*a]*CoshIntegral[3*b*x^2])/8 + (3*Sinh[a]*SinhIntegral[b*x^2])/8 + (Sinh[3*a]*SinhIntegral[3*b*x^2])/8

Rule 5424

Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinhIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CoshIntegral[d*x^n]/n, x]
  /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_))^(m_.),
x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x]
  /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3 \cosh(a + bx^2)}{4x} + \frac{\cosh(3a + 3bx^2)}{4x} \right) dx \\
&= \frac{1}{4} \int \frac{\cosh(3a + 3bx^2)}{x} dx + \frac{3}{4} \int \frac{\cosh(a + bx^2)}{x} dx \\
&= \frac{1}{4} (3 \cosh(a)) \int \frac{\cosh(bx^2)}{x} dx + \frac{1}{4} \cosh(3a) \int \frac{\cosh(3bx^2)}{x} dx \\
&\quad + \frac{1}{4} (3 \sinh(a)) \int \frac{\sinh(bx^2)}{x} dx + \frac{1}{4} \sinh(3a) \int \frac{\sinh(3bx^2)}{x} dx \\
&= \frac{3}{8} \cosh(a) \text{Chi}(bx^2) + \frac{1}{8} \cosh(3a) \text{Chi}(3bx^2) + \frac{3}{8} \sinh(a) \text{Shi}(bx^2) + \frac{1}{8} \sinh(3a) \text{Shi}(3bx^2)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.89

$$\int \frac{\cosh^3(a + bx^2)}{x} dx = \frac{1}{8} (3 \cosh(a) \text{Chi}(bx^2) + \cosh(3a) \text{Chi}(3bx^2) + 3 \sinh(a) \text{Shi}(bx^2) + \sinh(3a) \text{Shi}(3bx^2))$$

```
[In] Integrate[Cosh[a + b*x^2]^3/x, x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x^2] + Cosh[3*a]*CoshIntegral[3*b*x^2] + 3*Sinh[a]
*SinhIntegral[b*x^2] + Sinh[3*a]*SinhIntegral[3*b*x^2])/8
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.25

method	result	size
risch	$-\frac{e^{6a}e^{-3a} \operatorname{Ei}_1(-3bx^2)}{16} - \frac{3e^{4a}e^{-3a} \operatorname{Ei}_1(-bx^2)}{16} - \frac{3e^{2a}e^{-3a} \operatorname{Ei}_1(bx^2)}{16} - \frac{e^{-3a} \operatorname{Ei}_1(3bx^2)}{16}$	69

[In] `int(cosh(b*x^2+a)^3/x,x,method=_RETURNVERBOSE)`

[Out] $-1/16*\exp(6*a)*\exp(-3*a)*\operatorname{Ei}(1,-3*b*x^2)-3/16*\exp(4*a)*\exp(-3*a)*\operatorname{Ei}(1,-b*x^2)-3/16*\exp(2*a)*\exp(-3*a)*\operatorname{Ei}(1,b*x^2)-1/16*\exp(-3*a)*\operatorname{Ei}(1,3*b*x^2)$

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int \frac{\cosh^3(a+bx^2)}{x} dx = \frac{1}{16} (\operatorname{Ei}(3bx^2) + \operatorname{Ei}(-3bx^2)) \cosh(3a) + \frac{3}{16} (\operatorname{Ei}(bx^2) + \operatorname{Ei}(-bx^2)) \cosh(a) + \frac{1}{16} (\operatorname{Ei}(3bx^2) - \operatorname{Ei}(-3bx^2)) \sinh(3a) + \frac{3}{16} (\operatorname{Ei}(bx^2) - \operatorname{Ei}(-bx^2)) \sinh(a)$$

[In] `integrate(cosh(b*x^2+a)^3/x,x, algorithm="fricas")`

[Out] $1/16*(\operatorname{Ei}(3*b*x^2) + \operatorname{Ei}(-3*b*x^2))*\cosh(3*a) + 3/16*(\operatorname{Ei}(b*x^2) + \operatorname{Ei}(-b*x^2))*\cosh(a) + 1/16*(\operatorname{Ei}(3*b*x^2) - \operatorname{Ei}(-3*b*x^2))*\sinh(3*a) + 3/16*(\operatorname{Ei}(b*x^2) - \operatorname{Ei}(-b*x^2))*\sinh(a)$

Sympy [F]

$$\int \frac{\cosh^3(a+bx^2)}{x} dx = \int \frac{\cosh^3(a+bx^2)}{x} dx$$

[In] `integrate(cosh(b*x**2+a)**3/x,x)`

[Out] `Integral(cosh(a + b*x**2)**3/x, x)`

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^3(a + bx^2)}{x} dx = \frac{1}{16} \operatorname{Ei}(3bx^2) e^{(3a)} + \frac{3}{16} \operatorname{Ei}(-bx^2) e^{(-a)} \\ + \frac{1}{16} \operatorname{Ei}(-3bx^2) e^{(-3a)} + \frac{3}{16} \operatorname{Ei}(bx^2) e^a$$

[In] integrate(cosh(b*x^2+a)^3/x,x, algorithm="maxima")

[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) + 1/16*Ei(-3*b*x^2)*e^(-3*a) + 3/16*Ei(b*x^2)*e^a

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^3(a + bx^2)}{x} dx = \frac{1}{16} \operatorname{Ei}(3bx^2) e^{(3a)} + \frac{3}{16} \operatorname{Ei}(-bx^2) e^{(-a)} \\ + \frac{1}{16} \operatorname{Ei}(-3bx^2) e^{(-3a)} + \frac{3}{16} \operatorname{Ei}(bx^2) e^a$$

[In] integrate(cosh(b*x^2+a)^3/x,x, algorithm="giac")

[Out] 1/16*Ei(3*b*x^2)*e^(3*a) + 3/16*Ei(-b*x^2)*e^(-a) + 1/16*Ei(-3*b*x^2)*e^(-3*a) + 3/16*Ei(b*x^2)*e^a

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a + bx^2)}{x} dx = \int \frac{\cosh(bx^2 + a)^3}{x} dx$$

[In] int(cosh(a + b*x^2)^3/x,x)

[Out] int(cosh(a + b*x^2)^3/x, x)

3.20 $\int \frac{\cosh^3(a+bx^2)}{x^2} dx$

Optimal result	130
Rubi [A] (verified)	130
Mathematica [A] (verified)	132
Maple [A] (verified)	132
Fricas [B] (verification not implemented)	133
Sympy [F]	133
Maxima [A] (verification not implemented)	134
Giac [F]	134
Mupad [F(-1)]	134

Optimal result

Integrand size = 14, antiderivative size = 136

$$\int \frac{\cosh^3(a+bx^2)}{x^2} dx = -\frac{\cosh^3(a+bx^2)}{x} - \frac{3}{8}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx})$$

$$- \frac{1}{8}\sqrt{b}e^{-3a}\sqrt{3\pi}\operatorname{erf}(\sqrt{3}\sqrt{bx})$$

$$+ \frac{3}{8}\sqrt{b}e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx}) + \frac{1}{8}\sqrt{b}e^{3a}\sqrt{3\pi}\operatorname{erfi}(\sqrt{3}\sqrt{bx})$$

[Out] $-\cosh(b*x^2+a)^3/x-3/8*\operatorname{erf}(x*b^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}/\exp(a)+3/8*\exp(a)*\operatorname{erfi}(x*b^{(1/2)})*b^{(1/2)}*\pi^{(1/2)}-1/8*\operatorname{erf}(x*3^{(1/2)}*b^{(1/2)})*b^{(1/2)}*3^{(1/2)}*\pi^{(1/2)}/\exp(3*a)+1/8*\exp(3*a)*\operatorname{erfi}(x*3^{(1/2)}*b^{(1/2)})*b^{(1/2)}*3^{(1/2)}*\pi^{(1/2)}$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {5439, 5737, 5406, 2235, 2236}

$$\int \frac{\cosh^3(a+bx^2)}{x^2} dx = -\frac{3}{8}\sqrt{\pi}e^{-a}\sqrt{b}\operatorname{berf}(\sqrt{bx})$$

$$- \frac{1}{8}\sqrt{3\pi}e^{-3a}\sqrt{b}\operatorname{berf}(\sqrt{3}\sqrt{bx}) + \frac{3}{8}\sqrt{\pi}e^a\sqrt{b}\operatorname{berfi}(\sqrt{bx})$$

$$+ \frac{1}{8}\sqrt{3\pi}e^{3a}\sqrt{b}\operatorname{berfi}(\sqrt{3}\sqrt{bx}) - \frac{\cosh^3(a+bx^2)}{x}$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b*x^2]^3/x^2, x]$

[Out] $-(\text{Cosh}[a + b*x^2]^3/x) - (3*\text{Sqrt}[b]*\text{Sqrt}[\text{Pi}]*\text{Erf}[\text{Sqrt}[b]*x])/(8*E^a) - (\text{Sqrt}[b]*\text{Sqrt}[3*\text{Pi}]*\text{Erf}[\text{Sqrt}[3]*\text{Sqrt}[b]*x])/(8*E^{(3*a)}) + (3*\text{Sqrt}[b]*E^a*\text{Sqrt}[\text{Pi}]*\text{Erfi}[\text{Sqrt}[b]*x])/8 + (\text{Sqrt}[b]*E^{(3*a)}*\text{Sqrt}[3*\text{Pi}]*\text{Erfi}[\text{Sqrt}[3]*\text{Sqrt}[b]*x])/8$

Rule 2235

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x_Symbol] \rightarrow \text{Simp}[F^a*\text{Sqrt}[\text{Pi}]*(\text{Erfi}[(c + d*x)*\text{Rt}[b*\text{Log}[F], 2]]/(2*d*\text{Rt}[b*\text{Log}[F], 2]))], x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{PosQ}[b]$

Rule 2236

$\text{Int}[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2}), x_Symbol] \rightarrow \text{Simp}[F^a*\text{Sqrt}[\text{Pi}]*(\text{Erf}[(c + d*x)*\text{Rt}[(-b)*\text{Log}[F], 2]]/(2*d*\text{Rt}[(-b)*\text{Log}[F], 2]))], x] /; \text{FreeQ}\{F, a, b, c, d\}, x] \&\& \text{NegQ}[b]$

Rule 5406

$\text{Int}[\text{Sinh}[(c_.) + (d_.)*(x_)]^{(n_.)}, x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[E^{(c + d*x^n)}, x], x] - \text{Dist}[1/2, \text{Int}[E^{(-c - d*x^n)}, x], x] /; \text{FreeQ}\{c, d\}, x] \&\& \text{IGtQ}[n, 1]$

Rule 5439

$\text{Int}[\text{Cosh}[(a_.) + (b_.)*(x_)]^{(n_.)}]^{(p_.)}*(x_)]^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[-\text{Cosh}[a + b*x^n]^{p/((n-1)*x^{(n-1)})}, x] + \text{Dist}[b*n*(p/(n-1)), \text{Int}[\text{Cosh}[a + b*x^n]^{(p-1)}*\text{Sinh}[a + b*x^n], x], x] /; \text{FreeQ}\{a, b\}, x] \&\& \text{IntegersQ}[n, p] \&\& \text{EqQ}[m + n, 0] \&\& \text{GtQ}[p, 1] \&\& \text{NeQ}[n, 1]$

Rule 5737

$\text{Int}[\text{Cosh}[w_]^{(q_.)}*\text{Sinh}[v_]^{(p_.)}, x_Symbol] \rightarrow \text{Int}[\text{ExpandTrigReduce}[\text{Sinh}[v]^{p}*\text{Cosh}[w]^{q}, x], x] /; \text{IGtQ}[p, 0] \&\& \text{IGtQ}[q, 0] \&\& ((\text{PolynomialQ}[v, x] \&\& \text{PolynomialQ}[w, x]) \|\ (\text{BinomialQ}\{v, w\}, x] \&\& \text{IndependentQ}[\text{Cancel}[v/w], x]))$

Rubi steps

$$\begin{aligned} \text{integral} &= -\frac{\cosh^3(a + bx^2)}{x} + (6b) \int \cosh^2(a + bx^2) \sinh(a + bx^2) dx \\ &= -\frac{\cosh^3(a + bx^2)}{x} + (6b) \int \left(\frac{1}{4} \sinh(a + bx^2) + \frac{1}{4} \sinh(3a + 3bx^2) \right) dx \\ &= -\frac{\cosh^3(a + bx^2)}{x} + \frac{1}{2}(3b) \int \sinh(a + bx^2) dx + \frac{1}{2}(3b) \int \sinh(3a + 3bx^2) dx \end{aligned}$$

$$\begin{aligned}
&= -\frac{\cosh^3(a+bx^2)}{x} - \frac{1}{4}(3b) \int e^{-3a-3bx^2} dx - \frac{1}{4}(3b) \int e^{-a-bx^2} dx \\
&\quad + \frac{1}{4}(3b) \int e^{a+bx^2} dx + \frac{1}{4}(3b) \int e^{3a+3bx^2} dx \\
&= -\frac{\cosh^3(a+bx^2)}{x} - \frac{3}{8}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx}) - \frac{1}{8}\sqrt{b}e^{-3a}\sqrt{3\pi}\operatorname{erf}(\sqrt{3}\sqrt{bx}) \\
&\quad + \frac{3}{8}\sqrt{b}e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx}) + \frac{1}{8}\sqrt{b}e^{3a}\sqrt{3\pi}\operatorname{erfi}(\sqrt{3}\sqrt{bx})
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.50

$$\int \frac{\cosh^3(a+bx^2)}{x^2} dx$$

$$-6 \cosh(a+bx^2) - 2 \cosh(3(a+bx^2)) + 3\sqrt{b}\sqrt{\pi}x \cosh(a)\operatorname{erfi}(\sqrt{bx}) + \sqrt{b}\sqrt{3\pi}x \cosh(3a)\operatorname{erfi}(\sqrt{3}\sqrt{bx}) +$$

[In] Integrate[Cosh[a + b*x^2]^3/x^2,x]

[Out] (-6*Cosh[a + b*x^2] - 2*Cosh[3*(a + b*x^2)]) + 3*Sqrt[b]*Sqrt[Pi]*x*Cosh[a]*Erfi[Sqrt[b]*x] + Sqrt[b]*Sqrt[3*Pi]*x*Cosh[3*a]*Erfi[Sqrt[3]*Sqrt[b]*x] + 3*Sqrt[b]*Sqrt[Pi]*x*Erfi[Sqrt[b]*x]*Sinh[a] + 3*Sqrt[b]*Sqrt[Pi]*x*Erf[Sqrt[b]*x]*(-Cosh[a] + Sinh[a]) + Sqrt[b]*Sqrt[3*Pi]*x*Erfi[Sqrt[3]*Sqrt[b]*x]*Sinh[3*a] + Sqrt[b]*Sqrt[3*Pi]*x*Erf[Sqrt[3]*Sqrt[b]*x]*(-Cosh[3*a] + Sinh[3*a]))/(8*x)

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.10

method	result
risch	$-\frac{e^{-3a}e^{-3bx^2}}{8x} - \frac{e^{-3a}\sqrt{b}\sqrt{\pi}\sqrt{3}\operatorname{erf}(x\sqrt{3}\sqrt{b})}{8} - \frac{3e^{-a}e^{-bx^2}}{8x} - \frac{3\operatorname{erf}(x\sqrt{b})\sqrt{b}\sqrt{\pi}e^{-a}}{8} - \frac{e^{3a}e^{3bx^2}}{8x} + \frac{3e^{3a}b\sqrt{\pi}\operatorname{erf}(\sqrt{-3b}x)}{8\sqrt{-3b}}$

[In] int(cosh(b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)

[Out] -1/8/exp(a)^3/x*exp(-3*b*x^2)-1/8/exp(a)^3*b^(1/2)*Pi^(1/2)*3^(1/2)*erf(x*3^(1/2)*b^(1/2))-3/8/exp(a)/x*exp(-b*x^2)-3/8*erf(x*b^(1/2))*b^(1/2)*Pi^(1/2)/exp(a)-1/8*exp(a)^3/x*exp(3*b*x^2)+3/8*exp(a)^3*b*Pi^(1/2)/(-3*b)^(1/2)*erf((-3*b)^(1/2)*x)-3/8*exp(a)*exp(b*x^2)/x+3/8*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)*x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 891 vs. 2(98) = 196.

Time = 0.26 (sec) , antiderivative size = 891, normalized size of antiderivative = 6.55

$$\int \frac{\cosh^3(a + bx^2)}{x^2} dx = \text{Too large to display}$$

[In] integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="fricas")

[Out]
$$\begin{aligned} & -1/8*(\cosh(b*x^2 + a)^6 + 6*\cosh(b*x^2 + a)*\sinh(b*x^2 + a)^5 + \sinh(b*x^2 + a)^6 + 3*(5*\cosh(b*x^2 + a)^2 + 1)*\sinh(b*x^2 + a)^4 + 3*\cosh(b*x^2 + a)^4 + 4*(5*\cosh(b*x^2 + a)^3 + 3*\cosh(b*x^2 + a))*\sinh(b*x^2 + a)^3 + \sqrt{3}*\sqrt{\pi}*(x*\cosh(b*x^2 + a)^3*\cosh(3*a) + x*\cosh(b*x^2 + a)^3*\sinh(3*a) + (x*\cosh(3*a) + x*\sinh(3*a))*\sinh(b*x^2 + a)^3 + 3*(x*\cosh(b*x^2 + a)*\cosh(3*a) + x*\cosh(b*x^2 + a)*\sinh(3*a))*\sinh(b*x^2 + a)^2 + 3*(x*\cosh(b*x^2 + a)^2*\cosh(3*a) + x*\cosh(b*x^2 + a)^2*\sinh(3*a))*\sinh(b*x^2 + a)*\sqrt{-b}*\text{erf}(\sqrt{3}*\sqrt{-b}*x) + \sqrt{3}*\sqrt{\pi}*(x*\cosh(b*x^2 + a)^3*\cosh(3*a) - x*\cosh(b*x^2 + a)^3*\sinh(3*a) + (x*\cosh(3*a) - x*\sinh(3*a))*\sinh(b*x^2 + a)^3 + 3*(x*\cosh(b*x^2 + a)*\cosh(3*a) - x*\cosh(b*x^2 + a)*\sinh(3*a))*\sinh(b*x^2 + a)^2 + 3*(x*\cosh(b*x^2 + a)^2*\cosh(3*a) - x*\cosh(b*x^2 + a)^2*\sinh(3*a))*\sinh(b*x^2 + a)*\sqrt{b}*\text{erf}(\sqrt{3}*\sqrt{b}*x) + 3*\sqrt{\pi}*(x*\cosh(b*x^2 + a)^3*\cosh(a) + x*\cosh(b*x^2 + a)^3*\sinh(a) + (x*\cosh(a) + x*\sinh(a))*\sinh(b*x^2 + a)^3 + 3*(x*\cosh(b*x^2 + a)*\cosh(a) + x*\cosh(b*x^2 + a)*\sinh(a))*\sinh(b*x^2 + a)^2 + 3*(x*\cosh(b*x^2 + a)^2*\cosh(a) + x*\cosh(b*x^2 + a)^2*\sinh(a))*\sinh(b*x^2 + a)*\sqrt{-b}*\text{erf}(\sqrt{-b}*x) + 3*\sqrt{\pi}*(x*\cosh(b*x^2 + a)^3*\cosh(a) - x*\cosh(b*x^2 + a)^3*\sinh(a) + (x*\cosh(a) - x*\sinh(a))*\sinh(b*x^2 + a)^3 + 3*(x*\cosh(b*x^2 + a)*\cosh(a) - x*\cosh(b*x^2 + a)*\sinh(a))*\sinh(b*x^2 + a)^2 + 3*(x*\cosh(b*x^2 + a)^2*\cosh(a) - x*\cosh(b*x^2 + a)^2*\sinh(a))*\sinh(b*x^2 + a)*\sqrt{b}*\text{erf}(\sqrt{b}*x) + 3*(5*\cosh(b*x^2 + a)^4 + 6*\cosh(b*x^2 + a)^2 + 1)*\sinh(b*x^2 + a)^2 + 3*\cosh(b*x^2 + a)^2 + 6*(\cosh(b*x^2 + a)^5 + 2*\cosh(b*x^2 + a)^3 + \cosh(b*x^2 + a))*\sinh(b*x^2 + a) + 1)/(x*\cosh(b*x^2 + a)^3 + 3*x*\cosh(b*x^2 + a)^2*\sinh(b*x^2 + a) + 3*x*\cosh(b*x^2 + a)*\sinh(b*x^2 + a)^2 + x*\sinh(b*x^2 + a)^3) \end{aligned}$$

Sympy [F]

$$\int \frac{\cosh^3(a + bx^2)}{x^2} dx = \int \frac{\cosh^3(a + bx^2)}{x^2} dx$$

[In] integrate(cosh(b*x**2+a)**3/x**2,x)

[Out] Integral(cosh(a + b*x**2)**3/x**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^3(a + bx^2)}{x^2} dx = -\frac{\sqrt{3}\sqrt{bx^2}e^{(-3a)}\Gamma(-\frac{1}{2}, 3bx^2)}{16x} - \frac{\sqrt{3}\sqrt{-bx^2}e^{(3a)}\Gamma(-\frac{1}{2}, -3bx^2)}{16x} \\ - \frac{3\sqrt{bx^2}e^{(-a)}\Gamma(-\frac{1}{2}, bx^2)}{16x} - \frac{3\sqrt{-bx^2}e^a\Gamma(-\frac{1}{2}, -bx^2)}{16x}$$

[In] integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="maxima")

```
[Out] -1/16*sqrt(3)*sqrt(b*x^2)*e^(-3*a)*gamma(-1/2, 3*b*x^2)/x - 1/16*sqrt(3)*sqrt(-b*x^2)*e^(3*a)*gamma(-1/2, -3*b*x^2)/x - 3/16*sqrt(b*x^2)*e^(-a)*gamma(-1/2, b*x^2)/x - 3/16*sqrt(-b*x^2)*e^a*gamma(-1/2, -b*x^2)/x
```

Giac [F]

$$\int \frac{\cosh^3(a + bx^2)}{x^2} dx = \int \frac{\cosh(bx^2 + a)^3}{x^2} dx$$

[In] integrate(cosh(b*x^2+a)^3/x^2,x, algorithm="giac")

[Out] integrate(cosh(b*x^2 + a)^3/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a + bx^2)}{x^2} dx = \int \frac{\cosh(bx^2 + a)^3}{x^2} dx$$

[In] int(cosh(a + b*x^2)^3/x^2,x)

[Out] int(cosh(a + b*x^2)^3/x^2, x)

3.21 $\int \frac{\cosh^3(a+bx^2)}{x^3} dx$

Optimal result	135
Rubi [A] (verified)	135
Mathematica [A] (verified)	137
Maple [A] (verified)	138
Fricas [B] (verification not implemented)	138
Sympy [F]	138
Maxima [A] (verification not implemented)	139
Giac [B] (verification not implemented)	139
Mupad [F(-1)]	139

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int \frac{\cosh^3(a+bx^2)}{x^3} dx = -\frac{3 \cosh(a+bx^2)}{8x^2} - \frac{\cosh(3(a+bx^2))}{8x^2} + \frac{3}{8}b \operatorname{Chi}(bx^2) \sinh(a) + \frac{3}{8}b \operatorname{Chi}(3bx^2) \sinh(3a) + \frac{3}{8}b \cosh(a) \operatorname{Shi}(bx^2) + \frac{3}{8}b \cosh(3a) \operatorname{Shi}(3bx^2)$$

[Out] $-3/8*\cosh(b*x^2+a)/x^2-1/8*\cosh(3*b*x^2+3*a)/x^2+3/8*b*\cosh(a)*\operatorname{Shi}(b*x^2)+3/8*b*\cosh(3*a)*\operatorname{Shi}(3*b*x^2)+3/8*b*\operatorname{Chi}(b*x^2)*\sinh(a)+3/8*b*\operatorname{Chi}(3*b*x^2)*\sinh(3*a)$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {5449, 5429, 3378, 3384, 3379, 3382}

$$\int \frac{\cosh^3(a+bx^2)}{x^3} dx = \frac{3}{8}b \sinh(a) \operatorname{Chi}(bx^2) + \frac{3}{8}b \sinh(3a) \operatorname{Chi}(3bx^2) + \frac{3}{8}b \cosh(a) \operatorname{Shi}(bx^2) + \frac{3}{8}b \cosh(3a) \operatorname{Shi}(3bx^2) - \frac{3 \cosh(a+bx^2)}{8x^2} - \frac{\cosh(3(a+bx^2))}{8x^2}$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b*x^2]^3/x^3, x]$

[Out] $(-3*\operatorname{Cosh}[a + b*x^2])/(8*x^2) - \operatorname{Cosh}[3*(a + b*x^2)]/(8*x^2) + (3*b*\operatorname{CoshIntegral}[b*x^2]*\operatorname{Sinh}[a])/8 + (3*b*\operatorname{CoshIntegral}[3*b*x^2]*\operatorname{Sinh}[3*a])/8 + (3*b*\operatorname{Cosh}[a]*\operatorname{SinhIntegral}[b*x^2])/8 + (3*b*\operatorname{Cosh}[3*a]*\operatorname{SinhIntegral}[3*b*x^2])/8$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5449

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_.),
x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 1] && IGtQ[n, 0]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{3 \cosh(a + bx^2)}{4x^3} + \frac{\cosh(3a + 3bx^2)}{4x^3} \right) dx \\ &= \frac{1}{4} \int \frac{\cosh(3a + 3bx^2)}{x^3} dx + \frac{3}{4} \int \frac{\cosh(a + bx^2)}{x^3} dx \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{8} \text{Subst} \left(\int \frac{\cosh(3a + 3bx)}{x^2} dx, x, x^2 \right) + \frac{3}{8} \text{Subst} \left(\int \frac{\cosh(a + bx)}{x^2} dx, x, x^2 \right) \\
&= -\frac{3 \cosh(a + bx^2)}{8x^2} - \frac{\cosh(3(a + bx^2))}{8x^2} + \frac{1}{8} (3b) \text{Subst} \left(\int \frac{\sinh(a + bx)}{x} dx, x, x^2 \right) \\
&\quad + \frac{1}{8} (3b) \text{Subst} \left(\int \frac{\sinh(3a + 3bx)}{x} dx, x, x^2 \right) \\
&= -\frac{3 \cosh(a + bx^2)}{8x^2} - \frac{\cosh(3(a + bx^2))}{8x^2} + \frac{1}{8} (3b \cosh(a)) \text{Subst} \left(\int \frac{\sinh(bx)}{x} dx, x, x^2 \right) \\
&\quad + \frac{1}{8} (3b \cosh(3a)) \text{Subst} \left(\int \frac{\sinh(3bx)}{x} dx, x, x^2 \right) \\
&\quad + \frac{1}{8} (3b \sinh(a)) \text{Subst} \left(\int \frac{\cosh(bx)}{x} dx, x, x^2 \right) \\
&\quad + \frac{1}{8} (3b \sinh(3a)) \text{Subst} \left(\int \frac{\cosh(3bx)}{x} dx, x, x^2 \right) \\
&= -\frac{3 \cosh(a + bx^2)}{8x^2} - \frac{\cosh(3(a + bx^2))}{8x^2} + \frac{3}{8} b \text{Chi}(bx^2) \sinh(a) \\
&\quad + \frac{3}{8} b \text{Chi}(3bx^2) \sinh(3a) + \frac{3}{8} b \cosh(a) \text{Shi}(bx^2) + \frac{3}{8} b \cosh(3a) \text{Shi}(3bx^2)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.01

$$\int \frac{\cosh^3(a + bx^2)}{x^3} dx = \frac{-3 \cosh(a + bx^2) - \cosh(3(a + bx^2)) + 3bx^2 \text{Chi}(bx^2) \sinh(a) + 3bx^2 \text{Chi}(3bx^2) \sinh(3a) + 3bx^2 \cosh(a) \text{Shi}(bx^2) + 3bx^2 \cosh(3a) \text{Shi}(3bx^2)}{8x^2}$$

[In] Integrate[Cosh[a + b*x^2]^3/x^3,x]

[Out] (-3*Cosh[a + b*x^2] - Cosh[3*(a + b*x^2)] + 3*b*x^2*CoshIntegral[b*x^2]*Sinh[a] + 3*b*x^2*CoshIntegral[3*b*x^2]*Sinh[3*a] + 3*b*x^2*Cosh[a]*SinhIntegral[b*x^2] + 3*b*x^2*Cosh[3*a]*SinhIntegral[3*b*x^2])/(8*x^2)

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.33

method	result
risch	$-\frac{-3e^{-3a} \operatorname{Ei}_1(3bx^2)bx^2 - 3e^{-a} \operatorname{Ei}_1(bx^2)bx^2 + 3e^{3a} \operatorname{Ei}_1(-3bx^2)bx^2 + 3 \operatorname{Ei}_1(-bx^2)e^a bx^2 + e^{-3bx^2-3a} + 3e^{-bx^2-a} + e^{3bx^2+3a} + 3e^b}{16x^2}$

[In] `int(cosh(b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)`

[Out]
$$-1/16*(-3*\exp(-3*a)*\operatorname{Ei}(1,3*b*x^2)*b*x^2 - 3*\exp(-a)*\operatorname{Ei}(1,b*x^2)*b*x^2 + 3*\exp(3*a)*\operatorname{Ei}(1,-3*b*x^2)*b*x^2 + 3*\operatorname{Ei}(1,-b*x^2)*\exp(a)*b*x^2 + \exp(-3*b*x^2-3*a) + 3*\exp(-b*x^2-a) + \exp(3*b*x^2+3*a) + 3*\exp(b*x^2+a))/x^2$$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(80) = 160.

Time = 0.25 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^3(a + bx^2)}{x^3} dx = \frac{2 \cosh(bx^2 + a)^3 + 6 \cosh(bx^2 + a) \sinh(bx^2 + a)^2 - 3(bx^2 \operatorname{Ei}(3bx^2) - bx^2 \operatorname{Ei}(-3bx^2)) \cosh(3a) - 3(bx^2 \operatorname{Ei}(3bx^2) - bx^2 \operatorname{Ei}(-3bx^2)) \sinh(3a) - 3(bx^2 \operatorname{Ei}(bx^2) - bx^2 \operatorname{Ei}(-bx^2)) \cosh(a) - 3(bx^2 \operatorname{Ei}(bx^2) - bx^2 \operatorname{Ei}(-bx^2)) \sinh(a) + 6 \cosh(bx^2 + a)}{x^2}$$

[In] `integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="fricas")`

[Out]
$$-1/16*(2*\cosh(b*x^2 + a)^3 + 6*\cosh(b*x^2 + a)*\sinh(b*x^2 + a)^2 - 3*(b*x^2*\operatorname{Ei}(3*b*x^2) - b*x^2*\operatorname{Ei}(-3*b*x^2))*\cosh(3*a) - 3*(b*x^2*\operatorname{Ei}(b*x^2) - b*x^2*\operatorname{Ei}(-b*x^2))*\sinh(3*a) - 3*(b*x^2*\operatorname{Ei}(3*b*x^2) + b*x^2*\operatorname{Ei}(-3*b*x^2))*\cosh(a) - 3*(b*x^2*\operatorname{Ei}(b*x^2) + b*x^2*\operatorname{Ei}(-b*x^2))*\sinh(a) + 6*\cosh(b*x^2 + a))/x^2$$

Sympy [F]

$$\int \frac{\cosh^3(a + bx^2)}{x^3} dx = \int \frac{\cosh^3(a + bx^2)}{x^3} dx$$

[In] `integrate(cosh(b*x**2+a)**3/x**3,x)`

[Out] `Integral(cosh(a + b*x**2)**3/x**3, x)`

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.64

$$\int \frac{\cosh^3(a + bx^2)}{x^3} dx = -\frac{3}{16} be^{(-3a)}\Gamma(-1, 3bx^2) - \frac{3}{16} be^{(-a)}\Gamma(-1, bx^2) + \frac{3}{16} be^a\Gamma(-1, -bx^2) + \frac{3}{16} be^{(3a)}\Gamma(-1, -3bx^2)$$

[In] integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="maxima")

[Out] -3/16*b*e^(-3*a)*gamma(-1, 3*b*x^2) - 3/16*b*e^(-a)*gamma(-1, b*x^2) + 3/16*b*e^a*gamma(-1, -b*x^2) + 3/16*b*e^(3*a)*gamma(-1, -3*b*x^2)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. 2(80) = 160.

Time = 0.26 (sec) , antiderivative size = 224, normalized size of antiderivative = 2.46

$$\int \frac{\cosh^3(a + bx^2)}{x^3} dx = \frac{3(bx^2 + a)b^2\text{Ei}(3bx^2)e^{(3a)} - 3ab^2\text{Ei}(3bx^2)e^{(3a)} - 3(bx^2 + a)b^2\text{Ei}(-bx^2)e^{(-a)} + 3ab^2\text{Ei}(-bx^2)e^{(-a)} - 3(bx^2 + a)b^2\text{Ei}(-3bx^2)e^{(-3a)} + 3ab^2\text{Ei}(-3bx^2)e^{(-3a)} + 3(bx^2 + a)b^2\text{Ei}(bx^2)e^a - 3ab^2\text{Ei}(bx^2)e^a - b^2e^{(3bx^2 + 3a)} - 3b^2e^{(bx^2 + a)} - 3b^2e^{(-bx^2 - a)} - b^2e^{(-3bx^2 - 3a)}}{(bx^2 + a)^3}$$

[In] integrate(cosh(b*x^2+a)^3/x^3,x, algorithm="giac")

[Out] 1/16*(3*(b*x^2 + a)*b^2*Ei(3*b*x^2)*e^(3*a) - 3*a*b^2*Ei(3*b*x^2)*e^(3*a) - 3*(b*x^2 + a)*b^2*Ei(-b*x^2)*e^(-a) + 3*a*b^2*Ei(-b*x^2)*e^(-a) - 3*(b*x^2 + a)*b^2*Ei(-3*b*x^2)*e^(-3*a) + 3*a*b^2*Ei(-3*b*x^2)*e^(-3*a) + 3*(b*x^2 + a)*b^2*Ei(b*x^2)*e^a - 3*a*b^2*Ei(b*x^2)*e^a - b^2*e^(3*b*x^2 + 3*a) - 3*b^2*e^(b*x^2 + a) - 3*b^2*e^(-b*x^2 - a) - b^2*e^(-3*b*x^2 - 3*a))/(b^2*x^2)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a + bx^2)}{x^3} dx = \int \frac{\cosh(bx^2 + a)^3}{x^3} dx$$

[In] int(cosh(a + b*x^2)^3/x^3,x)

[Out] int(cosh(a + b*x^2)^3/x^3, x)

3.22 $\int x \cosh^7(a + bx^2) dx$

Optimal result	140
Rubi [A] (verified)	140
Mathematica [A] (verified)	141
Maple [A] (verified)	141
Fricas [B] (verification not implemented)	142
Sympy [A] (verification not implemented)	142
Maxima [B] (verification not implemented)	142
Giac [A] (verification not implemented)	143
Mupad [B] (verification not implemented)	143

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int x \cosh^7(a + bx^2) dx = \frac{\sinh(a + bx^2)}{2b} + \frac{\sinh^3(a + bx^2)}{2b} + \frac{3 \sinh^5(a + bx^2)}{10b} + \frac{\sinh^7(a + bx^2)}{14b}$$

[Out] 1/2*sinh(b*x^2+a)/b+1/2*sinh(b*x^2+a)^3/b+3/10*sinh(b*x^2+a)^5/b+1/14*sinh(b*x^2+a)^7/b

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2713}

$$\int x \cosh^7(a + bx^2) dx = \frac{\sinh^7(a + bx^2)}{14b} + \frac{3 \sinh^5(a + bx^2)}{10b} + \frac{\sinh^3(a + bx^2)}{2b} + \frac{\sinh(a + bx^2)}{2b}$$

[In] Int[x*Cosh[a + b*x^2]^7,x]

[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(2*b) + (3*Sinh[a + b*x^2]^5)/(10*b) + Sinh[a + b*x^2]^7/(14*b)

Rule 2713

Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] := Dist[-d^(-1), Subst[Int[Expand[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x] && IGtQ[(n - 1)/2, 0]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])

```

^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))

```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{1}{2} \text{Subst} \left(\int \cosh^7(a + bx) dx, x, x^2 \right) \\
&= \frac{i \text{Subst} \left(\int (1 - 3x^2 + 3x^4 - x^6) dx, x, -i \sinh(a + bx^2) \right)}{2b} \\
&= \frac{\sinh(a + bx^2)}{2b} + \frac{\sinh^3(a + bx^2)}{2b} + \frac{3 \sinh^5(a + bx^2)}{10b} + \frac{\sinh^7(a + bx^2)}{14b}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int x \cosh^7(a + bx^2) dx = \frac{\sinh(a + bx^2)}{2b} + \frac{\sinh^3(a + bx^2)}{2b} + \frac{3 \sinh^5(a + bx^2)}{10b} + \frac{\sinh^7(a + bx^2)}{14b}$$

[In] Integrate[x*Cosh[a + b*x^2]^7,x]

[Out] Sinh[a + b*x^2]/(2*b) + Sinh[a + b*x^2]^3/(2*b) + (3*Sinh[a + b*x^2]^5)/(10*b) + Sinh[a + b*x^2]^7/(14*b)

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{\left(\frac{16}{35} + \frac{\cosh(bx^2+a)^6}{7} + \frac{6 \cosh(bx^2+a)^4}{35} + \frac{8 \cosh(bx^2+a)^2}{35} \right) \sinh(bx^2+a)}{2b}$
default	$\frac{\left(\frac{16}{35} + \frac{\cosh(bx^2+a)^6}{7} + \frac{6 \cosh(bx^2+a)^4}{35} + \frac{8 \cosh(bx^2+a)^2}{35} \right) \sinh(bx^2+a)}{2b}$
parallelrisch	$\frac{5 \sinh(7bx^2+7a) + 49 \sinh(5bx^2+5a) + 245 \sinh(3bx^2+3a) + 1225 \sinh(bx^2+a)}{4480b}$
risch	$\frac{e^{7bx^2+7a}}{1792b} + \frac{7e^{5bx^2+5a}}{1280b} + \frac{7e^{3bx^2+3a}}{256b} + \frac{35e^{bx^2+a}}{256b} - \frac{35e^{-bx^2-a}}{256b} - \frac{7e^{-3bx^2-3a}}{256b} - \frac{7e^{-5bx^2-5a}}{1280b} - \frac{e^{-7bx^2-7a}}{1792b}$

[In] int(x*cosh(b*x^2+a)^7,x,method=_RETURNVERBOSE)

[Out] 1/2/b*(16/35+1/7*cosh(b*x^2+a)^6+6/35*cosh(b*x^2+a)^4+8/35*cosh(b*x^2+a)^2)*sinh(b*x^2+a)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. 2(59) = 118.

Time = 0.24 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.91

$$\int x \cosh^7(a + bx^2) dx$$

$$= \frac{5 \sinh(bx^2 + a)^7 + 7 \left(15 \cosh(bx^2 + a)^2 + 7 \right) \sinh(bx^2 + a)^5 + 35 \left(5 \cosh(bx^2 + a)^4 + 14 \cosh(bx^2 + a)^2 + 7 \right) \sinh(bx^2 + a)^3 + 35 \left(\cosh(bx^2 + a)^6 + 7 \cosh(bx^2 + a)^4 + 21 \cosh(bx^2 + a)^2 + 35 \right) \sinh(bx^2 + a)}{b}$$

[In] integrate(x*cosh(b*x^2+a)^7,x, algorithm="fricas")

[Out] 1/4480*(5*sinh(b*x^2 + a)^7 + 7*(15*cosh(b*x^2 + a)^2 + 7)*sinh(b*x^2 + a)^5 + 35*(5*cosh(b*x^2 + a)^4 + 14*cosh(b*x^2 + a)^2 + 7)*sinh(b*x^2 + a)^3 + 35*(cosh(b*x^2 + a)^6 + 7*cosh(b*x^2 + a)^4 + 21*cosh(b*x^2 + a)^2 + 35)*sinh(b*x^2 + a))/b

Sympy [A] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.40

$$\int x \cosh^7(a + bx^2) dx$$

$$= \begin{cases} -\frac{8 \sinh^7(a+bx^2)}{35b} + \frac{4 \sinh^5(a+bx^2) \cosh^2(a+bx^2)}{5b} - \frac{\sinh^3(a+bx^2) \cosh^4(a+bx^2)}{b} + \frac{\sinh(a+bx^2) \cosh^6(a+bx^2)}{2b} & \text{for } b \neq 0 \\ \frac{x^2 \cosh^7(a)}{2} & \text{otherwise} \end{cases}$$

[In] integrate(x*cosh(b*x**2+a)**7,x)

[Out] Piecewise((-8*sinh(a + b*x**2)**7/(35*b) + 4*sinh(a + b*x**2)**5*cosh(a + b*x**2)**2/(5*b) - sinh(a + b*x**2)**3*cosh(a + b*x**2)**4/b + sinh(a + b*x**2)**cosh(a + b*x**2)**6/(2*b), Ne(b, 0)), (x**2*cosh(a)**7/2, True))

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(59) = 118.

Time = 0.18 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.88

$$\int x \cosh^7(a + bx^2) dx = \frac{e^{(7bx^2+7a)}}{1792b} + \frac{7e^{(5bx^2+5a)}}{1280b} + \frac{7e^{(3bx^2+3a)}}{256b} + \frac{35e^{(bx^2+a)}}{256b} - \frac{35e^{(-bx^2-a)}}{256b} - \frac{7e^{(-3bx^2-3a)}}{256b} - \frac{7e^{(-5bx^2-5a)}}{1280b} - \frac{e^{(-7bx^2-7a)}}{1792b}$$

[In] integrate(x*cosh(b*x^2+a)^7,x, algorithm="maxima")

[Out] $1/1792e^{(7bx^2 + 7a)/b} + 7/1280e^{(5bx^2 + 5a)/b} + 7/256e^{(3bx^2 + 3a)/b} + 35/256e^{(bx^2 + a)/b} - 35/256e^{(-bx^2 - a)/b} - 7/256e^{(-3bx^2 - 3a)/b} - 7/1280e^{(-5bx^2 - 5a)/b} - 1/1792e^{(-7bx^2 - 7a)/b}$

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.61

$$\int x \cosh^7(a + bx^2) dx = \frac{\left(1225 e^{(6bx^2+6a)} + 245 e^{(4bx^2+4a)} + 49 e^{(2bx^2+2a)} + 5\right) e^{(-7bx^2-7a)} - 5 e^{(7bx^2+7a)} - 49 e^{(5bx^2+5a)} - 245 e^{(3bx^2+3a)} - 1225 e^{(bx^2+a)}}{8960 b}$$

[In] `integrate(x*cosh(b*x^2+a)^7,x, algorithm="giac")`

[Out] $-1/8960*((1225e^{(6bx^2 + 6a)} + 245e^{(4bx^2 + 4a)} + 49e^{(2bx^2 + 2a)} + 5)*e^{(-7bx^2 - 7a)} - 5e^{(7bx^2 + 7a)} - 49e^{(5bx^2 + 5a)} - 245e^{(3bx^2 + 3a)} - 1225e^{(bx^2 + a)})/b$

Mupad [B] (verification not implemented)

Time = 1.66 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\int x \cosh^7(a + bx^2) dx = \frac{5 \sinh(bx^2 + a)^7 + 21 \sinh(bx^2 + a)^5 + 35 \sinh(bx^2 + a)^3 + 35 \sinh(bx^2 + a)}{70 b}$$

[In] `int(x*cosh(a + b*x^2)^7,x)`

[Out] $(35*\sinh(a + b*x^2) + 35*\sinh(a + b*x^2)^3 + 21*\sinh(a + b*x^2)^5 + 5*\sinh(a + b*x^2)^7)/(70*b)$

3.23 $\int x^2 \cosh(x^3) dx$

Optimal result	144
Rubi [A] (verified)	144
Mathematica [A] (verified)	145
Maple [A] (verified)	145
Fricas [A] (verification not implemented)	146
Sympy [A] (verification not implemented)	146
Maxima [A] (verification not implemented)	146
Giac [B] (verification not implemented)	146
Mupad [B] (verification not implemented)	147

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int x^2 \cosh(x^3) dx = \frac{\sinh(x^3)}{3}$$

[Out] 1/3*sinh(x^3)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 2717}

$$\int x^2 \cosh(x^3) dx = \frac{\sinh(x^3)}{3}$$

[In] Int[x^2*Cosh[x^3],x]

[Out] Sinh[x^3]/3

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify

`[(m + 1)/n], 0))`

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{3} \text{Subst} \left(\int \cosh(x) dx, x, x^3 \right) \\ &= \frac{\sinh(x^3)}{3} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int x^2 \cosh(x^3) dx = \frac{\sinh(x^3)}{3}$$

`[In] Integrate[x^2*Cosh[x^3],x]`

`[Out] Sinh[x^3]/3`

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\sinh(x^3)}{3}$	7
default	$\frac{\sinh(x^3)}{3}$	7
meijerg	$\frac{\sinh(x^3)}{3}$	7
parallelrisc	$\frac{\sinh(x^3)}{3}$	7
risc	$\frac{e^{x^3}}{6} - \frac{e^{-x^3}}{6}$	16

`[In] int(x^2*cosh(x^3),x,method=_RETURNVERBOSE)`

`[Out] 1/3*sinh(x^3)`

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^2 \cosh(x^3) dx = \frac{1}{3} \sinh(x^3)$$

[In] integrate(x^2*cosh(x^3),x, algorithm="fricas")

[Out] 1/3*sinh(x^3)

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.62

$$\int x^2 \cosh(x^3) dx = \frac{\sinh(x^3)}{3}$$

[In] integrate(x**2*cosh(x**3),x)

[Out] sinh(x**3)/3

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^2 \cosh(x^3) dx = \frac{1}{3} \sinh(x^3)$$

[In] integrate(x^2*cosh(x^3),x, algorithm="maxima")

[Out] 1/3*sinh(x^3)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int x^2 \cosh(x^3) dx = -\frac{1}{6} e^{(-x^3)} + \frac{1}{6} e^{(x^3)}$$

[In] integrate(x^2*cosh(x^3),x, algorithm="giac")

[Out] -1/6*e^(-x^3) + 1/6*e^(x^3)

Mupad [B] (verification not implemented)

Time = 1.51 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int x^2 \cosh(x^3) dx = \frac{\sinh(x^3)}{3}$$

[In] int(x^2*cosh(x^3),x)

[Out] sinh(x^3)/3

3.24 $\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx$

Optimal result	148
Rubi [A] (verified)	148
Mathematica [A] (verified)	149
Maple [A] (verified)	149
Fricas [A] (verification not implemented)	150
Sympy [A] (verification not implemented)	150
Maxima [A] (verification not implemented)	150
Giac [B] (verification not implemented)	150
Mupad [B] (verification not implemented)	151

Optimal result

Integrand size = 8, antiderivative size = 8

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = -\frac{1}{5} \sinh\left(\frac{1}{x^5}\right)$$

[Out] -1/5*sinh(1/x^5)

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 2717}

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = -\frac{1}{5} \sinh\left(\frac{1}{x^5}\right)$$

[In] Int[Cosh[x^(-5)]/x^6,x]

[Out] -1/5*Sinh[x^(-5)]

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
```

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(\frac{1}{5}\text{Subst}\left(\int \cosh(x) dx, x, \frac{1}{x^5}\right)\right) \\ &= -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = -\frac{1}{5}\sinh\left(\frac{1}{x^5}\right)$$

```
[In] Integrate[Cosh[x^(-5)]/x^6,x]
```

```
[Out] -1/5*Sinh[x^(-5)]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
default	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
meijerg	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
parallelsch	$-\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$	7
risch	$-\frac{e^{\frac{1}{x^5}}}{10} + \frac{e^{-\frac{1}{x^5}}}{10}$	16

```
[In] int(cosh(1/x^5)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*sinh(1/x^5)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = -\frac{1}{5} \sinh\left(\frac{1}{x^5}\right)$$

[In] integrate(cosh(1/x^5)/x^6,x, algorithm="fricas")

[Out] -1/5*sinh(x^(-5))

Sympy [A] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = -\frac{\sinh\left(\frac{1}{x^5}\right)}{5}$$

[In] integrate(cosh(1/x**5)/x**6,x)

[Out] -sinh(x**(-5))/5

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 6, normalized size of antiderivative = 0.75

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = -\frac{1}{5} \sinh\left(\frac{1}{x^5}\right)$$

[In] integrate(cosh(1/x^5)/x^6,x, algorithm="maxima")

[Out] -1/5*sinh(x^(-5))

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15 vs. 2(6) = 12.

Time = 0.25 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = \frac{1}{10} e^{\left(-\frac{1}{x^5}\right)} - \frac{1}{10} e^{\left(\frac{1}{x^5}\right)}$$

[In] integrate(cosh(1/x^5)/x^6,x, algorithm="giac")

[Out] 1/10*e^(-1/x^5) - 1/10*e^(x^(-5))

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.88

$$\int \frac{\cosh\left(\frac{1}{x^5}\right)}{x^6} dx = \frac{e^{-\frac{1}{x^5}}}{10} - \frac{e^{\frac{1}{x^5}}}{10}$$

[In] `int(cosh(1/x^5)/x^6,x)`

[Out] `exp(-1/x^5)/10 - exp(1/x^5)/10`

3.25 $\int \cosh\left(a + \frac{b}{x}\right) dx$

Optimal result	152
Rubi [A] (verified)	152
Mathematica [A] (verified)	154
Maple [A] (verified)	154
Fricas [A] (verification not implemented)	154
Sympy [F]	155
Maxima [A] (verification not implemented)	155
Giac [B] (verification not implemented)	155
Mupad [F(-1)]	156

Optimal result

Integrand size = 8, antiderivative size = 33

$$\int \cosh\left(a + \frac{b}{x}\right) dx = x \cosh\left(a + \frac{b}{x}\right) - b \operatorname{Chi}\left(\frac{b}{x}\right) \sinh(a) - b \cosh(a) \operatorname{Shi}\left(\frac{b}{x}\right)$$

[Out] $x*\cosh(a+b/x)-b*\cosh(a)*\operatorname{Shi}(b/x)-b*\operatorname{Chi}(b/x)*\sinh(a)$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5411, 3378, 3384, 3379, 3382}

$$\int \cosh\left(a + \frac{b}{x}\right) dx = -b \sinh(a) \operatorname{Chi}\left(\frac{b}{x}\right) - b \cosh(a) \operatorname{Shi}\left(\frac{b}{x}\right) + x \cosh\left(a + \frac{b}{x}\right)$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b/x], x]$

[Out] $x*\operatorname{Cosh}[a + b/x] - b*\operatorname{CoshIntegral}[b/x]*\operatorname{Sinh}[a] - b*\operatorname{Cosh}[a]*\operatorname{SinhIntegral}[b/x]$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x/d), x] /; FreeQ[{c, d, e, f
```


, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5411

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.), x_Symbol] :> -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x] && ILtQ[n, 0] && IntegerQ[p]

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\text{Subst}\left(\int \frac{\cosh(a + bx)}{x^2} dx, x, \frac{1}{x}\right) \\
 &= x \cosh\left(a + \frac{b}{x}\right) - b \text{Subst}\left(\int \frac{\sinh(a + bx)}{x} dx, x, \frac{1}{x}\right) \\
 &= x \cosh\left(a + \frac{b}{x}\right) - (b \cosh(a)) \text{Subst}\left(\int \frac{\sinh(bx)}{x} dx, x, \frac{1}{x}\right) \\
 &\quad - (b \sinh(a)) \text{Subst}\left(\int \frac{\cosh(bx)}{x} dx, x, \frac{1}{x}\right) \\
 &= x \cosh\left(a + \frac{b}{x}\right) - b \text{Chi}\left(\frac{b}{x}\right) \sinh(a) - b \cosh(a) \text{Shi}\left(\frac{b}{x}\right)
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \cosh\left(a + \frac{b}{x}\right) dx = x \cosh\left(a + \frac{b}{x}\right) - b \operatorname{Chi}\left(\frac{b}{x}\right) \sinh(a) - b \cosh(a) \operatorname{Shi}\left(\frac{b}{x}\right)$$

[In] Integrate[Cosh[a + b/x], x]

[Out] x*Cosh[a + b/x] - b*CoshIntegral[b/x]*Sinh[a] - b*Cosh[a]*SinhIntegral[b/x]

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.70

method	result	size
risch	$-\frac{e^{-a} \operatorname{Ei}_1\left(\frac{b}{x}\right)b}{2} + \frac{e^{-\frac{ax+b}{x}}x}{2} + \frac{e^a \operatorname{Ei}_1\left(-\frac{b}{x}\right)b}{2} + \frac{e^{\frac{ax+b}{x}}x}{2}$	56
meijerg	$-\frac{i\sqrt{\pi} \cosh(a)b \left(\frac{4ix \cosh\left(\frac{b}{x}\right)}{b\sqrt{\pi}} - \frac{4i \operatorname{Shi}\left(\frac{b}{x}\right)}{\sqrt{\pi}} \right)}{4} - \frac{\sqrt{\pi} \sinh(a)b \left(\frac{4\gamma - 4 - 4\ln(x) + 4\ln(ib)}{\sqrt{\pi}} + \frac{4}{\sqrt{\pi}} - \frac{4x \sinh\left(\frac{b}{x}\right)}{\sqrt{\pi}b} + \frac{4 \operatorname{Chi}\left(\frac{b}{x}\right) - 4\ln\left(\frac{b}{x}\right) - 4\gamma}{\sqrt{\pi}} \right)}{4}$	11

[In] int(cosh(a+b/x), x, method=_RETURNVERBOSE)

[Out] -1/2*exp(-a)*Ei(1, b/x)*b+1/2*exp(-(a*x+b)/x)*x+1/2*exp(a)*Ei(1, -b/x)*b+1/2*exp((a*x+b)/x)*x

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.76

$$\int \cosh\left(a + \frac{b}{x}\right) dx = -\frac{1}{2} \left(b \operatorname{Ei}\left(\frac{b}{x}\right) - b \operatorname{Ei}\left(-\frac{b}{x}\right) \right) \cosh(a) + x \cosh\left(\frac{ax+b}{x}\right) - \frac{1}{2} \left(b \operatorname{Ei}\left(\frac{b}{x}\right) + b \operatorname{Ei}\left(-\frac{b}{x}\right) \right) \sinh(a)$$

[In] integrate(cosh(a+b/x), x, algorithm="fricas")

[Out] -1/2*(b*Ei(b/x) - b*Ei(-b/x))*cosh(a) + x*cosh((a*x + b)/x) - 1/2*(b*Ei(b/x) + b*Ei(-b/x))*sinh(a)

Sympy [F]

$$\int \cosh\left(a + \frac{b}{x}\right) dx = \int \cosh\left(a + \frac{b}{x}\right) dx$$

[In] integrate(cosh(a+b/x),x)

[Out] Integral(cosh(a + b/x), x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \cosh\left(a + \frac{b}{x}\right) dx = \frac{1}{2} \left(\operatorname{Ei}\left(-\frac{b}{x}\right) e^{(-a)} - \operatorname{Ei}\left(\frac{b}{x}\right) e^a \right) b + x \cosh\left(a + \frac{b}{x}\right)$$

[In] integrate(cosh(a+b/x),x, algorithm="maxima")

[Out] 1/2*(Ei(-b/x)*e^(-a) - Ei(b/x)*e^a)*b + x*cosh(a + b/x)

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(33) = 66.

Time = 0.28 (sec) , antiderivative size = 173, normalized size of antiderivative = 5.24

$$\int \cosh\left(a + \frac{b}{x}\right) dx = \frac{ab^2 \operatorname{Ei}\left(a - \frac{ax+b}{x}\right) e^{(-a)} - \frac{(ax+b)b^2 \operatorname{Ei}\left(a - \frac{ax+b}{x}\right) e^{(-a)}}{x} - b^2 e^{\left(-\frac{ax+b}{x}\right)}}{2\left(a - \frac{ax+b}{x}\right)b} - \frac{ab^2 \operatorname{Ei}\left(-a + \frac{ax+b}{x}\right) e^a - \frac{(ax+b)b^2 \operatorname{Ei}\left(-a + \frac{ax+b}{x}\right) e^a}{x} + b^2 e^{\left(\frac{ax+b}{x}\right)}}{2\left(a - \frac{ax+b}{x}\right)b}$$

[In] integrate(cosh(a+b/x),x, algorithm="giac")

[Out] 1/2*(a*b^2*Ei(a - (a*x + b)/x)*e^(-a) - (a*x + b)*b^2*Ei(a - (a*x + b)/x)*e^(-a)/x - b^2*e^(-(a*x + b)/x))/((a - (a*x + b)/x)*b) - 1/2*(a*b^2*Ei(-a + (a*x + b)/x)*e^a - (a*x + b)*b^2*Ei(-a + (a*x + b)/x)*e^a/x + b^2*e^((a*x + b)/x))/((a - (a*x + b)/x)*b)

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(a + \frac{b}{x}\right) dx = \int \cosh\left(a + \frac{b}{x}\right) dx$$

```
[In] int(cosh(a + b/x),x)
```

```
[Out] int(cosh(a + b/x), x)
```

$$3.26 \quad \int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx$$

Optimal result	157
Rubi [A] (verified)	157
Mathematica [A] (verified)	158
Maple [A] (verified)	158
Fricas [A] (verification not implemented)	159
Sympy [A] (verification not implemented)	159
Maxima [A] (verification not implemented)	159
Giac [B] (verification not implemented)	159
Mupad [F(-1)]	160

Optimal result

Integrand size = 12, antiderivative size = 21

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

[Out] $-\text{Chi}(b/x) * \cosh(a) - \text{Shi}(b/x) * \sinh(a)$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5427, 5425, 5424}

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

[In] $\text{Int}[\text{Cosh}[a + b/x]/x, x]$

[Out] $-(\text{Cosh}[a] * \text{CoshIntegral}[b/x]) - \text{Sinh}[a] * \text{SinhIntegral}[b/x]$

Rule 5424

$\text{Int}[\text{Sinh}[(d \cdot x)^n]/(x), x_Symbol] \rightarrow \text{Simp}[\text{SinhIntegral}[d \cdot x^n]/n, x]$
 /; FreeQ[{d, n}, x]

Rule 5425

$\text{Int}[\text{Cosh}[(d \cdot x)^n]/(x), x_Symbol] \rightarrow \text{Simp}[\text{CoshIntegral}[d \cdot x^n]/n, x]$
 /; FreeQ[{d, n}, x]

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a) \int \frac{\cosh\left(\frac{b}{x}\right)}{x} dx + \sinh(a) \int \frac{\sinh\left(\frac{b}{x}\right)}{x} dx \\ &= -\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\cosh(a)\text{Chi}\left(\frac{b}{x}\right) - \sinh(a)\text{Shi}\left(\frac{b}{x}\right)$$

```
[In] Integrate[Cosh[a + b/x]/x,x]
```

```
[Out] -(Cosh[a]*CoshIntegral[b/x]) - Sinh[a]*SinhIntegral[b/x]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.29

method	result	size
risch	$\frac{e^{-a} \text{Ei}_1\left(\frac{b}{x}\right)}{2} + \frac{e^a \text{Ei}_1\left(-\frac{b}{x}\right)}{2}$	27
meijerg	$-\frac{\sqrt{\pi} \cosh(a) \left(\frac{2\gamma - 2\ln(x) + 2\ln(ib)}{\sqrt{\pi}} + \frac{2 \text{Chi}\left(\frac{b}{x}\right) - 2\ln\left(\frac{b}{x}\right) - 2\gamma}{\sqrt{\pi}} \right)}{2} - \text{Shi}\left(\frac{b}{x}\right) \sinh(a)$	62

```
[In] int(cosh(a+b/x)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*exp(-a)*Ei(1,b/x)+1/2*exp(a)*Ei(1,-b/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.86

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\frac{1}{2} \left(\operatorname{Ei}\left(\frac{b}{x}\right) + \operatorname{Ei}\left(-\frac{b}{x}\right) \right) \cosh(a) - \frac{1}{2} \left(\operatorname{Ei}\left(\frac{b}{x}\right) - \operatorname{Ei}\left(-\frac{b}{x}\right) \right) \sinh(a)$$

[In] integrate(cosh(a+b/x)/x,x, algorithm="fricas")

[Out] -1/2*(Ei(b/x) + Ei(-b/x))*cosh(a) - 1/2*(Ei(b/x) - Ei(-b/x))*sinh(a)

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.81

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\sinh(a) \operatorname{Shi}\left(\frac{b}{x}\right) - \cosh(a) \operatorname{Chi}\left(\frac{b}{x}\right)$$

[In] integrate(cosh(a+b/x)/x,x)

[Out] -sinh(a)*Shi(b/x) - cosh(a)*Chi(b/x)

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.14

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\frac{1}{2} \operatorname{Ei}\left(-\frac{b}{x}\right) e^{(-a)} - \frac{1}{2} \operatorname{Ei}\left(\frac{b}{x}\right) e^a$$

[In] integrate(cosh(a+b/x)/x,x, algorithm="maxima")

[Out] -1/2*Ei(-b/x)*e^(-a) - 1/2*Ei(b/x)*e^a

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 43 vs. 2(21) = 42.

Time = 0.25 (sec) , antiderivative size = 43, normalized size of antiderivative = 2.05

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\frac{b \operatorname{Ei}\left(a - \frac{ax+b}{x}\right) e^{(-a)} + b \operatorname{Ei}\left(-a + \frac{ax+b}{x}\right) e^a}{2b}$$

[In] integrate(cosh(a+b/x)/x,x, algorithm="giac")

[Out] -1/2*(b*Ei(a - (a*x + b)/x)*e^(-a) + b*Ei(-a + (a*x + b)/x)*e^a)/b

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x} dx = -\cosh(a) \operatorname{coshint}\left(\frac{b}{x}\right) - \sinh(a) \operatorname{sinhint}\left(\frac{b}{x}\right)$$

```
[In] int(cosh(a + b/x)/x,x)
```

```
[Out] - cosh(a)*coshint(b/x) - sinh(a)*sinhint(b/x)
```


$$3.27 \quad \int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx$$

Optimal result	161
Rubi [A] (verified)	161
Mathematica [A] (verified)	162
Maple [A] (verified)	162
Fricas [A] (verification not implemented)	163
Sympy [A] (verification not implemented)	163
Maxima [A] (verification not implemented)	163
Giac [B] (verification not implemented)	164
Mupad [B] (verification not implemented)	164

Optimal result

Integrand size = 12, antiderivative size = 13

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$$

[Out] $-\sinh(a+b/x)/b$

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2717}

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$$

[In] `Int[Cosh[a + b/x]/x^2,x]`

[Out] $-(\text{Sinh}[a + b/x]/b)$

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 5429

`Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]`
`:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])`
`^p, x], x, x^n], x] /;` `FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify`

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \cosh(a + bx) dx, x, \frac{1}{x}\right) \\ &= -\frac{\sinh\left(a + \frac{b}{x}\right)}{b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$$

```
[In] Integrate[Cosh[a + b/x]/x^2,x]
```

```
[Out] -(Sinh[a + b/x]/b)
```

Maple [A] (verified)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$	14
default	$-\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$	14
parallelrisc	$-\frac{\sinh\left(\frac{ax+b}{x}\right)}{b}$	16
risc	$-\frac{e^{\frac{ax+b}{x}}}{2b} + \frac{e^{-\frac{ax+b}{x}}}{2b}$	33
meijerg	$-\frac{\cosh(a) \sinh\left(\frac{b}{x}\right)}{b} + \frac{\sqrt{\pi} \sinh(a) \left(\frac{1}{\sqrt{\pi}} - \frac{\cosh\left(\frac{b}{x}\right)}{\sqrt{\pi}}\right)}{b}$	39

```
[In] int(cosh(a+b/x)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -sinh(a+b/x)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{\sinh\left(\frac{ax+b}{x}\right)}{b}$$

[In] integrate(cosh(a+b/x)/x^2,x, algorithm="fricas")

[Out] -sinh((a*x + b)/x)/b

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = \begin{cases} -\frac{\sinh\left(a + \frac{b}{x}\right)}{b} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{x} & \text{otherwise} \end{cases}$$

[In] integrate(cosh(a+b/x)/x**2,x)

[Out] Piecewise((-sinh(a + b/x)/b, Ne(b, 0)), (-cosh(a)/x, True))

Maxima [A] (verification not implemented)

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$$

[In] integrate(cosh(a+b/x)/x^2,x, algorithm="maxima")

[Out] -sinh(a + b/x)/b

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 29 vs. 2(13) = 26.

Time = 0.30 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.23

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{e^{\left(\frac{ax+b}{x}\right)} - e^{\left(-\frac{ax+b}{x}\right)}}{2b}$$

[In] integrate(cosh(a+b/x)/x^2,x, algorithm="giac")

[Out] -1/2*(e^((a*x + b)/x) - e^(-(a*x + b)/x))/b

Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^2} dx = -\frac{\sinh\left(a + \frac{b}{x}\right)}{b}$$

[In] int(cosh(a + b/x)/x^2,x)

[Out] -sinh(a + b/x)/b

$$3.28 \quad \int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx$$

Optimal result	165
Rubi [A] (verified)	165
Mathematica [A] (verified)	166
Maple [A] (verified)	166
Fricas [A] (verification not implemented)	167
Sympy [A] (verification not implemented)	167
Maxima [C] (verification not implemented)	167
Giac [B] (verification not implemented)	168
Mupad [B] (verification not implemented)	168

Optimal result

Integrand size = 12, antiderivative size = 29

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \frac{\cosh\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sinh\left(a + \frac{b}{x}\right)}{bx}$$

[Out] $\cosh(a+b/x)/b^2 - \sinh(a+b/x)/b/x$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 3377, 2718}

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \frac{\cosh\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sinh\left(a + \frac{b}{x}\right)}{bx}$$

[In] $\text{Int}[\text{Cosh}[a + b/x]/x^3, x]$

[Out] $\text{Cosh}[a + b/x]/b^2 - \text{Sinh}[a + b/x]/(b*x)$

Rule 2718

$\text{Int}[\sin[(c_.) + (d_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-\text{Cos}[c + d*x]/d, x] /;$ $\text{FreeQ}\{c, d, x\}$

Rule 3377

$\text{Int}[((c_.) + (d_.)*(x_.))^{(m_.)} \sin[(e_.) + (f_.)*(x_.)], x_Symbol] \rightarrow \text{Simp}[-(c + d*x)^m * (\text{Cos}[e + f*x]/f), x] + \text{Dist}[d*(m/f), \text{Int}[(c + d*x)^{(m-1)} * \text{Cos}[e + f*x], x], x] /;$ $\text{FreeQ}\{c, d, e, f, x\} \ \&\& \ \text{GtQ}[m, 0]$

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int x \cosh(a + bx) dx, x, \frac{1}{x}\right) \\ &= -\frac{\sinh\left(a + \frac{b}{x}\right)}{bx} + \frac{\text{Subst}\left(\int \sinh(a + bx) dx, x, \frac{1}{x}\right)}{b} \\ &= \frac{\cosh\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sinh\left(a + \frac{b}{x}\right)}{bx} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \frac{x \cosh\left(a + \frac{b}{x}\right) - b \sinh\left(a + \frac{b}{x}\right)}{b^2 x}$$

[In] Integrate[Cosh[a + b/x]/x^3,x]

[Out] (x*Cosh[a + b/x] - b*Sinh[a + b/x])/(b^2*x)

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.48

method	result	size
parallelrisc	$\frac{2 \tanh\left(\frac{ax+b}{2x}\right)b-2x}{x b^2 \left(\tanh\left(\frac{ax+b}{2x}\right)^2-1\right)}$	43
derivativedivides	$-\frac{\left(a+\frac{b}{x}\right) \sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)-a \sinh\left(a+\frac{b}{x}\right)}{b^2}$	44
default	$-\frac{\left(a+\frac{b}{x}\right) \sinh\left(a+\frac{b}{x}\right)-\cosh\left(a+\frac{b}{x}\right)-a \sinh\left(a+\frac{b}{x}\right)}{b^2}$	44
risc	$-\frac{(-x+b)e^{\frac{ax+b}{x}}}{2b^2x} + \frac{(x+b)e^{-\frac{ax+b}{x}}}{2b^2x}$	47
meijerg	$\frac{2\sqrt{\pi} \cosh(a) \left(-\frac{1}{2\sqrt{\pi}} + \frac{\cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}} - \frac{b \sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi} x}\right)}{b^2} - \frac{\sinh(a) \left(\frac{\cosh\left(\frac{b}{x}\right)b}{x} - \sinh\left(\frac{b}{x}\right)\right)}{b^2}$	71

[In] `int(cosh(a+b/x)/x^3,x,method=_RETURNVERBOSE)`

[Out] $(2*\tanh(1/2*(a*x+b)/x)*b-2*x)/x/b^2/(\tanh(1/2*(a*x+b)/x)^2-1)$

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.14

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \frac{x \cosh\left(\frac{ax+b}{x}\right) - b \sinh\left(\frac{ax+b}{x}\right)}{b^2 x}$$

[In] `integrate(cosh(a+b/x)/x^3,x, algorithm="fricas")`

[Out] $(x*\cosh((a*x + b)/x) - b*\sinh((a*x + b)/x))/(b^2*x)$

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \begin{cases} -\frac{\sinh\left(a + \frac{b}{x}\right)}{bx} + \frac{\cosh\left(a + \frac{b}{x}\right)}{b^2} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{2x^2} & \text{otherwise} \end{cases}$$

[In] `integrate(cosh(a+b/x)/x**3,x)`

[Out] `Piecewise((-sinh(a + b/x)/(b*x) + cosh(a + b/x)/b**2, Ne(b, 0)), (-cosh(a)/(2*x**2), True))`

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.62

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \frac{1}{4} b \left(\frac{e^{(-a)} \Gamma\left(3, \frac{b}{x}\right)}{b^3} + \frac{e^a \Gamma\left(3, -\frac{b}{x}\right)}{b^3} \right) - \frac{\cosh\left(a + \frac{b}{x}\right)}{2x^2}$$

[In] `integrate(cosh(a+b/x)/x^3,x, algorithm="maxima")`

[Out] $1/4*b*(e^{(-a)}*\gamma(3, b/x)/b^3 + e^a*\gamma(3, -b/x)/b^3) - 1/2*cosh(a + b/x)/x^2$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. 2(29) = 58.

Time = 0.27 (sec) , antiderivative size = 93, normalized size of antiderivative = 3.21

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx$$

$$= \frac{ae^{\left(\frac{ax+b}{x}\right)} - ae^{\left(-\frac{ax+b}{x}\right)} - \frac{(ax+b)e^{\left(\frac{ax+b}{x}\right)}}{x} + \frac{(ax+b)e^{\left(-\frac{ax+b}{x}\right)}}{x} + e^{\left(\frac{ax+b}{x}\right)} + e^{\left(-\frac{ax+b}{x}\right)}}{2b^2}$$

[In] integrate(cosh(a+b/x)/x^3,x, algorithm="giac")

[Out] 1/2*(a*e^((a*x + b)/x) - a*e^(-(a*x + b)/x) - (a*x + b)*e^((a*x + b)/x)/x + (a*x + b)*e^(-(a*x + b)/x)/x + e^((a*x + b)/x) + e^(-(a*x + b)/x))/b^2

Mupad [B] (verification not implemented)

Time = 1.56 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^3} dx = \frac{\cosh\left(a + \frac{b}{x}\right)}{b^2} - \frac{\sinh\left(a + \frac{b}{x}\right)}{bx}$$

[In] int(cosh(a + b/x)/x^3,x)

[Out] cosh(a + b/x)/b^2 - sinh(a + b/x)/(b*x)

$$3.29 \quad \int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx$$

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Mupad [B] (verification not implemented)	172

Optimal result

Integrand size = 12, antiderivative size = 46

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \frac{2 \cosh\left(a + \frac{b}{x}\right)}{b^2 x} - \frac{2 \sinh\left(a + \frac{b}{x}\right)}{b^3} - \frac{\sinh\left(a + \frac{b}{x}\right)}{b x^2}$$

[Out] $2*\cosh(a+b/x)/b^2/x-2*\sinh(a+b/x)/b^3-\sinh(a+b/x)/b/x^2$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5429, 3377, 2717}

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = -\frac{2 \sinh\left(a + \frac{b}{x}\right)}{b^3} + \frac{2 \cosh\left(a + \frac{b}{x}\right)}{b^2 x} - \frac{\sinh\left(a + \frac{b}{x}\right)}{b x^2}$$

[In] Int[Cosh[a + b/x]/x^4,x]

[Out] $(2*\cosh[a + b/x])/(b^2*x) - (2*\sinh[a + b/x])/b^3 - \sinh[a + b/x]/(b*x^2)$

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co

`s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 5429

`Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))`

Rubi steps

$$\begin{aligned}
 \text{integral} &= -\text{Subst}\left(\int x^2 \cosh(a + bx) dx, x, \frac{1}{x}\right) \\
 &= -\frac{\sinh\left(a + \frac{b}{x}\right)}{bx^2} + \frac{2\text{Subst}\left(\int x \sinh(a + bx) dx, x, \frac{1}{x}\right)}{b} \\
 &= \frac{2 \cosh\left(a + \frac{b}{x}\right)}{b^2x} - \frac{\sinh\left(a + \frac{b}{x}\right)}{bx^2} - \frac{2\text{Subst}\left(\int \cosh(a + bx) dx, x, \frac{1}{x}\right)}{b^2} \\
 &= \frac{2 \cosh\left(a + \frac{b}{x}\right)}{b^2x} - \frac{2 \sinh\left(a + \frac{b}{x}\right)}{b^3} - \frac{\sinh\left(a + \frac{b}{x}\right)}{bx^2}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.85

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \frac{2bx \cosh\left(a + \frac{b}{x}\right) - (b^2 + 2x^2) \sinh\left(a + \frac{b}{x}\right)}{b^3x^2}$$

[In] `Integrate[Cosh[a + b/x]/x^4, x]`

[Out] `(2*b*x*Cosh[a + b/x] - (b^2 + 2*x^2)*Sinh[a + b/x])/(b^3*x^2)`

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

method	result
risch	$-\frac{(b^2-2bx+2x^2)e^{\frac{ax+b}{x}}}{2b^3x^2} + \frac{(b^2+2bx+2x^2)e^{-\frac{ax+b}{x}}}{2b^3x^2}$
parallelrisc	$\frac{-2 \tanh\left(\frac{ax+b}{2x}\right)^2 xb + 4 \tanh\left(\frac{ax+b}{2x}\right) x^2 + 2 \tanh\left(\frac{ax+b}{2x}\right) b^2 - 2bx}{x^2 b^3 \left(\tanh\left(\frac{ax+b}{2x}\right)^2 - 1\right)}$
derivativedivides	$-\frac{a^2 \sinh\left(a + \frac{b}{x}\right) - 2a \left(\left(a + \frac{b}{x}\right) \sinh\left(a + \frac{b}{x}\right) - \cosh\left(a + \frac{b}{x}\right) \right) + \left(a + \frac{b}{x}\right)^2 \sinh\left(a + \frac{b}{x}\right) - 2 \left(a + \frac{b}{x}\right) \cosh\left(a + \frac{b}{x}\right) + 2 \sinh\left(a + \frac{b}{x}\right)}{b^3}$
default	$-\frac{a^2 \sinh\left(a + \frac{b}{x}\right) - 2a \left(\left(a + \frac{b}{x}\right) \sinh\left(a + \frac{b}{x}\right) - \cosh\left(a + \frac{b}{x}\right) \right) + \left(a + \frac{b}{x}\right)^2 \sinh\left(a + \frac{b}{x}\right) - 2 \left(a + \frac{b}{x}\right) \cosh\left(a + \frac{b}{x}\right) + 2 \sinh\left(a + \frac{b}{x}\right)}{b^3}$
meijerg	$-\frac{4i\sqrt{\pi} \cosh(a) \left(\frac{ib \cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}x} - \frac{i \left(\frac{3b^2}{2x^2} + 3\right) \sinh\left(\frac{b}{x}\right)}{6\sqrt{\pi}} \right)}{b^3} - \frac{4\sqrt{\pi} \sinh(a) \left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(\frac{b^2}{2x^2} + 1\right) \cosh\left(\frac{b}{x}\right)}{2\sqrt{\pi}} - \frac{b \sinh\left(\frac{b}{x}\right)}{2\sqrt{\pi}x} \right)}{b^3}$

[In] `int(cosh(a+b/x)/x^4,x,method=_RETURNVERBOSE)`

[Out] $-1/2*(b^2-2*b*x+2*x^2)/b^3/x^2*\exp((a*x+b)/x)+1/2*(b^2+2*b*x+2*x^2)/b^3/x^2*\exp(-(a*x+b)/x)$

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \frac{2bx \cosh\left(\frac{ax+b}{x}\right) - (b^2 + 2x^2) \sinh\left(\frac{ax+b}{x}\right)}{b^3 x^2}$$

[In] `integrate(cosh(a+b/x)/x^4,x, algorithm="fricas")`

[Out] $(2*b*x*cosh((a*x + b)/x) - (b^2 + 2*x^2)*sinh((a*x + b)/x))/(b^3*x^2)$

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \begin{cases} -\frac{\sinh\left(a + \frac{b}{x}\right)}{bx^2} + \frac{2 \cosh\left(a + \frac{b}{x}\right)}{b^2 x} - \frac{2 \sinh\left(a + \frac{b}{x}\right)}{b^3} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{3x^3} & \text{otherwise} \end{cases}$$

[In] `integrate(cosh(a+b/x)/x**4,x)`

[Out] `Piecewise((-sinh(a + b/x)/(b*x**2) + 2*cosh(a + b/x)/(b**2*x) - 2*sinh(a + b/x)/b**3, Ne(b, 0)), (-cosh(a)/(3*x**3), True))`

Maxima [C] (verification not implemented)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.23 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.04

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \frac{1}{6} b \left(\frac{e^{(-a)} \Gamma\left(4, \frac{b}{x}\right)}{b^4} - \frac{e^a \Gamma\left(4, -\frac{b}{x}\right)}{b^4} \right) - \frac{\cosh\left(a + \frac{b}{x}\right)}{3 x^3}$$

[In] integrate(cosh(a+b/x)/x^4,x, algorithm="maxima")

[Out] 1/6*b*(e^(-a)*gamma(4, b/x)/b^4 - e^a*gamma(4, -b/x)/b^4) - 1/3*cosh(a + b/x)/x^3

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(46) = 92.

Time = 0.31 (sec) , antiderivative size = 216, normalized size of antiderivative = 4.70

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \frac{a^2 e^{\left(\frac{ax+b}{x}\right)} - a^2 e^{\left(-\frac{ax+b}{x}\right)} + 2 a e^{\left(\frac{ax+b}{x}\right)} - \frac{2(ax+b) a e^{\left(\frac{ax+b}{x}\right)}}{x} + 2 a e^{\left(-\frac{ax+b}{x}\right)} + \frac{2(ax+b) a e^{\left(-\frac{ax+b}{x}\right)}}{x} + \frac{(ax+b)^2 e^{\left(\frac{ax+b}{x}\right)}}{x^2} - \frac{(ax+b)^2 e^{\left(-\frac{ax+b}{x}\right)}}{x^2}}{2 b^3}$$

[In] integrate(cosh(a+b/x)/x^4,x, algorithm="giac")

[Out] -1/2*(a^2*e^((a*x + b)/x) - a^2*e^(-(a*x + b)/x) + 2*a*e^((a*x + b)/x) - 2*(a*x + b)*a*e^((a*x + b)/x)/x + 2*a*e^(-(a*x + b)/x) + 2*(a*x + b)*a*e^(-(a*x + b)/x)/x + (a*x + b)^2*e^((a*x + b)/x)/x^2 - 2*(a*x + b)*e^((a*x + b)/x)/x - (a*x + b)^2*e^(-(a*x + b)/x)/x^2 - 2*(a*x + b)*e^(-(a*x + b)/x)/x + 2*e^((a*x + b)/x) - 2*e^(-(a*x + b)/x))/b^3

Mupad [B] (verification not implemented)

Time = 1.60 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.43

$$\int \frac{\cosh\left(a + \frac{b}{x}\right)}{x^4} dx = \frac{e^{-a - \frac{b}{x}} \left(\frac{x}{b^2} + \frac{1}{2b} + \frac{x^2}{b^3}\right)}{x^2} - \frac{e^{a + \frac{b}{x}} \left(\frac{1}{2b} - \frac{x}{b^2} + \frac{x^2}{b^3}\right)}{x^2}$$

[In] int(cosh(a + b/x)/x^4,x)

[Out] (exp(- a - b/x)*(x/b^2 + 1/(2*b) + x^2/b^3))/x^2 - (exp(a + b/x)*(1/(2*b) - x/b^2 + x^2/b^3))/x^2

3.30 $\int \cosh\left(a + \frac{b}{x^2}\right) dx$

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Mathematica [A] (verified)	175
Maple [A] (verified)	175
Fricas [B] (verification not implemented)	175
Sympy [F]	176
Maxima [A] (verification not implemented)	176
Giac [F]	176
Mupad [F(-1)]	177

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = x \cosh\left(a + \frac{b}{x^2}\right) + \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{2}\sqrt{b}e^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)$$

[Out] $x*\cosh(a+b/x^2)+1/2*\operatorname{erf}(b^{(1/2)}/x)*b^{(1/2)}*\operatorname{Pi}^{(1/2)}/\exp(a)-1/2*\exp(a)*\operatorname{erfi}(b^{(1/2)}/x)*b^{(1/2)}*\operatorname{Pi}^{(1/2)}$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {5411, 5435, 5406, 2235, 2236}

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = \frac{1}{2}\sqrt{\pi}e^{-a}\sqrt{b}\operatorname{berf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{2}\sqrt{\pi}e^a\sqrt{b}\operatorname{berfi}\left(\frac{\sqrt{b}}{x}\right) + x \cosh\left(a + \frac{b}{x^2}\right)$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b/x^2], x]$

[Out] $x*\operatorname{Cosh}[a + b/x^2] + (\operatorname{Sqrt}[b]*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]/x])/(2*E^a) - (\operatorname{Sqrt}[b]*E^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]/x])/2$

Rule 2235

$\operatorname{Int}[(F_.)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2)}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5411

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.), x_Symbol] := -Subs
t[Int[(a + b*Cosh[c + d/x^n])^p/x^2, x], x, 1/x] /; FreeQ[{a, b, c, d}, x]
&& ILtQ[n, 0] && IntegerQ[p]
```

Rule 5435

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_)), x_Symbol] := Simp[(e*x
)^(m + 1)*(Cosh[c + d*x^n]/(e*(m + 1))), x] - Dist[d*(n/(e^n*(m + 1))), Int
[(e*x)^(m + n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0]
&& LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int \frac{\cosh(a + bx^2)}{x^2} dx, x, \frac{1}{x}\right) \\
&= x \cosh\left(a + \frac{b}{x^2}\right) - (2b)\text{Subst}\left(\int \sinh(a + bx^2) dx, x, \frac{1}{x}\right) \\
&= x \cosh\left(a + \frac{b}{x^2}\right) + b\text{Subst}\left(\int e^{-a-bx^2} dx, x, \frac{1}{x}\right) - b\text{Subst}\left(\int e^{a+bx^2} dx, x, \frac{1}{x}\right) \\
&= x \cosh\left(a + \frac{b}{x^2}\right) + \frac{1}{2}\sqrt{b}e^{-a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{b}}{x}\right) - \frac{1}{2}\sqrt{b}e^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = x \cosh\left(a + \frac{b}{x^2}\right) + \frac{1}{2}\sqrt{b}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) (\cosh(a) - \sinh(a)) - \frac{1}{2}\sqrt{b}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right) (\cosh(a) + \sinh(a))$$

`[In] Integrate[Cosh[a + b/x^2], x]`

```
[Out] x*Cosh[a + b/x^2] + (Sqrt[b]*Sqrt[Pi]*Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]))/2
- (Sqrt[b]*Sqrt[Pi]*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]))/2
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04

method	result
risch	$\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{b}\sqrt{\pi}e^{-a}}{2} + \frac{e^{-a}xe^{-\frac{b}{x^2}}}{2} + \frac{e^ae^{\frac{b}{x^2}}x}{2} - \frac{e^ab\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{2\sqrt{-b}}$
meijerg	$-\frac{\sqrt{\pi}\cosh(a)\sqrt{2}\sqrt{ib}\left(-\frac{2x\sqrt{2}e^{\frac{b}{x^2}}}{\sqrt{\pi}\sqrt{ib}} - \frac{2x\sqrt{2}e^{-\frac{b}{x^2}}}{\sqrt{\pi}\sqrt{ib}} - \frac{2\sqrt{2}\sqrt{b}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{ib}} + \frac{2\sqrt{2}\sqrt{b}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{\sqrt{ib}}\right)}{8} + \frac{i\sqrt{\pi}\sinh(a)\sqrt{2}\sqrt{ib}\left(\frac{2x\sqrt{2}\sqrt{ib}e^{-\frac{b}{x^2}}}{\sqrt{\pi}b}\right)}{8}$

`[In] int(cosh(a+b/x^2), x, method=_RETURNVERBOSE)`

```
[Out] 1/2*erf(b^(1/2)/x)*b^(1/2)*Pi^(1/2)/exp(a)+1/2/exp(a)*x*exp(-b/x^2)+1/2*exp
(a)*exp(b/x^2)*x-1/2*exp(a)*b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(49) = 98.

Time = 0.26 (sec) , antiderivative size = 225, normalized size of antiderivative = 3.36

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = \frac{x \cosh\left(\frac{ax^2+b}{x^2}\right)^2 + \sqrt{\pi}\left(\cosh(a)\cosh\left(\frac{ax^2+b}{x^2}\right) + \cosh\left(\frac{ax^2+b}{x^2}\right)\sinh(a) + (\cosh(a) + \sinh(a))\sinh\left(\frac{ax^2+b}{x^2}\right)\right)}{2}$$

`[In] integrate(cosh(a+b/x^2), x, algorithm="fricas")`

```
[Out] 1/2*(x*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) +
cosh((a*x^2 + b)/x^2)*sinh(a) + (cosh(a) + sinh(a))*sinh((a*x^2 + b)/x^2))*
sqrt(-b)*erf(sqrt(-b)/x) + sqrt(pi)*(cosh(a)*cosh((a*x^2 + b)/x^2) - cosh((
a*x^2 + b)/x^2)*sinh(a) + (cosh(a) - sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b
)*erf(sqrt(b)/x) + 2*x*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + x*sinh
((a*x^2 + b)/x^2)^2 + x)/(cosh((a*x^2 + b)/x^2) + sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = \int \cosh\left(a + \frac{b}{x^2}\right) dx$$

```
[In] integrate(cosh(a+b/x**2),x)
```

```
[Out] Integral(cosh(a + b/x**2), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.22 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = \frac{1}{2}b \left(\frac{\sqrt{\pi} \left(\operatorname{erf}\left(\sqrt{\frac{b}{x^2}}\right) - 1 \right) e^{-a}}{x \sqrt{\frac{b}{x^2}}} - \frac{\sqrt{\pi} \left(\operatorname{erf}\left(\sqrt{-\frac{b}{x^2}}\right) - 1 \right) e^a}{x \sqrt{-\frac{b}{x^2}}} \right) + x \cosh\left(a + \frac{b}{x^2}\right)$$

```
[In] integrate(cosh(a+b/x^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(sqrt(pi)*(erf(sqrt(b/x^2)) - 1)*e^(-a)/(x*sqrt(b/x^2)) - sqrt(pi)*(e
rf(sqrt(-b/x^2)) - 1)*e^a/(x*sqrt(-b/x^2))) + x*cosh(a + b/x^2)
```

Giac [F]

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = \int \cosh\left(a + \frac{b}{x^2}\right) dx$$

```
[In] integrate(cosh(a+b/x^2),x, algorithm="giac")
```

```
[Out] integrate(cosh(a + b/x^2), x)
```


Mupad [F(-1)]

Timed out.

$$\int \cosh\left(a + \frac{b}{x^2}\right) dx = \int \cosh\left(a + \frac{b}{x^2}\right) dx$$

```
[In] int(cosh(a + b/x^2), x)
```

```
[Out] int(cosh(a + b/x^2), x)
```

3.31 $\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx$

Optimal result	178
Rubi [A] (verified)	178
Mathematica [A] (verified)	179
Maple [A] (verified)	179
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Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = -\frac{1}{2} \cosh(a) \operatorname{Chi}\left(\frac{b}{x^2}\right) - \frac{1}{2} \sinh(a) \operatorname{Shi}\left(\frac{b}{x^2}\right)$$

[Out] $-1/2*\operatorname{Chi}(b/x^2)*\cosh(a)-1/2*\operatorname{Shi}(b/x^2)*\sinh(a)$

Rubi [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5427, 5425, 5424}

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = -\frac{1}{2} \cosh(a) \operatorname{Chi}\left(\frac{b}{x^2}\right) - \frac{1}{2} \sinh(a) \operatorname{Shi}\left(\frac{b}{x^2}\right)$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b/x^2]/x, x]$

[Out] $-1/2*(\operatorname{Cosh}[a]*\operatorname{CoshIntegral}[b/x^2]) - (\operatorname{Sinh}[a]*\operatorname{SinhIntegral}[b/x^2])/2$

Rule 5424

$\operatorname{Int}[\operatorname{Sinh}[(d_*)*(x_)^{(n)}]/(x_), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{SinhIntegral}[d*x^n]/n, x]$
 /; $\operatorname{FreeQ}[\{d, n\}, x]$

Rule 5425

$\operatorname{Int}[\operatorname{Cosh}[(d_*)*(x_)^{(n)}]/(x_), x_Symbol] \rightarrow \operatorname{Simp}[\operatorname{CoshIntegral}[d*x^n]/n, x]$
 /; $\operatorname{FreeQ}[\{d, n\}, x]$

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] :=> Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a) \int \frac{\cosh\left(\frac{b}{x^2}\right)}{x} dx + \sinh(a) \int \frac{\sinh\left(\frac{b}{x^2}\right)}{x} dx \\ &= -\frac{1}{2} \cosh(a) \text{Chi}\left(\frac{b}{x^2}\right) - \frac{1}{2} \sinh(a) \text{Shi}\left(\frac{b}{x^2}\right) \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = \frac{1}{2} \left(-\cosh(a) \text{Chi}\left(\frac{b}{x^2}\right) - \sinh(a) \text{Shi}\left(\frac{b}{x^2}\right) \right)$$

```
[In] Integrate[Cosh[a + b/x^2]/x,x]
```

```
[Out] (-(Cosh[a]*CoshIntegral[b/x^2]) - Sinh[a]*SinhIntegral[b/x^2])/2
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$\frac{e^{2a} e^{-a} \text{Ei}_1\left(-\frac{b}{x^2}\right)}{4} + \frac{e^{-a} \text{Ei}_1\left(\frac{b}{x^2}\right)}{4}$	33
meijerg	$-\frac{\sqrt{\pi} \cosh(a) \left(\frac{2\gamma - 4 \ln(x) + 2 \ln(ib)}{\sqrt{\pi}} + \frac{2 \text{Chi}\left(\frac{b}{x^2}\right) - 2 \ln\left(\frac{b}{x^2}\right) - 2\gamma}{\sqrt{\pi}} \right)}{4} - \frac{\text{Shi}\left(\frac{b}{x^2}\right) \sinh(a)}{2}$	62

```
[In] int(cosh(a+b/x^2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*exp(2*a)*exp(-a)*Ei(1,-b/x^2)+1/4*exp(-a)*Ei(1,b/x^2)
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = -\frac{1}{4} \left(\operatorname{Ei}\left(\frac{b}{x^2}\right) + \operatorname{Ei}\left(-\frac{b}{x^2}\right) \right) \cosh(a) \\ - \frac{1}{4} \left(\operatorname{Ei}\left(\frac{b}{x^2}\right) - \operatorname{Ei}\left(-\frac{b}{x^2}\right) \right) \sinh(a)$$

`[In] integrate(cosh(a+b/x^2)/x,x, algorithm="fricas")``[Out] -1/4*(Ei(b/x^2) + Ei(-b/x^2))*cosh(a) - 1/4*(Ei(b/x^2) - Ei(-b/x^2))*sinh(a)`**Sympy [F]**

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx$$

`[In] integrate(cosh(a+b/x**2)/x,x)``[Out] Integral(cosh(a + b/x**2)/x, x)`**Maxima [A] (verification not implemented)**

none

Time = 0.23 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = -\frac{1}{4} \operatorname{Ei}\left(-\frac{b}{x^2}\right) e^{(-a)} - \frac{1}{4} \operatorname{Ei}\left(\frac{b}{x^2}\right) e^a$$

`[In] integrate(cosh(a+b/x^2)/x,x, algorithm="maxima")``[Out] -1/4*Ei(-b/x^2)*e^(-a) - 1/4*Ei(b/x^2)*e^a`

Giac [F]

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx$$

[In] integrate(cosh(a+b/x^2)/x,x, algorithm="giac")

[Out] integrate(cosh(a + b/x^2)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x} dx = -\frac{\cosh(a) \operatorname{coshint}\left(\frac{b}{x^2}\right)}{2} - \frac{\sinh(a) \operatorname{sinhint}\left(\frac{b}{x^2}\right)}{2}$$

[In] int(cosh(a + b/x^2)/x,x)

[Out] - (cosh(a)*coshint(b/x^2))/2 - (sinh(a)*sinhint(b/x^2))/2

3.32 $\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx$

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Mupad [F(-1)]	185

Optimal result

Integrand size = 12, antiderivative size = 57

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = -\frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} - \frac{e^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

[Out] $-1/4*\operatorname{erf}(b^{(1/2)}/x)*\operatorname{Pi}^{(1/2)}/\exp(a)/b^{(1/2)}-1/4*\exp(a)*\operatorname{erfi}(b^{(1/2)}/x)*\operatorname{Pi}^{(1/2)}/b^{(1/2)}$

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5455, 5407, 2235, 2236}

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = -\frac{\sqrt{\pi}e^{-a}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} - \frac{\sqrt{\pi}e^a\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}}$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b/x^2]/x^2, x]$

[Out] $-1/4*(\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]/x])/(\operatorname{Sqrt}[b]*E^a) - (E^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]/x])/ (4*\operatorname{Sqrt}[b])$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5455

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbo
l] := -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\text{Subst}\left(\int \cosh(a + bx^2) dx, x, \frac{1}{x}\right) \\ &= -\left(\frac{1}{2}\text{Subst}\left(\int e^{-a-bx^2} dx, x, \frac{1}{x}\right)\right) - \frac{1}{2}\text{Subst}\left(\int e^{a+bx^2} dx, x, \frac{1}{x}\right) \\ &= -\frac{e^{-a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} - \frac{e^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{4\sqrt{b}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = -\frac{\sqrt{\pi}\left(\text{erf}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a) - \sinh(a)) + \text{erfi}\left(\frac{\sqrt{b}}{x}\right)(\cosh(a) + \sinh(a))\right)}{4\sqrt{b}}$$

```
[In] Integrate[Cosh[a + b/x^2]/x^2,x]
```

```
[Out] -1/4*(Sqrt[Pi]*(Erf[Sqrt[b]/x]*(Cosh[a] - Sinh[a]) + Erfi[Sqrt[b]/x]*(Cosh[
a] + Sinh[a]))) / Sqrt[b]
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.77

method	result	size
risch	$-\frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)\sqrt{\pi}e^{-a}}{4\sqrt{b}} - \frac{e^a\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{4\sqrt{-b}}$	44
meijerg	$\frac{i\sqrt{\pi}\cosh(a)\sqrt{2}\sqrt{ib}\left(\frac{\sqrt{ib}\sqrt{2}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}} + \frac{\sqrt{ib}\sqrt{2}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2\sqrt{b}}\right)}{4b} + \frac{\sqrt{\pi}\sinh(a)\sqrt{2}\sqrt{ib}\left(-\frac{(ib)^{\frac{3}{2}}\sqrt{2}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{2b^{\frac{3}{2}}} + \frac{(ib)^{\frac{3}{2}}\sqrt{2}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{2b^{\frac{3}{2}}}\right)}{4b}$	13

```
[In] int(cosh(a+b/x^2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*erf(b^(1/2)/x)*Pi^(1/2)/exp(a)/b^(1/2)-1/4*exp(a)*Pi^(1/2)/(-b)^(1/2)*
erf((-b)^(1/2)/x)
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx$$

$$= \frac{\sqrt{\pi}\sqrt{-b}(\cosh(a) + \sinh(a))\operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right) - \sqrt{\pi}\sqrt{b}(\cosh(a) - \sinh(a))\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{4b}$$

```
[In] integrate(cosh(a+b/x^2)/x^2,x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)/x) - sqrt(pi)*sqrt(
b)*(cosh(a) - sinh(a))*erf(sqrt(b)/x))/b
```

Sympy [F]

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx$$

```
[In] integrate(cosh(a+b/x**2)/x**2,x)
```

```
[Out] Integral(cosh(a + b/x**2)/x**2, x)
```


Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = \frac{1}{2} b \left(\frac{e^{(-a)} \Gamma\left(\frac{3}{2}, \frac{b}{x^2}\right)}{x^3 \left(\frac{b}{x^2}\right)^{\frac{3}{2}}} - \frac{e^a \Gamma\left(\frac{3}{2}, -\frac{b}{x^2}\right)}{x^3 \left(-\frac{b}{x^2}\right)^{\frac{3}{2}}} \right) - \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x}$$

[In] integrate(cosh(a+b/x^2)/x^2,x, algorithm="maxima")

[Out] 1/2*b*(e^(-a)*gamma(3/2, b/x^2)/(x^3*(b/x^2)^(3/2)) - e^a*gamma(3/2, -b/x^2)/(x^3*(-b/x^2)^(3/2))) - cosh(a + b/x^2)/x

Giac [F]

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx$$

[In] integrate(cosh(a+b/x^2)/x^2,x, algorithm="giac")

[Out] integrate(cosh(a + b/x^2)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^2} dx$$

[In] int(cosh(a + b/x^2)/x^2,x)

[Out] int(cosh(a + b/x^2)/x^2, x)

3.33 $\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx$

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Giac [B] (verification not implemented)	189
Mupad [B] (verification not implemented)	189

Optimal result

Integrand size = 12, antiderivative size = 15

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$$

[Out] $-1/2*\sinh(a+b/x^2)/b$

Rubi [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5429, 2717}

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$$

[In] `Int[Cosh[a + b/x^2]/x^3,x]`

[Out] $-1/2*\sinh[a + b/x^2]/b$

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_.)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 5429

`Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]`
`:= Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])`
`^p, x], x, x^n], x] /;` `FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify`

```
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rubi steps

$$\begin{aligned} \text{integral} &= -\left(\frac{1}{2}\text{Subst}\left(\int \cosh(a + bx) dx, x, \frac{1}{x^2}\right)\right) \\ &= -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$$

```
[In] Integrate[Cosh[a + b/x^2]/x^3,x]
```

```
[Out] -1/2*Sinh[a + b/x^2]/b
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.93

method	result	size
derivativedivides	$-\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$	14
default	$-\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$	14
parallelrisc	$-\frac{\sinh\left(\frac{ax^2+b}{x^2}\right)}{2b}$	18
risc	$-\frac{e^{\frac{ax^2+b}{x^2}}}{4b} + \frac{e^{-\frac{ax^2+b}{x^2}}}{4b}$	37
meijerg	$-\frac{\cosh(a)\sinh\left(\frac{b}{x^2}\right)}{2b} + \frac{\sqrt{\pi}\sinh(a)\left(\frac{1}{\sqrt{\pi}} - \frac{\cosh\left(\frac{b}{x^2}\right)}{\sqrt{\pi}}\right)}{2b}$	40

```
[In] int(cosh(a+b/x^2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*sinh(a+b/x^2)/b
```

Fricas [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{\sinh\left(\frac{ax^2+b}{x^2}\right)}{2b}$$

`[In] integrate(cosh(a+b/x^2)/x^3,x, algorithm="fricas")``[Out] -1/2*sinh((a*x^2 + b)/x^2)/b`**Sympy [A] (verification not implemented)**

Time = 0.53 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = \begin{cases} -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b} & \text{for } b \neq 0 \\ -\frac{\cosh(a)}{2x^2} & \text{otherwise} \end{cases}$$

`[In] integrate(cosh(a+b/x**2)/x**3,x)``[Out] Piecewise((-sinh(a + b/x**2)/(2*b), Ne(b, 0)), (-cosh(a)/(2*x**2), True))`**Maxima [A] (verification not implemented)**

none

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$$

`[In] integrate(cosh(a+b/x^2)/x^3,x, algorithm="maxima")``[Out] -1/2*sinh(a + b/x^2)/b`

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(13) = 26$.

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{e^{\left(\frac{ax^2+b}{x^2}\right)} - e^{\left(-\frac{ax^2+b}{x^2}\right)}}{4b}$$

[In] integrate(cosh(a+b/x^2)/x^3,x, algorithm="giac")

[Out] -1/4*(e^((a*x^2 + b)/x^2) - e^(-(a*x^2 + b)/x^2))/b

Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^3} dx = -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2b}$$

[In] int(cosh(a + b/x^2)/x^3,x)

[Out] -sinh(a + b/x^2)/(2*b)

3.34 $\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx$

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Optimal result

Integrand size = 12, antiderivative size = 75

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = -\frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{e^a\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} - \frac{\sinh\left(a + \frac{b}{x^2}\right)}{2bx}$$

[Out] $-1/2*\sinh(a+b/x^2)/b/x-1/8*\operatorname{erf}(b^{(1/2)}/x)*\operatorname{Pi}^{(1/2)}/b^{(3/2)}/\exp(a)+1/8*\exp(a)*\operatorname{erfi}(b^{(1/2)}/x)*\operatorname{Pi}^{(1/2)}/b^{(3/2)}$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {5455, 5433, 5406, 2235, 2236}

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = -\frac{\sqrt{\pi}e^{-a}\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{\sqrt{\pi}e^a\operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} - \frac{\sinh\left(a + \frac{b}{x^2}\right)}{2bx}$$

[In] $\operatorname{Int}[\operatorname{Cosh}[a + b/x^2]/x^4, x]$

[Out] $-1/8*(\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]/x])/(b^{(3/2)}*E^a) + (E^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]/x])/(8*b^{(3/2)}) - \operatorname{Sinh}[a + b/x^2]/(2*b*x)$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_.))^2)}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2]))], x] /; \operatorname{FreeQ}\{F, a, b, c, d\}, x] \ \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5406

```
Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5433

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[e^(
n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1
)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x]
&& IGtQ[n, 0] && LtQ[0, n, m + 1]
```

Rule 5455

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] := -Subst[Int[(a + b*Cosh[c + d/x^n])^p/x^(m + 2), x], x, 1/x] /; FreeQ[
{a, b, c, d}, x] && IntegerQ[p] && ILtQ[n, 0] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= -\text{Subst}\left(\int x^2 \cosh(a + bx^2) dx, x, \frac{1}{x}\right) \\
&= -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2bx} + \frac{\text{Subst}\left(\int \sinh(a + bx^2) dx, x, \frac{1}{x}\right)}{2b} \\
&= -\frac{\sinh\left(a + \frac{b}{x^2}\right)}{2bx} - \frac{\text{Subst}\left(\int e^{-a-bx^2} dx, x, \frac{1}{x}\right)}{4b} + \frac{\text{Subst}\left(\int e^{a+bx^2} dx, x, \frac{1}{x}\right)}{4b} \\
&= -\frac{e^{-a}\sqrt{\pi}\text{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} + \frac{e^a\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{3/2}} - \frac{\sinh\left(a + \frac{b}{x^2}\right)}{2bx}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = \frac{\sqrt{\pi} x \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) (-\cosh(a) + \sinh(a)) + \sqrt{\pi} x \operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right) (\cosh(a) + \sinh(a)) - 4\sqrt{b} \sinh\left(a + \frac{b}{x^2}\right)}{8b^{3/2}x}$$

[In] Integrate[Cosh[a + b/x^2]/x^4,x]

[Out] (Sqrt[Pi]*x*Erf[Sqrt[b]/x]*(-Cosh[a] + Sinh[a]) + Sqrt[Pi]*x*Erfi[Sqrt[b]/x]*(Cosh[a] + Sinh[a]) - 4*Sqrt[b]*Sinh[a + b/x^2])/(8*b^(3/2)*x)

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.09

method	result
risch	$\frac{e^{-a} e^{-\frac{b}{x^2}}}{4bx} - \frac{\operatorname{erf}\left(\frac{\sqrt{b}}{x}\right) \sqrt{\pi} e^{-a}}{8b^{\frac{3}{2}}} - \frac{e^a e^{\frac{b}{x^2}}}{4xb} + \frac{e^a \sqrt{\pi} \operatorname{erf}\left(\frac{\sqrt{-b}}{x}\right)}{8b\sqrt{-b}}$
meijerg	$\frac{\sqrt{\pi} \cosh(a) \sqrt{2} \sqrt{ib} \left(\frac{\sqrt{2} (ib)^{\frac{3}{2}} e^{\frac{b}{x^2}}}{4\sqrt{\pi} xb} - \frac{\sqrt{2} (ib)^{\frac{3}{2}} e^{-\frac{b}{x^2}}}{4\sqrt{\pi} xb} + \frac{(ib)^{\frac{3}{2}} \sqrt{2} \operatorname{erf}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}} - \frac{(ib)^{\frac{3}{2}} \sqrt{2} \operatorname{erfi}\left(\frac{\sqrt{b}}{x}\right)}{8b^{\frac{3}{2}}} \right)}{2b^2} - \frac{i\sqrt{\pi} \sinh(a) \sqrt{2} \sqrt{ib} \left(\frac{\sqrt{2} (ib)^{\frac{5}{2}} e^{-\frac{b}{x^2}}}{4\sqrt{\pi} x b^2} - \frac{\sqrt{2} (ib)^{\frac{5}{2}} e^{\frac{b}{x^2}}}{4\sqrt{\pi} x b^2} \right)}{2b^2}$

[In] int(cosh(a+b/x^2)/x^4,x,method=_RETURNVERBOSE)

[Out] 1/4/exp(a)/b/x*exp(-b/x^2)-1/8*erf(b^(1/2)/x)*Pi^(1/2)/b^(3/2)/exp(a)-1/4*exp(a)*exp(b/x^2)/x/b+1/8*exp(a)/b*Pi^(1/2)/(-b)^(1/2)*erf((-b)^(1/2)/x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(55) = 110.

Time = 0.25 (sec) , antiderivative size = 250, normalized size of antiderivative = 3.33

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = \frac{2b \cosh\left(\frac{ax^2+b}{x^2}\right)^2 + \sqrt{\pi} \left(x \cosh(a) \cosh\left(\frac{ax^2+b}{x^2}\right) + x \cosh\left(\frac{ax^2+b}{x^2}\right) \sinh(a) + (x \cosh(a) + x \sinh(a)) \sinh\left(\frac{ax^2+b}{x^2}\right) \right)}{8b^{3/2}x}$$

[In] integrate(cosh(a+b/x^2)/x^4,x, algorithm="fricas")


```
[Out] -1/8*(2*b*cosh((a*x^2 + b)/x^2)^2 + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x^2) + x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) + x*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(-b)*erf(sqrt(-b)/x) + sqrt(pi)*(x*cosh(a)*cosh((a*x^2 + b)/x^2) - x*cosh((a*x^2 + b)/x^2)*sinh(a) + (x*cosh(a) - x*sinh(a))*sinh((a*x^2 + b)/x^2))*sqrt(b)*erf(sqrt(b)/x) + 4*b*cosh((a*x^2 + b)/x^2)*sinh((a*x^2 + b)/x^2) + 2*b*sinh((a*x^2 + b)/x^2)^2 - 2*b)/(b^2*x*cosh((a*x^2 + b)/x^2) + b^2*x*sinh((a*x^2 + b)/x^2))
```

Sympy [F]

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx$$

```
[In] integrate(cosh(a+b/x**2)/x**4,x)
```

```
[Out] Integral(cosh(a + b/x**2)/x**4, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = \frac{1}{6} b \left(\frac{e^{(-a)} \Gamma\left(\frac{5}{2}, \frac{b}{x^2}\right)}{x^5 \left(\frac{b}{x^2}\right)^{\frac{5}{2}}} - \frac{e^a \Gamma\left(\frac{5}{2}, -\frac{b}{x^2}\right)}{x^5 \left(-\frac{b}{x^2}\right)^{\frac{5}{2}}} \right) - \frac{\cosh\left(a + \frac{b}{x^2}\right)}{3x^3}$$

```
[In] integrate(cosh(a+b/x^2)/x^4,x, algorithm="maxima")
```

```
[Out] 1/6*b*(e^(-a)*gamma(5/2, b/x^2)/(x^5*(b/x^2)^(5/2)) - e^a*gamma(5/2, -b/x^2)/(x^5*(-b/x^2)^(5/2))) - 1/3*cosh(a + b/x^2)/x^3
```

Giac [F]

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx$$

```
[In] integrate(cosh(a+b/x^2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(cosh(a + b/x^2)/x^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx = \int \frac{\cosh\left(a + \frac{b}{x^2}\right)}{x^4} dx$$

```
[In] int(cosh(a + b/x^2)/x^4,x)
```

```
[Out] int(cosh(a + b/x^2)/x^4, x)
```

3.35 $\int \cosh(a + bx^n) dx$

Optimal result	195
Rubi [A] (verified)	195
Mathematica [A] (verified)	196
Maple [C] (verified)	196
Fricas [F]	197
Sympy [F]	197
Maxima [A] (verification not implemented)	197
Giac [F]	197
Mupad [F(-1)]	198

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cosh(a + bx^n) dx = -\frac{e^a x(-bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x(bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, bx^n\right)}{2n}$$

[Out] $-1/2*\exp(a)*x*\text{GAMMA}(1/n, -b*x^n)/n/((-b*x^n)^{(1/n)}) - 1/2*x*\text{GAMMA}(1/n, b*x^n)/\exp(a)/n/((b*x^n)^{(1/n)})$

Rubi [A] (verified)

Time = 0.02 (sec), antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5415, 2239}

$$\int \cosh(a + bx^n) dx = -\frac{e^a x(-bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x(bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, bx^n\right)}{2n}$$

[In] $\text{Int}[\text{Cosh}[a + b*x^n], x]$

[Out] $-1/2*(E^a*x*\text{Gamma}[n^(-1), -(b*x^n)])/(n*(-(b*x^n))^{n^(-1)}) - (x*\text{Gamma}[n^(-1), b*x^n])/(2*E^a*n*(b*x^n)^{n^(-1)})$

Rule 2239

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}, x_Symbol] \rightarrow \text{Simp}[(-F^a)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*\text{Log}[F]]/(d*n*((-b)*(c + d*x)^n*\text{Log}[F])^{(1/n)})), x] /; \text{FreeQ}\{F, a, b, c, d, n\}, x \ \&\& \ !\text{IntegerQ}[2/n]$

Rule 5415

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \int e^{-a-bx^n} dx + \frac{1}{2} \int e^{a+bx^n} dx \\ &= -\frac{e^a x (-bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x (bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, bx^n\right)}{2n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \cosh(a + bx^n) dx = -\frac{e^a x (-bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -bx^n\right) + e^{-a} x (bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, bx^n\right)}{2n}$$

```
[In] Integrate[Cosh[a + b*x^n], x]
```

```
[Out] -1/2*((E^a*x*Gamma[n^(-1), -(b*x^n)])/(-(b*x^n))^n^(-1) + (x*Gamma[n^(-1),
b*x^n])/(E^a*(b*x^n)^n^(-1)))/n
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.10

method	result	size
meijerg	$x \operatorname{hypergeom}\left(\left[\frac{1}{2n}\right], \left[\frac{1}{2}, 1 + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \cosh(a) + \frac{x^{n+1} b \operatorname{hypergeom}\left(\left[\frac{1}{2} + \frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2} + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \sinh(a)}{n+1}$	74

```
[In] int(cosh(a+b*x^n), x, method=_RETURNVERBOSE)
```

```
[Out] x*hypergeom([1/2/n], [1/2, 1+1/2/n], 1/4*x^(2*n)*b^2)*cosh(a)+1/(n+1)*x^(n+1)*
b*hypergeom([1/2+1/2/n], [3/2, 3/2+1/2/n], 1/4*x^(2*n)*b^2)*sinh(a)
```

Fricas [F]

$$\int \cosh(a + bx^n) dx = \int \cosh(bx^n + a) dx$$

[In] integrate(cosh(a+b*x^n),x, algorithm="fricas")

[Out] integral(cosh(b*x^n + a), x)

Sympy [F]

$$\int \cosh(a + bx^n) dx = \int \cosh(a + bx^n) dx$$

[In] integrate(cosh(a+b*x**n),x)

[Out] Integral(cosh(a + b*x**n), x)

Maxima [A] (verification not implemented)

none

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \cosh(a + bx^n) dx = -\frac{xe^{(-a)}\Gamma\left(\frac{1}{n}, bx^n\right)}{2 (bx^n)^{\left(\frac{1}{n}\right)} n} - \frac{xe^a\Gamma\left(\frac{1}{n}, -bx^n\right)}{2 (-bx^n)^{\left(\frac{1}{n}\right)} n}$$

[In] integrate(cosh(a+b*x^n),x, algorithm="maxima")

[Out] -1/2*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 1/2*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^(1/n)*n)

Giac [F]

$$\int \cosh(a + bx^n) dx = \int \cosh(bx^n + a) dx$$

[In] integrate(cosh(a+b*x^n),x, algorithm="giac")

[Out] integrate(cosh(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int \cosh(a + bx^n) dx = \int \cosh(a + bx^n) dx$$

```
[In] int(cosh(a + b*x^n), x)
```

```
[Out] int(cosh(a + b*x^n), x)
```

3.36 $\int \frac{\cosh(a+bx^n)}{x} dx$

Optimal result	199
Rubi [A] (verified)	199
Mathematica [A] (verified)	200
Maple [A] (verified)	200
Fricas [B] (verification not implemented)	201
Sympy [F]	201
Maxima [A] (verification not implemented)	201
Giac [F]	202
Mupad [F(-1)]	202

Optimal result

Integrand size = 12, antiderivative size = 25

$$\int \frac{\cosh(a+bx^n)}{x} dx = \frac{\cosh(a)\text{Chi}(bx^n)}{n} + \frac{\sinh(a)\text{Shi}(bx^n)}{n}$$

[Out] Chi(b*x^n)*cosh(a)/n+Shi(b*x^n)*sinh(a)/n

Rubi [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {5427, 5425, 5424}

$$\int \frac{\cosh(a+bx^n)}{x} dx = \frac{\cosh(a)\text{Chi}(bx^n)}{n} + \frac{\sinh(a)\text{Shi}(bx^n)}{n}$$

[In] Int[Cosh[a + b*x^n]/x,x]

[Out] (Cosh[a]*CoshIntegral[b*x^n])/n + (Sinh[a]*SinhIntegral[b*x^n])/n

Rule 5424

Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinhIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5425

Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CoshIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \cosh(a) \int \frac{\cosh(bx^n)}{x} dx + \sinh(a) \int \frac{\sinh(bx^n)}{x} dx \\ &= \frac{\cosh(a)\text{Chi}(bx^n)}{n} + \frac{\sinh(a)\text{Shi}(bx^n)}{n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(a + bx^n)}{x} dx = \frac{\cosh(a)\text{Chi}(bx^n) + \sinh(a)\text{Shi}(bx^n)}{n}$$

```
[In] Integrate[Cosh[a + b*x^n]/x,x]
```

```
[Out] (Cosh[a]*CoshIntegral[b*x^n] + Sinh[a]*SinhIntegral[b*x^n])/n
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.32

method	result	size
risch	$-\frac{e^{-a} \text{Ei}_1(bx^n)}{2n} - \frac{e^a \text{Ei}_1(-bx^n)}{2n}$	33
meijerg	$\frac{\sqrt{\pi} \left(\frac{2\gamma + 2n \ln(x) + 2 \ln(ib)}{\sqrt{\pi}} + \frac{2 \text{Chi}(bx^n) - 2 \ln(bx^n) - 2\gamma}{\sqrt{\pi}} \right) \cosh(a)}{2n} + \frac{\text{Shi}(bx^n) \sinh(a)}{n}$	68

```
[In] int(cosh(a+b*x^n)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/n*exp(-a)*Ei(1,b*x^n)-1/2/n*exp(a)*Ei(1,-b*x^n)
```


Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.26 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \frac{\cosh(a + bx^n)}{x} dx = \frac{(\cosh(a) + \sinh(a))\text{Ei}(b \cosh(n \log(x)) + b \sinh(n \log(x))) + (\cosh(a) - \sinh(a))\text{Ei}(-b \cosh(n \log(x)))}{2n}$$

[In] integrate(cosh(a+b*x^n)/x,x, algorithm="fricas")

[Out] 1/2*((cosh(a) + sinh(a))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + (cosh(a) - sinh(a))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))))/n

Sympy [F]

$$\int \frac{\cosh(a + bx^n)}{x} dx = \int \frac{\cosh(a + bx^n)}{x} dx$$

[In] integrate(cosh(a+b*x**n)/x,x)

[Out] Integral(cosh(a + b*x**n)/x, x)

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.20

$$\int \frac{\cosh(a + bx^n)}{x} dx = \frac{\text{Ei}(-bx^n) e^{(-a)}}{2n} + \frac{\text{Ei}(bx^n) e^a}{2n}$$

[In] integrate(cosh(a+b*x^n)/x,x, algorithm="maxima")

[Out] 1/2*Ei(-b*x^n)*e^(-a)/n + 1/2*Ei(b*x^n)*e^a/n

Giac [F]

$$\int \frac{\cosh(a + bx^n)}{x} dx = \int \frac{\cosh(bx^n + a)}{x} dx$$

[In] integrate(cosh(a+b*x^n)/x,x, algorithm="giac")

[Out] integrate(cosh(b*x^n + a)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a + bx^n)}{x} dx = \int \frac{\cosh(a + b x^n)}{x} dx$$

[In] int(cosh(a + b*x^n)/x,x)

[Out] int(cosh(a + b*x^n)/x, x)

3.37 $\int \cosh^2(a + bx^n) dx$

Optimal result	203
Rubi [A] (verified)	203
Mathematica [A] (verified)	204
Maple [F]	205
Fricas [F]	205
Sympy [F]	205
Maxima [A] (verification not implemented)	205
Giac [F]	206
Mupad [F(-1)]	206

Optimal result

Integrand size = 10, antiderivative size = 89

$$\int \cosh^2(a + bx^n) dx = \frac{x}{2} - \frac{2^{-2-\frac{1}{n}} e^{2a} x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -2bx^n)}{n} - \frac{2^{-2-\frac{1}{n}} e^{-2a} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, 2bx^n)}{n}$$

[Out] $1/2*x-2^{(-2-1/n)}*\exp(2*a)*x*\text{GAMMA}(1/n, -2*b*x^n)/n/((-b*x^n)^{(1/n)})-2^{(-2-1/n)}*x*\text{GAMMA}(1/n, 2*b*x^n)/\exp(2*a)/n/((b*x^n)^{(1/n)})$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5417, 5415, 2239}

$$\int \cosh^2(a + bx^n) dx = -\frac{e^{2a} 2^{-\frac{1}{n}-2} x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -2bx^n)}{n} - \frac{e^{-2a} 2^{-\frac{1}{n}-2} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, 2bx^n)}{n} + \frac{x}{2}$$

[In] Int[Cosh[a + b*x^n]^2, x]

[Out] $x/2 - (2^{(-2 - n^{-1})} * E^{(2*a)} * x * \text{Gamma}[n^{-1}, -2*b*x^n]) / (n * (-b*x^n)^{n^{-1}(-1)}) - (2^{(-2 - n^{-1})} * x * \text{Gamma}[n^{-1}, 2*b*x^n]) / (E^{(2*a)} * n * (b*x^n)^{n^{-1}(-1)})$

Rule 2239

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^(n_)), x_Symbol] := Simp[(-F^a)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log

$[F])^{(1/n)})$, $x]$ /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]

Rule 5415

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]

Rule 5417

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_), x_Symbol] := Int[ExpandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{1}{2} + \frac{1}{2} \cosh(2a + 2bx^n) \right) dx \\
 &= \frac{x}{2} + \frac{1}{2} \int \cosh(2a + 2bx^n) dx \\
 &= \frac{x}{2} + \frac{1}{4} \int e^{-2a-2bx^n} dx + \frac{1}{4} \int e^{2a+2bx^n} dx \\
 &= \frac{x}{2} - \frac{2^{-2-\frac{1}{n}} e^{2a} x (-bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -2bx^n\right)}{n} - \frac{2^{-2-\frac{1}{n}} e^{-2a} x (bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, 2bx^n\right)}{n}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.91

$$\begin{aligned}
 &\int \cosh^2(a + bx^n) dx \\
 &= -\frac{x \left(-2n + 2^{-1/n} e^{2a} (-bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, -2bx^n\right) + 2^{-1/n} e^{-2a} (bx^n)^{-1/n} \Gamma\left(\frac{1}{n}, 2bx^n\right) \right)}{4n}
 \end{aligned}$$

[In] Integrate[Cosh[a + b*x^n]^2,x]

[Out] -1/4*(x*(-2*n + (E^(2*a)*Gamma[n^(-1), -2*b*x^n])/(2^n^(-1)*(-(b*x^n))^n^(-1))) + Gamma[n^(-1), 2*b*x^n]/(2^n^(-1)*E^(2*a)*(b*x^n)^n^(-1)))/n

Maple [F]

$$\int \cosh(a + bx^n)^2 dx$$

[In] int(cosh(a+b*x^n)^2,x)

[Out] int(cosh(a+b*x^n)^2,x)

Fricas [F]

$$\int \cosh^2(a + bx^n) dx = \int \cosh(bx^n + a)^2 dx$$

[In] integrate(cosh(a+b*x^n)^2,x, algorithm="fricas")

[Out] integral(cosh(b*x^n + a)^2, x)

Sympy [F]

$$\int \cosh^2(a + bx^n) dx = \int \cosh^2(a + bx^n) dx$$

[In] integrate(cosh(a+b*x**n)**2,x)

[Out] Integral(cosh(a + b*x**n)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.76

$$\int \cosh^2(a + bx^n) dx = \frac{1}{2}x - \frac{xe^{(-2a)}\Gamma\left(\frac{1}{n}, 2bx^n\right)}{4(2bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{xe^{(2a)}\Gamma\left(\frac{1}{n}, -2bx^n\right)}{4(-2bx^n)^{\left(\frac{1}{n}\right)}n}$$

[In] integrate(cosh(a+b*x^n)^2,x, algorithm="maxima")

[Out] 1/2*x - 1/4*x*e^(-2*a)*gamma(1/n, 2*b*x^n)/((2*b*x^n)^(1/n)*n) - 1/4*x*e^(2*a)*gamma(1/n, -2*b*x^n)/((-2*b*x^n)^(1/n)*n)

Giac [F]

$$\int \cosh^2(a + bx^n) dx = \int \cosh(bx^n + a)^2 dx$$

[In] integrate(cosh(a+b*x^n)^2,x, algorithm="giac")

[Out] integrate(cosh(b*x^n + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int \cosh^2(a + bx^n) dx = \int \cosh(a + bx^n)^2 dx$$

[In] int(cosh(a + b*x^n)^2,x)

[Out] int(cosh(a + b*x^n)^2, x)

3.38 $\int \frac{\cosh^2(a+bx^n)}{x} dx$

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Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \frac{\cosh^2(a+bx^n)}{x} dx = \frac{\cosh(2a)\text{Chi}(2bx^n)}{2n} + \frac{\log(x)}{2} + \frac{\sinh(2a)\text{Shi}(2bx^n)}{2n}$$

[Out] $1/2*\text{Chi}(2*b*x^n)*\cosh(2*a)/n+1/2*\ln(x)+1/2*\text{Shi}(2*b*x^n)*\sinh(2*a)/n$

Rubi [A] (verified)

Time = 0.05 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5471, 5427, 5425, 5424}

$$\int \frac{\cosh^2(a+bx^n)}{x} dx = \frac{\cosh(2a)\text{Chi}(2bx^n)}{2n} + \frac{\sinh(2a)\text{Shi}(2bx^n)}{2n} + \frac{\log(x)}{2}$$

[In] `Int[Cosh[a + b*x^n]^2/x,x]`

[Out] $(\text{Cosh}[2*a]*\text{CoshIntegral}[2*b*x^n])/(2*n) + \text{Log}[x]/2 + (\text{Sinh}[2*a]*\text{SinhIntegral}[2*b*x^n])/(2*n)$

Rule 5424

`Int[Sinh[(d.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinhIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]`

Rule 5425

`Int[Cosh[(d.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CoshIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]`

Rule 5427

```
Int[Cosh[(c_) + (d_)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5471

```
Int[((a_) + Cosh[(c_) + (d_)*(x_)^(n_)]*(b_))^(p_)*((e_)*(x_))^(m_),
x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{1}{2x} + \frac{\cosh(2a + 2bx^n)}{2x} \right) dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \int \frac{\cosh(2a + 2bx^n)}{x} dx \\
&= \frac{\log(x)}{2} + \frac{1}{2} \cosh(2a) \int \frac{\cosh(2bx^n)}{x} dx + \frac{1}{2} \sinh(2a) \int \frac{\sinh(2bx^n)}{x} dx \\
&= \frac{\cosh(2a)\text{Chi}(2bx^n)}{2n} + \frac{\log(x)}{2} + \frac{\sinh(2a)\text{Shi}(2bx^n)}{2n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^2(a + bx^n)}{x} dx = \frac{\cosh(2a)\text{Chi}(2bx^n) + n \log(x) + \sinh(2a)\text{Shi}(2bx^n)}{2n}$$

```
[In] Integrate[Cosh[a + b*x^n]^2/x, x]
```

```
[Out] (Cosh[2*a]*CoshIntegral[2*b*x^n] + n*Log[x] + Sinh[2*a]*SinhIntegral[2*b*x^n])/(2*n)
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.93

method	result	size
risch	$\frac{\ln(x)}{2} - \frac{e^{-2a} \text{Ei}_1(2bx^n)}{4n} - \frac{e^{2a} \text{Ei}_1(-2bx^n)}{4n}$	40

[In] `int(cosh(a+b*x^n)^2/x,x,method=_RETURNVERBOSE)`

[Out] $1/2*\ln(x)-1/4/n*\exp(-2*a)*\text{Ei}(1,2*b*x^n)-1/4/n*\exp(2*a)*\text{Ei}(1,-2*b*x^n)$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.60

$$\int \frac{\cosh^2(a + bx^n)}{x} dx = \frac{(\cosh(2a) + \sinh(2a))\text{Ei}(2b \cosh(n \log(x)) + 2b \sinh(n \log(x))) + (\cosh(2a) - \sinh(2a))\text{Ei}(-2b \cosh(n \log(x)) - 2b \sinh(n \log(x)))}{4n}$$

[In] `integrate(cosh(a+b*x^n)^2/x,x, algorithm="fricas")`

[Out] $1/4*((\cosh(2*a) + \sinh(2*a))*\text{Ei}(2*b*\cosh(n*\log(x)) + 2*b*\sinh(n*\log(x))) + (\cosh(2*a) - \sinh(2*a))*\text{Ei}(-2*b*\cosh(n*\log(x)) - 2*b*\sinh(n*\log(x))) + 2*n*\log(x))/n$

Sympy [F]

$$\int \frac{\cosh^2(a + bx^n)}{x} dx = \int \frac{\cosh^2(a + bx^n)}{x} dx$$

[In] `integrate(cosh(a+b*x**n)**2/x,x)`

[Out] `Integral(cosh(a + b*x**n)**2/x, x)`

Maxima [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.86

$$\int \frac{\cosh^2(a + bx^n)}{x} dx = \frac{\text{Ei}(2bx^n)e^{2a}}{4n} + \frac{\text{Ei}(-2bx^n)e^{-2a}}{4n} + \frac{1}{2} \log(x)$$

[In] `integrate(cosh(a+b*x^n)^2/x,x, algorithm="maxima")`

[Out] $1/4*\text{Ei}(2*b*x^n)*e^{(2*a)}/n + 1/4*\text{Ei}(-2*b*x^n)*e^{(-2*a)}/n + 1/2*\log(x)$

Giac [F]

$$\int \frac{\cosh^2(a + bx^n)}{x} dx = \int \frac{\cosh(bx^n + a)^2}{x} dx$$

[In] integrate(cosh(a+b*x^n)^2/x,x, algorithm="giac")

[Out] integrate(cosh(b*x^n + a)^2/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a + bx^n)}{x} dx = \int \frac{\cosh(a + bx^n)^2}{x} dx$$

[In] int(cosh(a + b*x^n)^2/x,x)

[Out] int(cosh(a + b*x^n)^2/x, x)

3.39 $\int \cosh^3(a + bx^n) dx$

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Giac [F]	214
Mupad [F(-1)]	214

Optimal result

Integrand size = 10, antiderivative size = 150

$$\int \cosh^3(a + bx^n) dx = -\frac{3^{-1/n} e^{3a} x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -3bx^n)}{8n} - \frac{3e^a x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -bx^n)}{8n} \\ - \frac{3e^{-a} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, bx^n)}{8n} - \frac{3^{-1/n} e^{-3a} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, 3bx^n)}{8n}$$

[Out] $-1/8*\exp(3*a)*x*\text{GAMMA}(1/n, -3*b*x^n)/(3^{(1/n)})/n/((-b*x^n)^{(1/n)}) - 3/8*\exp(a)*x*\text{GAMMA}(1/n, -b*x^n)/n/((-b*x^n)^{(1/n)}) - 3/8*x*\text{GAMMA}(1/n, b*x^n)/\exp(a)/n/(b*x^n)^{(1/n)} - 1/8*x*\text{GAMMA}(1/n, 3*b*x^n)/(3^{(1/n)})/\exp(3*a)/n/(b*x^n)^{(1/n)}$

Rubi [A] (verified)

Time = 0.06 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {5417, 5415, 2239}

$$\int \cosh^3(a + bx^n) dx = -\frac{e^{3a} 3^{-1/n} x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -3bx^n)}{8n} - \frac{3e^a x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -bx^n)}{8n} \\ - \frac{3e^{-a} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, bx^n)}{8n} - \frac{e^{-3a} 3^{-1/n} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, 3bx^n)}{8n}$$

[In] Int[Cosh[a + b*x^n]^3, x]

[Out] $-1/8*(E^{(3*a)}*x*\text{Gamma}[n^{(-1)}, -3*b*x^n])/(3^n^{(-1)}*n*(-(b*x^n))^{n^{(-1)}}) - (3*E^a*x*\text{Gamma}[n^{(-1)}, -(b*x^n)])/(8*n*(-(b*x^n))^{n^{(-1)}}) - (3*x*\text{Gamma}[n^{(-1)}, b*x^n])/(8*E^a*n*(b*x^n)^{n^{(-1)}}) - (x*\text{Gamma}[n^{(-1)}, 3*b*x^n])/(8*3^n^{(-1)}*E^{(3*a)}*n*(b*x^n)^{n^{(-1)}})$

Rule 2239

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)), x_Symbol] := Simp[(-F^a
)*(c + d*x)*(Gamma[1/n, (-b)*(c + d*x)^n*Log[F]]/(d*n*((-b)*(c + d*x)^n*Log
[F]))^(1/n)), x] /; FreeQ[{F, a, b, c, d, n}, x] && !IntegerQ[2/n]
```

Rule 5415

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d, n}, x]
```

Rule 5417

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_), x_Symbol] := Int[Ex
pandTrigReduce[(a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3}{4} \cosh(a + bx^n) + \frac{1}{4} \cosh(3a + 3bx^n) \right) dx \\
&= \frac{1}{4} \int \cosh(3a + 3bx^n) dx + \frac{3}{4} \int \cosh(a + bx^n) dx \\
&= \frac{1}{8} \int e^{-3a-3bx^n} dx + \frac{1}{8} \int e^{3a+3bx^n} dx + \frac{3}{8} \int e^{-a-bx^n} dx + \frac{3}{8} \int e^{a+bx^n} dx \\
&= -\frac{3^{-1/n} e^{3a} x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -3bx^n)}{8n} - \frac{3e^a x (-bx^n)^{-1/n} \Gamma(\frac{1}{n}, -bx^n)}{8n} \\
&\quad - \frac{3e^{-a} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, bx^n)}{8n} - \frac{3^{-1/n} e^{-3a} x (bx^n)^{-1/n} \Gamma(\frac{1}{n}, 3bx^n)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.92

$$\int \cosh^3(a + bx^n) dx = \frac{3^{-1/n} e^{-3a} x (-b^2 x^{2n})^{-1/n} \left(e^{6a} (bx^n)^{\frac{1}{n}} \Gamma(\frac{1}{n}, -3bx^n) + 3^{1+\frac{1}{n}} e^{4a} (bx^n)^{\frac{1}{n}} \Gamma(\frac{1}{n}, -bx^n) + (-bx^n)^{\frac{1}{n}} \left(3^{1+\frac{1}{n}} e^{2a} \Gamma(\frac{1}{n}, bx^n) + 3^{-1+\frac{1}{n}} e^{-2a} \Gamma(\frac{1}{n}, 3bx^n) \right) \right)}{8n}$$

```
[In] Integrate[Cosh[a + b*x^n]^3, x]
```

```
[Out] -1/8*(x*(E^(6*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -3*b*x^n] + 3^(1 + n^(-1))*E^(
4*a)*(b*x^n)^n^(-1)*Gamma[n^(-1), -(b*x^n)] + (-(b*x^n))^n^(-1)*(3^(1 + n^
(-1))*E^(2*a)*Gamma[n^(-1), b*x^n] + Gamma[n^(-1), 3*b*x^n]))) / (3^n^(-1)*E^
(3*a)*n*(-(b^2*x^(2*n)))^n^(-1))
```

Maple [F]

$$\int \cosh(a + bx^n)^3 dx$$

[In] int(cosh(a+b*x^n)^3,x)

[Out] int(cosh(a+b*x^n)^3,x)

Fricas [F]

$$\int \cosh^3(a + bx^n) dx = \int \cosh(bx^n + a)^3 dx$$

[In] integrate(cosh(a+b*x^n)^3,x, algorithm="fricas")

[Out] integral(cosh(b*x^n + a)^3, x)

Sympy [F]

$$\int \cosh^3(a + bx^n) dx = \int \cosh^3(a + bx^n) dx$$

[In] integrate(cosh(a+b*x**n)**3,x)

[Out] Integral(cosh(a + b*x**n)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.15 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.83

$$\int \cosh^3(a + bx^n) dx = -\frac{xe^{(-3a)}\Gamma\left(\frac{1}{n}, 3bx^n\right)}{8(3bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{3xe^{(-a)}\Gamma\left(\frac{1}{n}, bx^n\right)}{8(bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{3xe^a\Gamma\left(\frac{1}{n}, -bx^n\right)}{8(-bx^n)^{\left(\frac{1}{n}\right)}n} - \frac{xe^{(3a)}\Gamma\left(\frac{1}{n}, -3bx^n\right)}{8(-3bx^n)^{\left(\frac{1}{n}\right)}n}$$

[In] integrate(cosh(a+b*x^n)^3,x, algorithm="maxima")

[Out] -1/8*x*e^(-3*a)*gamma(1/n, 3*b*x^n)/((3*b*x^n)^(1/n)*n) - 3/8*x*e^(-a)*gamma(1/n, b*x^n)/((b*x^n)^(1/n)*n) - 3/8*x*e^a*gamma(1/n, -b*x^n)/((-b*x^n)^(1/n)*n) - 1/8*x*e^(3*a)*gamma(1/n, -3*b*x^n)/((-3*b*x^n)^(1/n)*n)

Giac [F]

$$\int \cosh^3(a + bx^n) dx = \int \cosh(bx^n + a)^3 dx$$

[In] integrate(cosh(a+b*x^n)^3,x, algorithm="giac")

[Out] integrate(cosh(b*x^n + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int \cosh^3(a + bx^n) dx = \int \cosh(a + bx^n)^3 dx$$

[In] int(cosh(a + b*x^n)^3,x)

[Out] int(cosh(a + b*x^n)^3, x)

3.40 $\int \frac{\cosh^3(a+bx^n)}{x} dx$

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Maxima [A] (verification not implemented)	218
Giac [F]	218
Mupad [F(-1)]	218

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\cosh^3(a+bx^n)}{x} dx = \frac{3 \cosh(a)\text{Chi}(bx^n)}{4n} + \frac{\cosh(3a)\text{Chi}(3bx^n)}{4n} + \frac{3 \sinh(a)\text{Shi}(bx^n)}{4n} + \frac{\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

[Out] 3/4*Chi(b*x^n)*cosh(a)/n+1/4*Chi(3*b*x^n)*cosh(3*a)/n+3/4*Shi(b*x^n)*sinh(a)/n+1/4*Shi(3*b*x^n)*sinh(3*a)/n

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5471, 5427, 5425, 5424}

$$\int \frac{\cosh^3(a+bx^n)}{x} dx = \frac{3 \cosh(a)\text{Chi}(bx^n)}{4n} + \frac{\cosh(3a)\text{Chi}(3bx^n)}{4n} + \frac{3 \sinh(a)\text{Shi}(bx^n)}{4n} + \frac{\sinh(3a)\text{Shi}(3bx^n)}{4n}$$

[In] Int[Cosh[a + b*x^n]^3/x,x]

[Out] (3*Cosh[a]*CoshIntegral[b*x^n])/(4*n) + (Cosh[3*a]*CoshIntegral[3*b*x^n])/(4*n) + (3*Sinh[a]*SinhIntegral[b*x^n])/(4*n) + (Sinh[3*a]*SinhIntegral[3*b*x^n])/(4*n)

Rule 5424

Int[Sinh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[SinhIntegral[d*x^n]/n, x] /; FreeQ[{d, n}, x]

Rule 5425

```
Int[Cosh[(d_.)*(x_)^(n_)]/(x_), x_Symbol] := Simp[CoshIntegral[d*x^n]/n, x]
  /; FreeQ[{d, n}, x]
```

Rule 5427

```
Int[Cosh[(c_) + (d_.)*(x_)^(n_)]/(x_), x_Symbol] := Dist[Cosh[c], Int[Cosh[
d*x^n]/x, x], x] + Dist[Sinh[c], Int[Sinh[d*x^n]/x, x], x] /; FreeQ[{c, d,
n}, x]
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_)^(m_.),
x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x
] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{3 \cosh(a + bx^n)}{4x} + \frac{\cosh(3a + 3bx^n)}{4x} \right) dx \\
&= \frac{1}{4} \int \frac{\cosh(3a + 3bx^n)}{x} dx + \frac{3}{4} \int \frac{\cosh(a + bx^n)}{x} dx \\
&= \frac{1}{4} (3 \cosh(a)) \int \frac{\cosh(bx^n)}{x} dx + \frac{1}{4} \cosh(3a) \int \frac{\cosh(3bx^n)}{x} dx \\
&\quad + \frac{1}{4} (3 \sinh(a)) \int \frac{\sinh(bx^n)}{x} dx + \frac{1}{4} \sinh(3a) \int \frac{\sinh(3bx^n)}{x} dx \\
&= \frac{3 \cosh(a) \text{Chi}(bx^n)}{4n} + \frac{\cosh(3a) \text{Chi}(3bx^n)}{4n} + \frac{3 \sinh(a) \text{Shi}(bx^n)}{4n} + \frac{\sinh(3a) \text{Shi}(3bx^n)}{4n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.78

$$\begin{aligned}
&\int \frac{\cosh^3(a + bx^n)}{x} dx \\
&= \frac{3 \cosh(a) \text{Chi}(bx^n) + \cosh(3a) \text{Chi}(3bx^n) + 3 \sinh(a) \text{Shi}(bx^n) + \sinh(3a) \text{Shi}(3bx^n)}{4n}
\end{aligned}$$

```
[In] Integrate[Cosh[a + b*x^n]^3/x, x]
```

```
[Out] (3*Cosh[a]*CoshIntegral[b*x^n] + Cosh[3*a]*CoshIntegral[3*b*x^n] + 3*Sinh[a]
]*SinhIntegral[b*x^n] + Sinh[3*a]*SinhIntegral[3*b*x^n])/(4*n)
```


Maple [A] (verified)

Time = 0.89 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{e^{-3a} \operatorname{Ei}_1(3bx^n)}{8n} - \frac{3e^{-a} \operatorname{Ei}_1(bx^n)}{8n} - \frac{e^{3a} \operatorname{Ei}_1(-3bx^n)}{8n} - \frac{3e^a \operatorname{Ei}_1(-bx^n)}{8n}$	67

[In] int(cosh(a+b*x^n)^3/x,x,method=_RETURNVERBOSE)

[Out] $-\frac{1}{8n} \exp(-3a) \operatorname{Ei}(1, 3bx^n) - \frac{3}{8n} \exp(-a) \operatorname{Ei}(1, bx^n) - \frac{1}{8n} \exp(3a) \operatorname{Ei}(1, -3bx^n) - \frac{3}{8n} \exp(a) \operatorname{Ei}(1, -bx^n)$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^3(a + bx^n)}{x} dx = \frac{(\cosh(3a) + \sinh(3a)) \operatorname{Ei}(3b \cosh(n \log(x)) + 3b \sinh(n \log(x))) + 3(\cosh(a) + \sinh(a)) \operatorname{Ei}(b \cosh(n \log(x)) + b \sinh(n \log(x))) + 3(\cosh(a) - \sinh(a)) \operatorname{Ei}(-b \cosh(n \log(x)) - b \sinh(n \log(x))) + (\cosh(3a) - \sinh(3a)) \operatorname{Ei}(-3b \cosh(n \log(x)) - 3b \sinh(n \log(x)))}{n}$$

[In] integrate(cosh(a+b*x^n)^3/x,x, algorithm="fricas")

[Out] $\frac{1}{8} * ((\cosh(3a) + \sinh(3a)) * \operatorname{Ei}(3b * \cosh(n * \log(x)) + 3b * \sinh(n * \log(x))) + 3 * (\cosh(a) + \sinh(a)) * \operatorname{Ei}(b * \cosh(n * \log(x)) + b * \sinh(n * \log(x))) + 3 * (\cosh(a) - \sinh(a)) * \operatorname{Ei}(-b * \cosh(n * \log(x)) - b * \sinh(n * \log(x))) + (\cosh(3a) - \sinh(3a)) * \operatorname{Ei}(-3b * \cosh(n * \log(x)) - 3b * \sinh(n * \log(x)))) / n$

Sympy [F]

$$\int \frac{\cosh^3(a + bx^n)}{x} dx = \int \frac{\cosh^3(a + bx^n)}{x} dx$$

[In] integrate(cosh(a+b*x**n)**3/x,x)

[Out] Integral(cosh(a + b*x**n)**3/x, x)

Maxima [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^3(a + bx^n)}{x} dx = \frac{\text{Ei}(3bx^n) e^{(3a)}}{8n} + \frac{3 \text{Ei}(-bx^n) e^{(-a)}}{8n} + \frac{\text{Ei}(-3bx^n) e^{(-3a)}}{8n} + \frac{3 \text{Ei}(bx^n) e^a}{8n}$$

[In] integrate(cosh(a+b*x^n)^3/x,x, algorithm="maxima")

[Out] 1/8*Ei(3*b*x^n)*e^(3*a)/n + 3/8*Ei(-b*x^n)*e^(-a)/n + 1/8*Ei(-3*b*x^n)*e^(-3*a)/n + 3/8*Ei(b*x^n)*e^a/n

Giac [F]

$$\int \frac{\cosh^3(a + bx^n)}{x} dx = \int \frac{\cosh(bx^n + a)^3}{x} dx$$

[In] integrate(cosh(a+b*x^n)^3/x,x, algorithm="giac")

[Out] integrate(cosh(b*x^n + a)^3/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a + bx^n)}{x} dx = \int \frac{\cosh(a + bx^n)^3}{x} dx$$

[In] int(cosh(a + b*x^n)^3/x,x)

[Out] int(cosh(a + b*x^n)^3/x, x)

3.41 $\int (ex)^m (b \cosh (c + dx^n))^p dx$

Optimal result	219
Rubi [N/A]	219
Mathematica [N/A]	220
Maple [N/A] (verified)	220
Fricas [N/A]	220
Sympy [N/A]	220
Maxima [N/A]	221
Giac [N/A]	221
Mupad [N/A]	221

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (ex)^m (b \cosh (c + dx^n))^p dx = \text{Int}((ex)^m (b \cosh (c + dx^n))^p, x)$$

[Out] Unintegrable((e*x)^m*(b*cosh(c+d*x^n))^p,x)

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (b \cosh (c + dx^n))^p dx = \int (ex)^m (b \cosh (c + dx^n))^p dx$$

[In] Int[(e*x)^m*(b*Cosh[c + d*x^n])^p,x]

[Out] Defer[Int] [(e*x)^m*(b*Cosh[c + d*x^n])^p, x]

Rubi steps

$$\text{integral} = \int (ex)^m (b \cosh (c + dx^n))^p dx$$

Mathematica [N/A]

Not integrable

Time = 4.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(c + dx^n))^p dx$$

[In] Integrate[(e*x)^m*(b*Cosh[c + d*x^n])^p,x]

[Out] Integrate[(e*x)^m*(b*Cosh[c + d*x^n])^p, x]

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (ex)^m (b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^m*(b*cosh(c+d*x^n))^p,x)

[Out] int((e*x)^m*(b*cosh(c+d*x^n))^p,x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^m*(b*cosh(d*x^n + c))^p, x)

Sympy [N/A]

Not integrable

Time = 8.30 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (ex)^m (b \cosh(c + dx^n))^p dx = \int (b \cosh(c + dx^n))^p (ex)^m dx$$

[In] integrate((e*x)**m*(b*cosh(c+d*x**n))**p,x)

[Out] Integral((b*cosh(c + d*x**n))**p*(e*x)**m, x)

Maxima [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^m*(b*cosh(d*x^n + c))^p, x)

Giac [N/A]

Not integrable

Time = 1.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^m*(b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^m*(b*cosh(d*x^n + c))^p, x)

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (ex)^m (b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^m*(b*cosh(c + d*x^n))^p,x)

[Out] int((e*x)^m*(b*cosh(c + d*x^n))^p, x)

3.42 $\int (ex)^m (a + b \cosh(c + dx^n))^p dx$

Optimal result	222
Rubi [N/A]	222
Mathematica [N/A]	223
Maple [N/A] (verified)	223
Fricas [N/A]	223
Sympy [N/A]	223
Maxima [N/A]	224
Giac [N/A]	224
Mupad [N/A]	224

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \text{Int}((ex)^m (a + b \cosh(c + dx^n))^p, x)$$

[Out] Unintegrable((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)

Rubi [N/A]

Not integrable

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (a + b \cosh(c + dx^n))^p dx$$

[In] Int[(e*x)^m*(a + b*Cosh[c + d*x^n])^p,x]

[Out] Defer[Int][(e*x)^m*(a + b*Cosh[c + d*x^n])^p, x]

Rubi steps

$$\text{integral} = \int (ex)^m (a + b \cosh(c + dx^n))^p dx$$

Mathematica [N/A]

Not integrable

Time = 6.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (a + b \cosh(c + dx^n))^p dx$$

[In] Integrate[(e*x)^m*(a + b*Cosh[c + d*x^n])^p,x]

[Out] Integrate[(e*x)^m*(a + b*Cosh[c + d*x^n])^p, x]

Maple [N/A] (verified)

Not integrable

Time = 0.04 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)

[Out] int((e*x)^m*(a+b*cosh(c+d*x^n))^p,x)

Fricas [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)

Sympy [N/A]

Not integrable

Time = 25.34 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (a + b \cosh(c + dx^n))^p dx$$

[In] integrate((e*x)**m*(a+b*cosh(c+d*x**n))**p,x)

[Out] Integral((e*x)**m*(a + b*cosh(c + d*x**n))**p, x)

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)

Giac [N/A]

Not integrable

Time = 7.71 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^m*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^m*(b*cosh(d*x^n + c) + a)^p, x)

Mupad [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (ex)^m (a + b \cosh(c + dx^n))^p dx = \int (ex)^m (a + b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^m*(a + b*cosh(c + d*x^n))^p,x)

[Out] int((e*x)^m*(a + b*cosh(c + d*x^n))^p, x)

3.43 $\int (ex)^{-1+n} (b \cosh (c + dx^n))^p dx$

Optimal result	225
Rubi [A] (verified)	225
Mathematica [A] (verified)	226
Maple [F]	227
Fricas [F]	227
Sympy [F]	227
Maxima [F]	227
Giac [F]	228
Mupad [F(-1)]	228

Optimal result

Integrand size = 20, antiderivative size = 95

$$\int (ex)^{-1+n} (b \cosh (c + dx^n))^p dx = \frac{x^{-n}(ex)^n (b \cosh (c + dx^n))^{1+p} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cosh^2(c + dx^n)\right) \sinh(c + dx^n)}{bden(1+p)\sqrt{-\sinh^2(c + dx^n)}}$$

[Out] $-(e*x)^n*(b*\cosh(c+d*x^n))^{p+1}*\operatorname{hypergeom}([1/2, 1/2+1/2*p], [3/2+1/2*p], \cosh(c+d*x^n)^2)*\sinh(c+d*x^n)/b/d/e/n/(p+1)/(x^n)/(-\sinh(c+d*x^n)^2)^{1/2}$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {5431, 5429, 2722}

$$\int (ex)^{-1+n} (b \cosh (c + dx^n))^p dx = \frac{x^{-n}(ex)^n \sinh(c + dx^n) (b \cosh (c + dx^n))^{p+1} \operatorname{Hypergeometric2F1}\left(\frac{1}{2}, \frac{p+1}{2}, \frac{p+3}{2}, \cosh^2(dx^n + c)\right)}{bden(p+1)\sqrt{-\sinh^2(c + dx^n)}}$$

[In] $\operatorname{Int}[(e*x)^{-1+n}*(b*\operatorname{Cosh}[c + d*x^n])^p, x]$

[Out] $-\left(\left(\left(e*x\right)^n*(b*\operatorname{Cosh}[c + d*x^n])^{(1+p)}*\operatorname{Hypergeometric2F1}\left[\frac{1}{2}, \frac{(1+p)}{2}, \frac{(3+p)}{2}, \operatorname{Cosh}[c + d*x^n]^2\right]*\operatorname{Sinh}[c + d*x^n]\right)/(b*d*e*n*(1+p)*x^n*\operatorname{Sqrt}[-\operatorname{Sin h}[c + d*x^n]^2])\right)$

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] := Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
&& !IntegerQ[2*n]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5431

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_.), x
_Symbol] := Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{(x^{-n}(ex)^n) \int x^{-1+n} (b \cosh(c + dx^n))^p dx}{e} \\ &= \frac{(x^{-n}(ex)^n) \text{Subst}(\int (b \cosh(c + dx))^p dx, x, x^n)}{en} \\ &= \frac{x^{-n}(ex)^n (b \cosh(c + dx^n))^{1+p} \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cosh^2(c + dx^n)\right) \sinh(c + dx^n)}{bden(1+p)\sqrt{-\sinh^2(c + dx^n)}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.95

$$\begin{aligned} &\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx \\ &= \frac{x^{1-n}(ex)^{-1+n} (b \cosh(c + dx^n))^p \coth(c + dx^n) \text{Hypergeometric2F1}\left(\frac{1}{2}, \frac{1+p}{2}, \frac{3+p}{2}, \cosh^2(c + dx^n)\right) \sqrt{-\sinh^2(c + dx^n)}}{dn(1+p)} \end{aligned}$$

```
[In] Integrate[(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p,x]
```

```
[Out] (x^(1 - n)*(e*x)^(-1 + n)*(b*Cosh[c + d*x^n])^p*Coth[c + d*x^n]*Hypergeomet
ric2F1[1/2, (1 + p)/2, (3 + p)/2, Cosh[c + d*x^n]^2]*Sqrt[-Sinh[c + d*x^n]^
2])/(d*n*(1 + p))
```

Maple [F]

$$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x)

[Out] int((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x)

Fricas [F]

$$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)

Sympy [F]

$$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx = \int (b \cosh(c + dx^n))^p (ex)^{n-1} dx$$

[In] integrate((e*x)**(-1+n)*(b*cosh(c+d*x**n))**p,x)

[Out] Integral((b*cosh(c + d*x**n))**p*(e*x)**(n - 1), x)

Maxima [F]

$$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)

Giac [F]

$$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^(-1+n)*(b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c))^p, x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(n - 1)*(b*cosh(c + d*x^n))^p,x)

[Out] int((e*x)^(n - 1)*(b*cosh(c + d*x^n))^p, x)

3.44 $\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx$

Optimal result	229
Rubi [N/A]	229
Mathematica [N/A]	230
Maple [N/A] (verified)	230
Fricas [N/A]	230
Sympy [N/A]	230
Maxima [N/A]	231
Giac [N/A]	231
Mupad [N/A]	231

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \frac{x^{-2n}(ex)^{2n} \text{Int}(x^{-1+2n}(b \cosh(c + dx^n))^p, x)}{e}$$

[Out] $(e*x)^{(2*n)}*Unintegrable(x^{(-1+2*n)}*(b*\cosh(c+d*x^n))^p,x)/e/(x^{(2*n)})$

Rubi [N/A]

Not integrable

Time = 0.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx$$

[In] $\text{Int}[(e*x)^{(-1 + 2*n)}*(b*\text{Cosh}[c + d*x^n])^p,x]$

[Out] $((e*x)^{(2*n)}*Defer[\text{Int}[x^{(-1 + 2*n)}*(b*\text{Cosh}[c + d*x^n])^p, x]])/(e*x^{(2*n)})$

Rubi steps

$$\text{integral} = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(b \cosh(c + dx^n))^p dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx$$

[In] Integrate[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p,x]

[Out] Integrate[(e*x)^(-1 + 2*n)*(b*Cosh[c + d*x^n])^p, x]

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int (ex)^{2n-1} (b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(2*n-1)*(b*cosh(c+d*x^n))^p,x)

[Out] int((e*x)^(2*n-1)*(b*cosh(c+d*x^n))^p,x)

Fricas [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)

Sympy [N/A]

Not integrable

Time = 7.94 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (b \cosh(c + dx^n))^p (ex)^{2n-1} dx$$

[In] integrate((e*x)**(-1+2*n)*(b*cosh(c+d*x**n))**p,x)

[Out] Integral((b*cosh(c + d*x**n))**p*(e*x)**(2*n - 1), x)

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)

Giac [N/A]

Not integrable

Time = 1.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(dx^n + c))^p dx$$

[In] integrate((e*x)^(-1+2*n)*(b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c))^p, x)

Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int (ex)^{-1+2n} (b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(2*n - 1)*(b*cosh(c + d*x^n))^p,x)

[Out] int((e*x)^(2*n - 1)*(b*cosh(c + d*x^n))^p, x)

3.45 $\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$

Optimal result	232
Rubi [A] (verified)	232
Mathematica [A] (verified)	234
Maple [F]	234
Fricas [F]	235
Sympy [F]	235
Maxima [F]	235
Giac [F]	235
Mupad [F(-1)]	236

Optimal result

Integrand size = 22, antiderivative size = 131

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$$

$$= \frac{\sqrt{2}x^{-n}(ex)^n \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, \frac{1}{2}(1 - \cosh(c + dx^n)), \frac{b(1 - \cosh(c + dx^n))}{a+b}\right) (a + b \cosh(c + dx^n))^p \left(\frac{a+b \cosh(c + dx^n)}{a+b}\right)}{den \sqrt{1 + \cosh(c + dx^n)}}$$

[Out] (e*x)^n*AppellF1(1/2, -p, 1/2, 3/2, b*(1-cosh(c+d*x^n))/(a+b), 1/2-1/2*cosh(c+d*x^n))*(a+b*cosh(c+d*x^n))^p*sinh(c+d*x^n)*2^(1/2)/d/e/n/(x^n)/(((a+b*cosh(c+d*x^n))/(a+b))^p)/(1+cosh(c+d*x^n))^(1/2)

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {5431, 5429, 2744, 144, 143}

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$$

$$= \frac{\sqrt{2}x^{-n}(ex)^n \sinh(c + dx^n) (a + b \cosh(c + dx^n))^p \left(\frac{a+b \cosh(c + dx^n)}{a+b}\right)^{-p} \operatorname{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, \frac{1}{2}(1 - \cosh(c + dx^n)), \frac{b(1 - \cosh(c + dx^n))}{a+b}\right)}{den \sqrt{\cosh(c + dx^n) + 1}}$$

[In] Int[(e*x)^(-1 + n)*(a + b*Cosh[c + d*x^n])^p,x]

[Out] (Sqrt[2]*(e*x)^n*AppellF1[1/2, 1/2, -p, 3/2, (1 - Cosh[c + d*x^n])/2, (b*(1 - Cosh[c + d*x^n]))/(a + b)]*(a + b*Cosh[c + d*x^n])^p*Sinh[c + d*x^n])/(d*e*n*x^n*Sqrt[1 + Cosh[c + d*x^n]]*((a + b*Cosh[c + d*x^n]))/(a + b))^p

Rule 143

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Simp[((a + b*x)^(m + 1)/(b*(m + 1)*(b/(b*c - a*d))^n*(b
/(b*e - a*f))^p))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c - a*d
)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n, p},
x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b*c - a*d)
, 0] && GtQ[b/(b*e - a*f), 0] && !(GtQ[d/(d*a - c*b), 0] && GtQ[d/(d*e - c
*f), 0] && SimplrQ[c + d*x, a + b*x]) && !(GtQ[f/(f*a - e*b), 0] && GtQ[f
/(f*c - e*d), 0] && SimplrQ[e + f*x, a + b*x])
```

Rule 144

```
Int[((a_) + (b_)*(x_))^(m_)*((c_) + (d_)*(x_))^(n_)*((e_) + (f_)*(x_))
^(p_), x_Symbol] := Dist[(e + f*x)^FracPart[p]/((b/(b*e - a*f))^IntPart[p]*
(b*((e + f*x)/(b*e - a*f)))^FracPart[p]), Int[(a + b*x)^m*(c + d*x)^n*(b*(e
/(b*e - a*f)) + b*f*(x/(b*e - a*f)))^p, x], x] /; FreeQ[{a, b, c, d, e, f,
m, n, p}, x] && !IntegerQ[m] && !IntegerQ[n] && !IntegerQ[p] && GtQ[b/(b
*c - a*d), 0] && !GtQ[b/(b*e - a*f), 0]
```

Rule 2744

```
Int[((a_) + (b_)*sin[(c_) + (d_)*(x_)])^(n_), x_Symbol] := Dist[Cos[c +
d*x]/(d*Sqrt[1 + Sin[c + d*x]]*Sqrt[1 - Sin[c + d*x]]), Subst[Int[(a + b*x)
^n/(Sqrt[1 + x]*Sqrt[1 - x]), x], x, Sin[c + d*x]], x] /; FreeQ[{a, b, c, d
, n}, x] && NeQ[a^2 - b^2, 0] && !IntegerQ[2*n]
```

Rule 5429

```
Int[((a_) + Cosh[(c_) + (d_)*(x_)^(n_)])*(b_)^(p_)*(x_)^(m_), x_Symbo
l] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5431

```
Int[((a_) + Cosh[(c_) + (d_)*(x_)^(n_)])*(b_)^(p_)*((e_)*(x_))^(m_), x
_Symbol] := Dist[e^IntPart[m]*((e*x)^FracPart[m]/x^FracPart[m]), Int[x^m*(a
+ b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n, p}, x] && In
tegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\text{integral} = \frac{(x^{-n}(ex)^n) \int x^{-1+n} (a + b \cosh(c + dx^n))^p dx}{e}$$

$$\begin{aligned}
&= \frac{(x^{-n}(ex)^n) \text{Subst}\left(\int (a + b \cosh(c + dx))^p dx, x, x^n\right)}{en} \\
&= \frac{(x^{-n}(ex)^n \sinh(c + dx^n)) \text{Subst}\left(\int \frac{(a+bx)^p}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cosh(c + dx^n)\right)}{den \sqrt{1 - \cosh(c + dx^n)} \sqrt{1 + \cosh(c + dx^n)}} \\
&= \frac{\left(x^{-n}(ex)^n (a + b \cosh(c + dx^n))^p \left(-\frac{a+b \cosh(c+dx^n)}{-a-b}\right)^{-p} \sinh(c + dx^n)\right) \text{Subst}\left(\int \frac{\left(-\frac{a}{-a-b} - \frac{bx}{-a-b}\right)^p}{\sqrt{1-x}\sqrt{1+x}} dx, x, \cosh(c + dx^n)\right)}{den \sqrt{1 - \cosh(c + dx^n)} \sqrt{1 + \cosh(c + dx^n)}} \\
&= \frac{\sqrt{2} x^{-n} (ex)^n \text{AppellF1}\left(\frac{1}{2}, \frac{1}{2}, -p, \frac{3}{2}, \frac{1}{2}(1 - \cosh(c + dx^n)), \frac{b(1 - \cosh(c + dx^n))}{a+b}\right) (a + b \cosh(c + dx^n))^p}{den \sqrt{1 + \cosh(c + dx^n)}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13

$$\begin{aligned}
&\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx \\
&= \frac{x^{-n} (ex)^n \text{AppellF1}\left(1 + p, \frac{1}{2}, \frac{1}{2}, 2 + p, \frac{a+b \cosh(c+dx^n)}{a+b}, \frac{a+b \cosh(c+dx^n)}{a-b}\right) \sqrt{-\frac{b(-1+\cosh(c+dx^n))}{a+b}} \sqrt{\frac{b(1+\cosh(c+dx^n))}{-a+b}} (a + b \cosh(c + dx^n))^p}{bden(1 + p)}
\end{aligned}$$

[In] Integrate[(e*x)^(-1 + n)*(a + b*Cosh[c + d*x^n])^p,x]

[Out] ((e*x)^n*AppellF1[1 + p, 1/2, 1/2, 2 + p, (a + b*Cosh[c + d*x^n])/(a + b), (a + b*Cosh[c + d*x^n])/(a - b)]*Sqrt[-((b*(-1 + Cosh[c + d*x^n]))/(a + b))]*Sqrt[(b*(1 + Cosh[c + d*x^n]))/(-a + b)]*(a + b*Cosh[c + d*x^n])^(1 + p)*Csch[c + d*x^n])/(b*d*e*n*(1 + p)*x^n)

Maple [F]

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x)

[Out] int((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x)

Fricas [F]

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)

Sympy [F]

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (a + b \cosh(c + dx^n))^p dx$$

[In] integrate((e*x)**(-1+n)*(a+b*cosh(c+d*x**n))**p,x)

[Out] Integral((e*x)**(n - 1)*(a + b*cosh(c + d*x**n))**p, x)

Maxima [F]

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)

Giac [F]

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^(-1+n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^(n - 1)*(b*cosh(d*x^n + c) + a)^p, x)

Mupad [F(-1)]

Timed out.

$$\int (ex)^{-1+n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{n-1} (a + b \cosh(c + dx^n))^p dx$$

```
[In] int((e*x)^(n - 1)*(a + b*cosh(c + d*x^n))^p, x)
```

```
[Out] int((e*x)^(n - 1)*(a + b*cosh(c + d*x^n))^p, x)
```

3.46 $\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx$

Optimal result	237
Rubi [N/A]	237
Mathematica [N/A]	238
Maple [N/A] (verified)	238
Fricas [N/A]	238
Sympy [N/A]	238
Maxima [N/A]	239
Giac [N/A]	239
Mupad [N/A]	239

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \frac{x^{-2n}(ex)^{2n} \text{Int}(x^{-1+2n}(a + b \cosh(c + dx^n))^p, x)}{e}$$

[Out] $(e*x)^{(2*n)}*Unintegrable(x^{(-1+2*n)}*(a+b*\cosh(c+d*x^n))^p,x)/e/(x^{(2*n)})$

Rubi [N/A]

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx$$

[In] $\text{Int}[(e*x)^{(-1 + 2*n)}*(a + b*\text{Cosh}[c + d*x^n])^p,x]$

[Out] $((e*x)^{(2*n)}*Defer[\text{Int}[x^{(-1 + 2*n)}*(a + b*\text{Cosh}[c + d*x^n])^p, x]])/(e*x^{(2*n)})$

Rubi steps

$$\text{integral} = \frac{(x^{-2n}(ex)^{2n}) \int x^{-1+2n}(a + b \cosh(c + dx^n))^p dx}{e}$$

Mathematica [N/A]

Not integrable

Time = 6.37 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx$$

[In] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p,x]

[Out] Integrate[(e*x)^(-1 + 2*n)*(a + b*Cosh[c + d*x^n])^p, x]

Maple [N/A] (verified)

Not integrable

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int (ex)^{2n-1} (a + b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(2*n-1)*(a+b*cosh(c+d*x^n))^p,x)

[Out] int((e*x)^(2*n-1)*(a+b*cosh(c+d*x^n))^p,x)

Fricas [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="fricas")

[Out] integral((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)

Sympy [N/A]

Not integrable

Time = 24.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (a + b \cosh(c + dx^n))^p dx$$

[In] integrate((e*x)**(-1+2*n)*(a+b*cosh(c+d*x**n))**p,x)

[Out] Integral((e*x)**(2*n - 1)*(a + b*cosh(c + d*x**n))**p, x)

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="maxima")

[Out] integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)

Giac [N/A]

Not integrable

Time = 7.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (b \cosh(dx^n + c) + a)^p dx$$

[In] integrate((e*x)^(-1+2*n)*(a+b*cosh(c+d*x^n))^p,x, algorithm="giac")

[Out] integrate((e*x)^(2*n - 1)*(b*cosh(d*x^n + c) + a)^p, x)

Mupad [N/A]

Not integrable

Time = 1.55 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int (ex)^{-1+2n} (a + b \cosh(c + dx^n))^p dx = \int (ex)^{2n-1} (a + b \cosh(c + dx^n))^p dx$$

[In] int((e*x)^(2*n - 1)*(a + b*cosh(c + d*x^n))^p,x)

[Out] int((e*x)^(2*n - 1)*(a + b*cosh(c + d*x^n))^p, x)

3.47 $\int x^m \cosh(a + bx^n) dx$

Optimal result	240
Rubi [A] (verified)	240
Mathematica [A] (verified)	241
Maple [C] (verified)	241
Fricas [F]	242
Sympy [F]	242
Maxima [A] (verification not implemented)	242
Giac [F]	243
Mupad [F(-1)]	243

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x^m \cosh(a + bx^n) dx = -\frac{e^a x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{2n}$$

[Out] $-1/2*\exp(a)*x^{(1+m)}*GAMMA((1+m)/n, -b*x^n)/n/((-b*x^n)^{((1+m)/n)}) - 1/2*x^{(1+m)}*GAMMA((1+m)/n, b*x^n)/\exp(a)/n/((b*x^n)^{((1+m)/n)})$

Rubi [A] (verified)

Time = 0.05 (sec), antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {5469, 2250}

$$\int x^m \cosh(a + bx^n) dx = -\frac{e^a x^{m+1} (-bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x^{m+1} (bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, bx^n\right)}{2n}$$

[In] $\text{Int}[x^m \text{Cosh}[a + b*x^n], x]$

[Out] $-1/2*(E^a*x^{(1+m)}*Gamma[(1+m)/n, -(b*x^n)])/(n*(-(b*x^n))^{((1+m)/n)}) - (x^{(1+m)}*Gamma[(1+m)/n, b*x^n])/(2*E^a*n*(b*x^n)^{((1+m)/n)})$

Rule 2250

$\text{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{(n_.)})}*((e_.) + (f_.)*(x_))^{(m_.)}, x_Symbol] \rightarrow \text{Simp}[(-F^a)*((e + f*x)^{(m + 1)})/(f*n*((-b)*(c + d*x)^n*\text{Log}[$

$F])^{((m + 1)/n)} * \text{Gamma}[(m + 1)/n, (-b)*(c + d*x)^n * \text{Log}[F]], x] /; \text{FreeQ}[\{F, a, b, c, d, e, f, m, n\}, x] \&\& \text{EqQ}[d*e - c*f, 0]$

Rule 5469

$\text{Int}[\text{Cosh}[(c_.) + (d_.)*(x_)^(n_.)]*((e_.)*(x_)^(m_.)), x_Symbol] \rightarrow \text{Dist}[1/2, \text{Int}[(e*x)^m * E^{(c + d*x^n)}, x], x] + \text{Dist}[1/2, \text{Int}[(e*x)^m * E^{-(c - d*x^n)}, x], x] /; \text{FreeQ}[\{c, d, e, m, n\}, x]$

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{1}{2} \int e^{-a-bx^n} x^m dx + \frac{1}{2} \int e^{a+bx^n} x^m dx \\ &= -\frac{e^a x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{2n} - \frac{e^{-a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{2n} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.94

$$\begin{aligned} &\int x^m \cosh(a + bx^n) dx \\ &= -\frac{e^a x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right) + e^{-a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{2n} \end{aligned}$$

[In] Integrate[x^m*Cosh[a + b*x^n],x]

[Out] $-1/2*((E^a*x^{(1+m)}*\text{Gamma}[(1+m)/n, -(b*x^n)])/(-(b*x^n))^{((1+m)/n)} + (x^{(1+m)}*\text{Gamma}[(1+m)/n, b*x^n])/(E^a*(b*x^n)^{((1+m)/n)}))/n$

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.30 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

method	result
meijerg	$\frac{x^{1+m} \text{hypergeom}\left(\left[\frac{m}{2n} + \frac{1}{2n}\right], \left[\frac{1}{2}, 1 + \frac{m}{2n} + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \cosh(a)}{1+m} + \frac{x^{n+m+1} b \text{hypergeom}\left(\left[\frac{1}{2} + \frac{m}{2n} + \frac{1}{2n}\right], \left[\frac{3}{2}, \frac{3}{2} + \frac{m}{2n} + \frac{1}{2n}\right], \frac{x^{2n} b^2}{4}\right) \sinh(a)}{n+m+1}$

[In] int(x^m*cosh(a+b*x^n),x,method=_RETURNVERBOSE)

[Out] $1/(1+m)*x^{(1+m)}*hypergeom([1/2/n*m+1/2/n], [1/2, 1+1/2/n*m+1/2/n], 1/4*x^{(2*n)}*b^2)*cosh(a)+1/(n+m+1)*x^{(n+m+1)}*b*hypergeom([1/2+1/2/n*m+1/2/n], [3/2, 3/2+1/2/n*m+1/2/n], 1/4*x^{(2*n)}*b^2)*sinh(a)$

Fricas [F]

$$\int x^m \cosh(a + bx^n) dx = \int x^m \cosh(bx^n + a) dx$$

[In] `integrate(x^m*cosh(a+b*x^n),x, algorithm="fricas")`

[Out] `integral(x^m*cosh(b*x^n + a), x)`

Sympy [F]

$$\int x^m \cosh(a + bx^n) dx = \int x^m \cosh(a + bx^n) dx$$

[In] `integrate(x**m*cosh(a+b*x**n),x)`

[Out] `Integral(x**m*cosh(a + b*x**n), x)`

Maxima [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.96

$$\int x^m \cosh(a + bx^n) dx = -\frac{x^{m+1}e^{(-a)}\Gamma\left(\frac{m+1}{n}, bx^n\right)}{2 (bx^n)^{\frac{m+1}{n}} n} - \frac{x^{m+1}e^a\Gamma\left(\frac{m+1}{n}, -bx^n\right)}{2 (-bx^n)^{\frac{m+1}{n}} n}$$

[In] `integrate(x^m*cosh(a+b*x^n),x, algorithm="maxima")`

[Out] $-1/2*x^{(m + 1)}*e^{(-a)}*\gamma((m + 1)/n, b*x^n)/((b*x^n)^{((m + 1)/n)*n}) - 1/2*x^{(m + 1)}*e^a*\gamma((m + 1)/n, -b*x^n)/((-b*x^n)^{((m + 1)/n)*n})$

Giac [F]

$$\int x^m \cosh(a + bx^n) dx = \int x^m \cosh(bx^n + a) dx$$

[In] integrate(x^m*cosh(a+b*x^n),x, algorithm="giac")

[Out] integrate(x^m*cosh(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a + bx^n) dx = \int x^m \cosh(a + bx^n) dx$$

[In] int(x^m*cosh(a + b*x^n),x)

[Out] int(x^m*cosh(a + b*x^n), x)

3.48 $\int x^m \cosh^2(a + bx^n) dx$

Optimal result	244
Rubi [A] (verified)	244
Mathematica [A] (verified)	245
Maple [F]	246
Fricas [F]	246
Sympy [F]	246
Maxima [A] (verification not implemented)	246
Giac [F]	247
Mupad [F(-1)]	247

Optimal result

Integrand size = 14, antiderivative size = 128

$$\int x^m \cosh^2(a + bx^n) dx = \frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a} x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2bx^n\right)}{n} - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2bx^n\right)}{n}$$

[Out] $\frac{1}{2} x^{1+m} / (1+m) - \exp(2a) x^{1+m} \text{GAMMA}\left(\frac{1+m}{n}, -2b x^n\right) / (2^{\frac{1+m+2n}{n}} / n) / ((-b x^n)^{\frac{1+m}{n}}) - x^{1+m} \text{GAMMA}\left(\frac{1+m}{n}, 2b x^n\right) / (2^{\frac{1+m+2n}{n}} / n) / \exp(2a) / n / ((b x^n)^{\frac{1+m}{n}})$

Rubi [A] (verified)

Time = 0.12 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {5471, 5469, 2250}

$$\int x^m \cosh^2(a + bx^n) dx = -\frac{e^{2a} 2^{-\frac{m+2n+1}{n}} x^{m+1} (-bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -2bx^n\right)}{n} - \frac{e^{-2a} 2^{-\frac{m+2n+1}{n}} x^{m+1} (bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, 2bx^n\right)}{n} + \frac{x^{m+1}}{2(m+1)}$$

[In] $\text{Int}[x^m \text{Cosh}[a + b x^n]^2, x]$

[Out] $x^{1+m} / (2(1+m)) - (E^{(2a)} x^{1+m} \text{Gamma}[(1+m)/n, -2b x^n]) / (2^{\frac{1+m+2n}{n}} / n * (-b x^n)^{\frac{1+m}{n}}) - (x^{1+m} \text{Gamma}[(1+m)/n, 2b x^n]) / (2^{\frac{1+m+2n}{n}} / n * E^{(2a)} x^{1+m} (b x^n)^{\frac{1+m}{n}})$

Rule 2250

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)))*((e_.) + (f_.)*(x_)^(m_.), x_Symbol] := Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[F]))^(m + 1)/n))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]
```

Rule 5469

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Dist[1/2, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)]*(b_.))^(p_)*((e_.)*(x_)^(m_.), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
 \text{integral} &= \int \left(\frac{x^m}{2} + \frac{1}{2} x^m \cosh(2a + 2bx^n) \right) dx \\
 &= \frac{x^{1+m}}{2(1+m)} + \frac{1}{2} \int x^m \cosh(2a + 2bx^n) dx \\
 &= \frac{x^{1+m}}{2(1+m)} + \frac{1}{4} \int e^{-2a-2bx^n} x^m dx + \frac{1}{4} \int e^{2a+2bx^n} x^m dx \\
 &= \frac{x^{1+m}}{2(1+m)} - \frac{2^{-\frac{1+m+2n}{n}} e^{2a} x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2bx^n\right)}{n} \\
 &\quad - \frac{2^{-\frac{1+m+2n}{n}} e^{-2a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2bx^n\right)}{n}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.91

$$\int x^m \cosh^2(a + bx^n) dx = \frac{x^{1+m} \left(-2n + 2^{-\frac{1+m}{n}} e^{2a} (1+m) (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -2bx^n\right) + 2^{-\frac{1+m}{n}} e^{-2a} (1+m) (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 2bx^n\right) \right)}{4(1+m)n}$$

[In] Integrate[x^m*Cosh[a + b*x^n]^2,x]

[Out] -1/4*(x^(1 + m)*(-2*n + (E^(2*a))*(1 + m)*Gamma[(1 + m)/n, -2*b*x^n])/(2^((1 + m)/n)*(-b*x^n)^((1 + m)/n)) + ((1 + m)*Gamma[(1 + m)/n, 2*b*x^n])/(2^((1 + m)/n)*E^(2*a)*(b*x^n)^((1 + m)/n)))/((1 + m)*n)

Maple [F]

$$\int x^m \cosh(a + b x^n)^2 dx$$

[In] int(x^m*cosh(a+b*x^n)^2,x)

[Out] int(x^m*cosh(a+b*x^n)^2,x)

Fricas [F]

$$\int x^m \cosh^2(a + b x^n) dx = \int x^m \cosh(b x^n + a)^2 dx$$

[In] integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="fricas")

[Out] integral(x^m*cosh(b*x^n + a)^2, x)

Sympy [F]

$$\int x^m \cosh^2(a + b x^n) dx = \int x^m \cosh^2(a + b x^n) dx$$

[In] integrate(x**m*cosh(a+b*x**n)**2,x)

[Out] Integral(x**m*cosh(a + b*x**n)**2, x)

Maxima [A] (verification not implemented)

none

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

$$\int x^m \cosh^2(a + b x^n) dx$$

$$= -\frac{x^{m+1} e^{(-2a)} \Gamma\left(\frac{m+1}{n}, 2 b x^n\right)}{4 (2 b x^n)^{\frac{m+1}{n}} n} - \frac{x^{m+1} e^{(2a)} \Gamma\left(\frac{m+1}{n}, -2 b x^n\right)}{4 (-2 b x^n)^{\frac{m+1}{n}} n} + \frac{x^{m+1}}{2(m+1)}$$

[In] integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="maxima")

[Out] -1/4*x^(m + 1)*e^(-2*a)*gamma((m + 1)/n, 2*b*x^n)/((2*b*x^n)^((m + 1)/n)*n)
 - 1/4*x^(m + 1)*e^(2*a)*gamma((m + 1)/n, -2*b*x^n)/((-2*b*x^n)^((m + 1)/n)
 *n) + 1/2*x^(m + 1)/(m + 1)

Giac [F]

$$\int x^m \cosh^2(a + bx^n) dx = \int x^m \cosh(bx^n + a)^2 dx$$

[In] integrate(x^m*cosh(a+b*x^n)^2,x, algorithm="giac")

[Out] integrate(x^m*cosh(b*x^n + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a + bx^n) dx = \int x^m \cosh(a + bx^n)^2 dx$$

[In] int(x^m*cosh(a + b*x^n)^2,x)

[Out] int(x^m*cosh(a + b*x^n)^2, x)

3.49 $\int x^m \cosh^3(a + bx^n) dx$

Optimal result	248
Rubi [A] (verified)	248
Mathematica [A] (verified)	250
Maple [F]	250
Fricas [F]	250
Sympy [F]	251
Maxima [A] (verification not implemented)	251
Giac [F]	251
Mupad [F(-1)]	252

Optimal result

Integrand size = 14, antiderivative size = 200

$$\int x^m \cosh^3(a + bx^n) dx = -\frac{3^{-\frac{1+m}{n}} e^{3a} x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -3bx^n\right)}{8n} - \frac{3e^a x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{8n} - \frac{3e^{-a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{8n} - \frac{3^{-\frac{1+m}{n}} e^{-3a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 3bx^n\right)}{8n}$$

```
[Out] -1/8*exp(3*a)*x^(1+m)*GAMMA((1+m)/n,-3*b*x^n)/(3^((1+m)/n))/n/((-b*x^n)^((1+m)/n))-3/8*exp(a)*x^(1+m)*GAMMA((1+m)/n,-b*x^n)/n/((-b*x^n)^((1+m)/n))-3/8*x^(1+m)*GAMMA((1+m)/n,b*x^n)/exp(a)/n/((b*x^n)^((1+m)/n))-1/8*x^(1+m)*GAMMA((1+m)/n,3*b*x^n)/(3^((1+m)/n))/exp(3*a)/n/((b*x^n)^((1+m)/n))
```

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used

= {5471, 5469, 2250}

$$\int x^m \cosh^3(a + bx^n) dx = -\frac{e^{3a} 3^{-\frac{m+1}{n}} x^{m+1} (-bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -3bx^n\right)}{8n} - \frac{3e^a x^{m+1} (-bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, -bx^n\right)}{8n} - \frac{3e^{-a} x^{m+1} (bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, bx^n\right)}{8n} - \frac{e^{-3a} 3^{-\frac{m+1}{n}} x^{m+1} (bx^n)^{-\frac{m+1}{n}} \Gamma\left(\frac{m+1}{n}, 3bx^n\right)}{8n}$$

[In] Int[x^m*Cosh[a + b*x^n]^3,x]

[Out] -1/8*(E^(3*a)*x^(1 + m)*Gamma[(1 + m)/n, -3*b*x^n])/(3^((1 + m)/n)*n*(-(b*x^n)^((1 + m)/n)) - (3*E^a*x^(1 + m)*Gamma[(1 + m)/n, -(b*x^n)])/(8*n*(-(b*x^n)^((1 + m)/n)) - (3*x^(1 + m)*Gamma[(1 + m)/n, b*x^n])/(8*E^a*n*(b*x^n)^((1 + m)/n)) - (x^(1 + m)*Gamma[(1 + m)/n, 3*b*x^n])/(8*3^((1 + m)/n)*E^(3*a)*n*(b*x^n)^((1 + m)/n))

Rule 2250

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^(n_)))*((e_.) + (f_.)*(x_)^(m_.), x_Symbol] :> Simp[(-F^a)*((e + f*x)^(m + 1)/(f*n*((-b)*(c + d*x)^n*Log[F])^((m + 1)/n)))*Gamma[(m + 1)/n, (-b)*(c + d*x)^n*Log[F]], x] /; FreeQ[{F, a, b, c, d, e, f, m, n}, x] && EqQ[d*e - c*f, 0]

Rule 5469

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] :> Dist[1/2, Int[(e*x)^m*E^(c + d*x^n), x], x] + Dist[1/2, Int[(e*x)^m*E^(-c - d*x^n), x], x] /; FreeQ[{c, d, e, m, n}, x]

Rule 5471

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_)*((e_.)*(x_)^(m_.), x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\begin{aligned} \text{integral} &= \int \left(\frac{3}{4} x^m \cosh(a + bx^n) + \frac{1}{4} x^m \cosh(3a + 3bx^n) \right) dx \\ &= \frac{1}{4} \int x^m \cosh(3a + 3bx^n) dx + \frac{3}{4} \int x^m \cosh(a + bx^n) dx \end{aligned}$$

$$\begin{aligned}
&= \frac{1}{8} \int e^{-3a-3bx^n} x^m dx + \frac{1}{8} \int e^{3a+3bx^n} x^m dx + \frac{3}{8} \int e^{-a-bx^n} x^m dx + \frac{3}{8} \int e^{a+bx^n} x^m dx \\
&= -\frac{3^{-\frac{1+m}{n}} e^{3a} x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -3bx^n\right)}{8n} - \frac{3e^a x^{1+m} (-bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right)}{8n} \\
&\quad - \frac{3e^{-a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right)}{8n} - \frac{3^{-\frac{1+m}{n}} e^{-3a} x^{1+m} (bx^n)^{-\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 3bx^n\right)}{8n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.91

$$\int x^m \cosh^3(a + bx^n) dx = \frac{3^{-\frac{1+m}{n}} e^{-3a} x^{1+m} (-b^2 x^{2n})^{-\frac{1+m}{n}} \left(e^{6a} (bx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -3bx^n\right) + 3^{\frac{1+m+n}{n}} e^{4a} (bx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, -bx^n\right) + (-bx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, bx^n\right) + 3e^{2a} (bx^n)^{\frac{1+m}{n}} \Gamma\left(\frac{1+m}{n}, 3bx^n\right) \right)}{8n}$$

[In] Integrate[x^m*Cosh[a + b*x^n]^3,x]

[Out] -1/8*(x^(1 + m)*(E^(6*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -3*b*x^n] + 3^((1 + m + n)/n)*E^(4*a)*(b*x^n)^((1 + m)/n)*Gamma[(1 + m)/n, -(b*x^n)] + (-b*x^n)^((1 + m)/n)*(3^((1 + m + n)/n)*E^(2*a)*Gamma[(1 + m)/n, b*x^n] + Gamma[(1 + m)/n, 3*b*x^n]))/(3^((1 + m)/n)*E^(3*a)*n*(-(b^2*x^(2*n)))^((1 + m)/n))

Maple [F]

$$\int x^m \cosh(a + bx^n)^3 dx$$

[In] int(x^m*cosh(a+b*x^n)^3,x)

[Out] int(x^m*cosh(a+b*x^n)^3,x)

Fricas [F]

$$\int x^m \cosh^3(a + bx^n) dx = \int x^m \cosh(bx^n + a)^3 dx$$

[In] integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="fricas")

[Out] integral(x^m*cosh(b*x^n + a)^3, x)

Sympy [F]

$$\int x^m \cosh^3(a + bx^n) dx = \int x^m \cosh^3(a + bx^n) dx$$

[In] integrate(x**m*cosh(a+b*x**n)**3,x)

[Out] Integral(x**m*cosh(a + b*x**n)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.16 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.86

$$\int x^m \cosh^3(a + bx^n) dx = -\frac{x^{m+1}e^{(-3a)}\Gamma\left(\frac{m+1}{n}, 3bx^n\right)}{8(3bx^n)^{\frac{m+1}{n}}n} - \frac{3x^{m+1}e^{(-a)}\Gamma\left(\frac{m+1}{n}, bx^n\right)}{8(bx^n)^{\frac{m+1}{n}}n} \\ - \frac{3x^{m+1}e^a\Gamma\left(\frac{m+1}{n}, -bx^n\right)}{8(-bx^n)^{\frac{m+1}{n}}n} - \frac{x^{m+1}e^{(3a)}\Gamma\left(\frac{m+1}{n}, -3bx^n\right)}{8(-3bx^n)^{\frac{m+1}{n}}n}$$

[In] integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="maxima")

[Out] -1/8*x^(m + 1)*e^(-3*a)*gamma((m + 1)/n, 3*b*x^n)/((3*b*x^n)^((m + 1)/n)*n) - 3/8*x^(m + 1)*e^(-a)*gamma((m + 1)/n, b*x^n)/((b*x^n)^((m + 1)/n)*n) - 3/8*x^(m + 1)*e^a*gamma((m + 1)/n, -b*x^n)/((-b*x^n)^((m + 1)/n)*n) - 1/8*x^(m + 1)*e^(3*a)*gamma((m + 1)/n, -3*b*x^n)/((-3*b*x^n)^((m + 1)/n)*n)

Giac [F]

$$\int x^m \cosh^3(a + bx^n) dx = \int x^m \cosh(bx^n + a)^3 dx$$

[In] integrate(x^m*cosh(a+b*x^n)^3,x, algorithm="giac")

[Out] integrate(x^m*cosh(b*x^n + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^3(a + bx^n) dx = \int x^m \cosh(a + bx^n)^3 dx$$

```
[In] int(x^m*cosh(a + b*x^n)^3,x)
```

```
[Out] int(x^m*cosh(a + b*x^n)^3, x)
```

3.50 $\int x^{-1-n} \cosh(a + bx^n) dx$

Optimal result	253
Rubi [A] (verified)	253
Mathematica [A] (verified)	255
Maple [A] (verified)	255
Fricas [B] (verification not implemented)	255
Sympy [F]	256
Maxima [A] (verification not implemented)	256
Giac [F]	256
Mupad [F(-1)]	256

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int x^{-1-n} \cosh(a + bx^n) dx = -\frac{x^{-n} \cosh(a + bx^n)}{n} + \frac{b \operatorname{Chi}(bx^n) \sinh(a)}{n} + \frac{b \cosh(a) \operatorname{Shi}(bx^n)}{n}$$

[Out] $-\cosh(a+bx^n)/n/(x^n)+b*\cosh(a)*\operatorname{Shi}(bx^n)/n+b*\operatorname{Chi}(bx^n)*\sinh(a)/n$

Rubi [A] (verified)

Time = 0.07 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5429, 3378, 3384, 3379, 3382}

$$\int x^{-1-n} \cosh(a + bx^n) dx = \frac{b \sinh(a) \operatorname{Chi}(bx^n)}{n} + \frac{b \cosh(a) \operatorname{Shi}(bx^n)}{n} - \frac{x^{-n} \cosh(a + bx^n)}{n}$$

[In] $\operatorname{Int}[x^{(-1 - n)}*\operatorname{Cosh}[a + b*x^n], x]$

[Out] $-(\operatorname{Cosh}[a + b*x^n]/(n*x^n)) + (b*\operatorname{CoshIntegral}[b*x^n]*\operatorname{Sinh}[a])/n + (b*\operatorname{Cosh}[a]*\operatorname{SinhIntegral}[b*x^n])/n$

Rule 3378

```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c
+ d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c
+ d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1
]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
```

, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-n} \cosh(a + bx^n)}{n} + \frac{b \text{Subst}\left(\int \frac{\sinh(a+bx)}{x} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-n} \cosh(a + bx^n)}{n} + \frac{(b \cosh(a)) \text{Subst}\left(\int \frac{\sinh(bx)}{x} dx, x, x^n\right)}{n} \\
 &\quad + \frac{(b \sinh(a)) \text{Subst}\left(\int \frac{\cosh(bx)}{x} dx, x, x^n\right)}{n} \\
 &= -\frac{x^{-n} \cosh(a + bx^n)}{n} + \frac{b \text{Chi}(bx^n) \sinh(a)}{n} + \frac{b \cosh(a) \text{Shi}(bx^n)}{n}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int x^{-1-n} \cosh(a + bx^n) dx$$

$$= \frac{x^{-n}(-\cosh(a + bx^n) + bx^n \text{Chi}(bx^n) \sinh(a) + bx^n \cosh(a) \text{Shi}(bx^n))}{n}$$

[In] Integrate[x^(-1 - n)*Cosh[a + b*x^n],x]

[Out] (-Cosh[a + b*x^n] + b*x^n*CoshIntegral[b*x^n]*Sinh[a] + b*x^n*Cosh[a]*SinhIntegral[b*x^n])/(n*x^n)

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

method	result	size
risch	$-\frac{(-b e^{-a} \text{Ei}_1(b x^n) x^n + b e^a \text{Ei}_1(-b x^n) x^n + e^{-a-b x^n} + e^{a+b x^n}) x^{-n}}{2n}$	63

[In] int(x^(-1-n)*cosh(a+b*x^n),x,method=_RETURNVERBOSE)

[Out] -1/2*(-b*exp(-a)*Ei(1,b*x^n)*x^n+b*exp(a)*Ei(1,-b*x^n)*x^n+exp(-a-b*x^n)+exp(a+b*x^n))/n/(x^n)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(45) = 90.

Time = 0.26 (sec) , antiderivative size = 140, normalized size of antiderivative = 3.11

$$\int x^{-1-n} \cosh(a + bx^n) dx$$

$$= \frac{((b \cosh(a) + b \sinh(a)) \cosh(n \log(x)) + (b \cosh(a) + b \sinh(a)) \sinh(n \log(x))) \text{Ei}(b \cosh(n \log(x)) + b \sinh(n \log(x))) - ((b \cosh(a) - b \sinh(a)) \cosh(n \log(x)) + (b \cosh(a) - b \sinh(a)) \sinh(n \log(x))) \text{Ei}(-b \cosh(n \log(x)) - b \sinh(n \log(x))) - 2 \cosh(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a)}{n \cosh(n \log(x)) + n \sinh(n \log(x))}$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="fricas")

[Out] 1/2*(((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) - ((b*cosh(a) - b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x)))) - 2*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))

Sympy [F]

$$\int x^{-1-n} \cosh(a + bx^n) dx = \int x^{-n-1} \cosh(a + bx^n) dx$$

[In] integrate(x**(-1-n)*cosh(a+b*x**n),x)

[Out] Integral(x**(-n - 1)*cosh(a + b*x**n), x)

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76

$$\int x^{-1-n} \cosh(a + bx^n) dx = -\frac{be^{(-a)}\Gamma(-1, bx^n)}{2n} + \frac{be^a\Gamma(-1, -bx^n)}{2n}$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="maxima")

[Out] -1/2*b*e^(-a)*gamma(-1, b*x^n)/n + 1/2*b*e^a*gamma(-1, -b*x^n)/n

Giac [F]

$$\int x^{-1-n} \cosh(a + bx^n) dx = \int x^{-n-1} \cosh(bx^n + a) dx$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n),x, algorithm="giac")

[Out] integrate(x^(-n - 1)*cosh(b*x^n + a), x)

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n} \cosh(a + bx^n) dx = \int \frac{\cosh(a + bx^n)}{x^{n+1}} dx$$

[In] int(cosh(a + b*x^n)/x^(n + 1),x)

[Out] int(cosh(a + b*x^n)/x^(n + 1), x)

3.51 $\int x^{-1-n} \cosh^2(a + bx^n) dx$

Optimal result	257
Rubi [A] (verified)	257
Mathematica [A] (verified)	259
Maple [A] (verified)	259
Fricas [B] (verification not implemented)	260
Sympy [F]	260
Maxima [A] (verification not implemented)	260
Giac [F]	261
Mupad [F(-1)]	261

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int x^{-1-n} \cosh^2(a + bx^n) dx = -\frac{x^{-n}}{2n} - \frac{x^{-n} \cosh(2(a + bx^n))}{2n} + \frac{b \operatorname{Chi}(2bx^n) \sinh(2a)}{n} + \frac{b \cosh(2a) \operatorname{Shi}(2bx^n)}{n}$$

[Out] $-1/2/n/(x^n) - 1/2*\cosh(2*a+2*b*x^n)/n/(x^n) + b*\cosh(2*a)*\operatorname{Shi}(2*b*x^n)/n + b*\operatorname{Chi}(2*b*x^n)*\sinh(2*a)/n$

Rubi [A] (verified)

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5471, 5429, 3378, 3384, 3379, 3382}

$$\int x^{-1-n} \cosh^2(a + bx^n) dx = \frac{b \sinh(2a) \operatorname{Chi}(2bx^n)}{n} + \frac{b \cosh(2a) \operatorname{Shi}(2bx^n)}{n} - \frac{x^{-n} \cosh(2(a + bx^n))}{2n} - \frac{x^{-n}}{2n}$$

[In] $\operatorname{Int}[x^{(-1 - n)}*\operatorname{Cosh}[a + b*x^n]^2, x]$

[Out] $-1/2*1/(n*x^n) - \operatorname{Cosh}[2*(a + b*x^n)]/(2*n*x^n) + (b*\operatorname{CoshIntegral}[2*b*x^n]*\operatorname{Shi}[2*a])/n + (b*\operatorname{Cosh}[2*a]*\operatorname{ShiIntegral}[2*b*x^n])/n$

Rule 3378

$\operatorname{Int}[(c_.) + (d_.)*(x_.)^{(m_)}*\sin[(e_.) + (f_.)*(x_.)], x_Symbol] := \operatorname{Simp}[(c + d*x)^{(m + 1)}*(\operatorname{Sin}[e + f*x]/(d*(m + 1))), x] - \operatorname{Dist}[f/(d*(m + 1)), \operatorname{Int}[(c + d*x)^{(m + 1)}*\operatorname{Cos}[e + f*x], x], x] /; \operatorname{FreeQ}\{c, d, e, f\}, x \&\& \operatorname{LtQ}[m, -1$

]

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f,
fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz},
x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5429

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])
^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify
[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify
[(m + 1)/n], 0]))
```

Rule 5471

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*((e_.)*(x_)^(m_.),
x_Symbol] :> Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x]
/; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \int \left(\frac{x^{-1-n}}{2} + \frac{1}{2} x^{-1-n} \cosh(2a + 2bx^n) \right) dx \\
&= -\frac{x^{-n}}{2n} + \frac{1}{2} \int x^{-1-n} \cosh(2a + 2bx^n) dx \\
&= -\frac{x^{-n}}{2n} + \frac{\text{Subst}\left(\int \frac{\cosh(2a+2bx)}{x^2} dx, x, x^n\right)}{2n}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{x^{-n}}{2n} - \frac{x^{-n} \cosh(2(a + bx^n))}{2n} + \frac{b \operatorname{Subst}\left(\int \frac{\sinh(2a+2bx)}{x} dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}}{2n} - \frac{x^{-n} \cosh(2(a + bx^n))}{2n} + \frac{(b \cosh(2a)) \operatorname{Subst}\left(\int \frac{\sinh(2bx)}{x} dx, x, x^n\right)}{n} \\
&\quad + \frac{(b \sinh(2a)) \operatorname{Subst}\left(\int \frac{\cosh(2bx)}{x} dx, x, x^n\right)}{n} \\
&= -\frac{x^{-n}}{2n} - \frac{x^{-n} \cosh(2(a + bx^n))}{2n} + \frac{b \operatorname{Chi}(2bx^n) \sinh(2a)}{n} + \frac{b \cosh(2a) \operatorname{Shi}(2bx^n)}{n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.81

$$\begin{aligned}
&\int x^{-1-n} \cosh^2(a + bx^n) dx \\
&= \frac{x^{-n} (-\cosh^2(a + bx^n) + bx^n \operatorname{Chi}(2bx^n) \sinh(2a) + bx^n \cosh(2a) \operatorname{Shi}(2bx^n))}{n}
\end{aligned}$$

[In] Integrate[x^(-1 - n)*Cosh[a + b*x^n]^2,x]

[Out] (-Cosh[a + b*x^n]^2 + b*x^n*CoshIntegral[2*b*x^n]*Sinh[2*a] + b*x^n*Cosh[2*a]*SinhIntegral[2*b*x^n])/(n*x^n)

Maple [A] (verified)

Time = 1.35 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.12

method	result	size
risch	$\frac{(2be^{-2a} \operatorname{Ei}_1(2bx^n)x^n - 2be^{2a} \operatorname{Ei}_1(-2bx^n)x^n - e^{-2a-2bx^n} - e^{2a+2bx^n} - 2)x^{-n}}{4n}$	75

[In] int(x^(-1-n)*cosh(a+b*x^n)^2,x,method=_RETURNVERBOSE)

[Out] 1/4*(2*b*exp(-2*a)*Ei(1,2*b*x^n)*x^n-2*b*exp(2*a)*Ei(1,-2*b*x^n)*x^n-exp(-2*a-2*b*x^n)-exp(2*a+2*b*x^n)-2)/(x^n)/n

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(64) = 128$.

Time = 0.26 (sec) , antiderivative size = 182, normalized size of antiderivative = 2.72

$$\int x^{-1-n} \cosh^2(a + bx^n) dx$$

$$= \frac{((b \cosh(2a) + b \sinh(2a)) \cosh(n \log(x)) + (b \cosh(2a) + b \sinh(2a)) \sinh(n \log(x))) \text{Ei}(2b \cosh(n \log(x))$$

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(((b*cosh(2*a) + b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) + b*sinh(2*a))*sinh(n*log(x)))*Ei(2*b*cosh(n*log(x)) + 2*b*sinh(n*log(x))) - ((b*cosh(2*a) - b*sinh(2*a))*cosh(n*log(x)) + (b*cosh(2*a) - b*sinh(2*a))*sinh(n*log(x)))*Ei(-2*b*cosh(n*log(x)) - 2*b*sinh(n*log(x))) - cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2 - sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2 - 1)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F]

$$\int x^{-1-n} \cosh^2(a + bx^n) dx = \int x^{-n-1} \cosh^2(a + bx^n) dx$$

```
[In] integrate(x**(-1-n)*cosh(a+b*x**n)**2,x)
```

```
[Out] Integral(x**(-n - 1)*cosh(a + b*x**n)**2, x)
```

Maxima [A] (verification not implemented)

none

Time = 0.24 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.70

$$\int x^{-1-n} \cosh^2(a + bx^n) dx = -\frac{be^{(-2a)}\Gamma(-1, 2bx^n)}{2n} + \frac{be^{(2a)}\Gamma(-1, -2bx^n)}{2n} - \frac{1}{2nx^n}$$

```
[In] integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="maxima")
```

```
[Out] -1/2*b*e^(-2*a)*gamma(-1, 2*b*x^n)/n + 1/2*b*e^(2*a)*gamma(-1, -2*b*x^n)/n - 1/2/(n*x^n)
```

Giac [F]

$$\int x^{-1-n} \cosh^2(a + bx^n) dx = \int x^{-n-1} \cosh(bx^n + a)^2 dx$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n)^2,x, algorithm="giac")

[Out] integrate(x^(-n - 1)*cosh(b*x^n + a)^2, x)

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n} \cosh^2(a + bx^n) dx = \int \frac{\cosh(a + bx^n)^2}{x^{n+1}} dx$$

[In] int(cosh(a + b*x^n)^2/x^(n + 1),x)

[Out] int(cosh(a + b*x^n)^2/x^(n + 1), x)

3.52 $\int x^{-1-n} \cosh^3(a + bx^n) dx$

Optimal result	262
Rubi [A] (verified)	262
Mathematica [A] (verified)	264
Maple [A] (verified)	265
Fricas [B] (verification not implemented)	265
Sympy [F]	266
Maxima [A] (verification not implemented)	266
Giac [F]	266
Mupad [F(-1)]	266

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = -\frac{3x^{-n} \cosh(a + bx^n)}{4n} - \frac{x^{-n} \cosh(3(a + bx^n))}{4n} \\ + \frac{3b \operatorname{Chi}(bx^n) \sinh(a)}{4n} + \frac{3b \operatorname{Chi}(3bx^n) \sinh(3a)}{4n} \\ + \frac{3b \cosh(a) \operatorname{Shi}(bx^n)}{4n} + \frac{3b \cosh(3a) \operatorname{Shi}(3bx^n)}{4n}$$

[Out] $-3/4*\cosh(a+b*x^n)/n/(x^n)-1/4*\cosh(3*a+3*b*x^n)/n/(x^n)+3/4*b*\cosh(a)*\operatorname{Shi}(b*x^n)/n+3/4*b*\cosh(3*a)*\operatorname{Shi}(3*b*x^n)/n+3/4*b*\operatorname{Chi}(b*x^n)*\sinh(a)/n+3/4*b*\operatorname{Chi}(3*b*x^n)*\sinh(3*a)/n$

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5471, 5429, 3378, 3384, 3379, 3382}

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = \frac{3b \sinh(a) \operatorname{Chi}(bx^n)}{4n} + \frac{3b \sinh(3a) \operatorname{Chi}(3bx^n)}{4n} \\ + \frac{3b \cosh(a) \operatorname{Shi}(bx^n)}{4n} + \frac{3b \cosh(3a) \operatorname{Shi}(3bx^n)}{4n} \\ - \frac{3x^{-n} \cosh(a + bx^n)}{4n} - \frac{x^{-n} \cosh(3(a + bx^n))}{4n}$$

[In] $\operatorname{Int}[x^{(-1-n)}*\operatorname{Cosh}[a + b*x^n]^3,x]$

[Out] $(-3*\operatorname{Cosh}[a + b*x^n])/(4*n*x^n) - \operatorname{Cosh}[3*(a + b*x^n)]/(4*n*x^n) + (3*b*\operatorname{CoshIntegral}[b*x^n]*\operatorname{Sinh}[a])/(4*n) + (3*b*\operatorname{CoshIntegral}[3*b*x^n]*\operatorname{Sinh}[3*a])/(4*n)$

+ (3*b*Cosh[a]*SinhIntegral[b*x^n])/(4*n) + (3*b*Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n)

Rule 3378

Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(c + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Dist[f/(d*(m + 1)), Int[(c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -1]

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 5471

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*((e_.)*(x_)^(m_.)), x_Symbol] := Int[ExpandTrigReduce[(e*x)^m, (a + b*Cosh[c + d*x^n])^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rubi steps

$$\text{integral} = \int \left(\frac{3}{4} x^{-1-n} \cosh(a + bx^n) + \frac{1}{4} x^{-1-n} \cosh(3a + 3bx^n) \right) dx$$

$$\begin{aligned}
&= \frac{1}{4} \int x^{-1-n} \cosh(3a + 3bx^n) dx + \frac{3}{4} \int x^{-1-n} \cosh(a + bx^n) dx \\
&= \frac{\text{Subst}\left(\int \frac{\cosh(3a+3bx)}{x^2} dx, x, x^n\right)}{4n} + \frac{3\text{Subst}\left(\int \frac{\cosh(a+bx)}{x^2} dx, x, x^n\right)}{4n} \\
&= -\frac{3x^{-n} \cosh(a + bx^n)}{4n} - \frac{x^{-n} \cosh(3(a + bx^n))}{4n} \\
&\quad + \frac{(3b)\text{Subst}\left(\int \frac{\sinh(a+bx)}{x} dx, x, x^n\right)}{4n} + \frac{(3b)\text{Subst}\left(\int \frac{\sinh(3a+3bx)}{x} dx, x, x^n\right)}{4n} \\
&= -\frac{3x^{-n} \cosh(a + bx^n)}{4n} - \frac{x^{-n} \cosh(3(a + bx^n))}{4n} \\
&\quad + \frac{(3b \cosh(a))\text{Subst}\left(\int \frac{\sinh(bx)}{x} dx, x, x^n\right)}{4n} \\
&\quad + \frac{(3b \cosh(3a))\text{Subst}\left(\int \frac{\sinh(3bx)}{x} dx, x, x^n\right)}{4n} \\
&\quad + \frac{(3b \sinh(a))\text{Subst}\left(\int \frac{\cosh(bx)}{x} dx, x, x^n\right)}{4n} \\
&\quad + \frac{(3b \sinh(3a))\text{Subst}\left(\int \frac{\cosh(3bx)}{x} dx, x, x^n\right)}{4n} \\
&= -\frac{3x^{-n} \cosh(a + bx^n)}{4n} - \frac{x^{-n} \cosh(3(a + bx^n))}{4n} + \frac{3b\text{Chi}(bx^n) \sinh(a)}{4n} \\
&\quad + \frac{3b\text{Chi}(3bx^n) \sinh(3a)}{4n} + \frac{3b \cosh(a)\text{Shi}(bx^n)}{4n} + \frac{3b \cosh(3a)\text{Shi}(3bx^n)}{4n}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.86

$$\begin{aligned}
&\int x^{-1-n} \cosh^3(a + bx^n) dx \\
&= \frac{x^{-n}(-3 \cosh(a + bx^n) - \cosh(3(a + bx^n)) + 3bx^n \text{Chi}(bx^n) \sinh(a) + 3bx^n \text{Chi}(3bx^n) \sinh(3a) + 3bx^n \cosh(a) \text{Shi}(bx^n) + 3bx^n \cosh(3a) \text{Shi}(3bx^n))}{4n}
\end{aligned}$$

[In] Integrate[x^(-1 - n)*Cosh[a + b*x^n]^3,x]

[Out] (-3*Cosh[a + b*x^n] - Cosh[3*(a + b*x^n)] + 3*b*x^n*CoshIntegral[b*x^n]*Sinh[a] + 3*b*x^n*CoshIntegral[3*b*x^n]*Sinh[3*a] + 3*b*x^n*Cosh[a]*SinhIntegral[b*x^n] + 3*b*x^n*Cosh[3*a]*SinhIntegral[3*b*x^n])/(4*n*x^n)

Maple [A] (verified)

Time = 7.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{(3be^a \operatorname{Ei}_1(-bx^n)x^n - 3be^{-3a} \operatorname{Ei}_1(3bx^n)x^n - 3be^{-a} \operatorname{Ei}_1(bx^n)x^n + 3be^{3a} \operatorname{Ei}_1(-3bx^n)x^n + e^{-3a-3bx^n} + 3e^{-a-bx^n} + e^{3a+3bx^n} + 3e^{a+3bx^n})}{8n}$

[In] int(x^(-1-n)*cosh(a+b*x^n)^3,x,method=_RETURNVERBOSE)

```
[Out] -1/8*(3*b*exp(a)*Ei(1,-b*x^n)*x^n-3*b*exp(-3*a)*Ei(1,3*b*x^n)*x^n-3*b*exp(-a)*Ei(1,b*x^n)*x^n+3*b*exp(3*a)*Ei(1,-3*b*x^n)*x^n+exp(-3*a-3*b*x^n)+3*exp(-a-b*x^n)+exp(3*a+3*b*x^n)+3*exp(a+b*x^n))/(x^n)/n
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(102) = 204.

Time = 0.26 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.83

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = \frac{2 \cosh(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a)^3 + 6 \cosh(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a) \sinh(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a)^2 - 3((b \cosh(3a) + b \sinh(3a)) \cosh(n \log(x)) + (b \cosh(3a) + b \sinh(3a)) \sinh(n \log(x))) \operatorname{Ei}(3b \cosh(n \log(x)) + 3b \sinh(n \log(x))) - 3((b \cosh(a) + b \sinh(a)) \cosh(n \log(x)) + (b \cosh(a) + b \sinh(a)) \sinh(n \log(x))) \operatorname{Ei}(b \cosh(n \log(x)) + b \sinh(n \log(x))) + 3((b \cosh(a) - b \sinh(a)) \cosh(n \log(x)) + (b \cosh(a) - b \sinh(a)) \sinh(n \log(x))) \operatorname{Ei}(-b \cosh(n \log(x)) - b \sinh(n \log(x))) + 3((b \cosh(3a) - b \sinh(3a)) \cosh(n \log(x)) + (b \cosh(3a) - b \sinh(3a)) \sinh(n \log(x))) \operatorname{Ei}(-3b \cosh(n \log(x)) - 3b \sinh(n \log(x))) + 6 \cosh(b \cosh(n \log(x)) + b \sinh(n \log(x)) + a) / (n \cosh(n \log(x)) + n \sinh(n \log(x)))$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="fricas")

```
[Out] -1/8*(2*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^3 + 6*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)*sinh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)^2 - 3*((b*cosh(3*a) + b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) + b*sinh(3*a))*sinh(n*log(x)))*Ei(3*b*cosh(n*log(x)) + 3*b*sinh(n*log(x))) - 3*((b*cosh(a) + b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) + b*sinh(a))*sinh(n*log(x)))*Ei(b*cosh(n*log(x)) + b*sinh(n*log(x))) + 3*((b*cosh(a) - b*sinh(a))*cosh(n*log(x)) + (b*cosh(a) - b*sinh(a))*sinh(n*log(x)))*Ei(-b*cosh(n*log(x)) - b*sinh(n*log(x))) + 3*((b*cosh(3*a) - b*sinh(3*a))*cosh(n*log(x)) + (b*cosh(3*a) - b*sinh(3*a))*sinh(n*log(x)))*Ei(-3*b*cosh(n*log(x)) - 3*b*sinh(n*log(x))) + 6*cosh(b*cosh(n*log(x)) + b*sinh(n*log(x)) + a)/(n*cosh(n*log(x)) + n*sinh(n*log(x)))
```

Sympy [F]

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = \int x^{-n-1} \cosh^3(a + bx^n) dx$$

[In] integrate(x**(-1-n)*cosh(a+b*x**n)**3,x)

[Out] Integral(x**(-n - 1)*cosh(a + b*x**n)**3, x)

Maxima [A] (verification not implemented)

none

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.62

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = -\frac{3be^{(-3a)}\Gamma(-1, 3bx^n)}{8n} - \frac{3be^{(-a)}\Gamma(-1, bx^n)}{8n} + \frac{3be^a\Gamma(-1, -bx^n)}{8n} + \frac{3be^{(3a)}\Gamma(-1, -3bx^n)}{8n}$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="maxima")

[Out] -3/8*b*e^(-3*a)*gamma(-1, 3*b*x^n)/n - 3/8*b*e^(-a)*gamma(-1, b*x^n)/n + 3/8*b*e^a*gamma(-1, -b*x^n)/n + 3/8*b*e^(3*a)*gamma(-1, -3*b*x^n)/n

Giac [F]

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = \int x^{-n-1} \cosh(bx^n + a)^3 dx$$

[In] integrate(x^(-1-n)*cosh(a+b*x^n)^3,x, algorithm="giac")

[Out] integrate(x^(-n - 1)*cosh(b*x^n + a)^3, x)

Mupad [F(-1)]

Timed out.

$$\int x^{-1-n} \cosh^3(a + bx^n) dx = \int \frac{\cosh(a + bx^n)^3}{x^{n+1}} dx$$

[In] int(cosh(a + b*x^n)^3/x^(n + 1),x)

[Out] int(cosh(a + b*x^n)^3/x^(n + 1), x)

3.53 $\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx$

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Giac [A] (verification not implemented)	270
Mupad [F(-1)]	270

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erf}(\sqrt{bx^{n/2}})}{2\sqrt{bn}} + \frac{e^a\sqrt{\pi}\operatorname{erfi}(\sqrt{bx^{n/2}})}{2\sqrt{bn}}$$

[Out] $1/2*\operatorname{erf}(x^{(1/2)*n}*b^{(1/2)})*\operatorname{Pi}^{(1/2)}/\exp(a)/n/b^{(1/2)}+1/2*\exp(a)*\operatorname{erfi}(x^{(1/2)*n}*b^{(1/2)})*\operatorname{Pi}^{(1/2)}/n/b^{(1/2)}$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5465, 5407, 2235, 2236}

$$\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx = \frac{\sqrt{\pi}e^{-a}\operatorname{erf}(\sqrt{bx^{n/2}})}{2\sqrt{bn}} + \frac{\sqrt{\pi}e^a\operatorname{erfi}(\sqrt{bx^{n/2}})}{2\sqrt{bn}}$$

[In] $\operatorname{Int}[x^{(-1 + n/2)}*\operatorname{Cosh}[a + b*x^n], x]$

[Out] $(\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erf}[\operatorname{Sqrt}[b]*x^{(n/2)}])/(2*\operatorname{Sqrt}[b]*E^a*n) + (E^a*\operatorname{Sqrt}[\operatorname{Pi}]*\operatorname{Erfi}[\operatorname{Sqrt}[b]*x^{(n/2)}])/(2*\operatorname{Sqrt}[b]*n)$

Rule 2235

$\operatorname{Int}[(F_)^{((a_.) + (b_.)*((c_.) + (d_.)*(x_))^{2})}, x_Symbol] \rightarrow \operatorname{Simp}[F^a*\operatorname{Sqrt}[\operatorname{Pi}]*(\operatorname{Erfi}[(c + d*x)*\operatorname{Rt}[b*\operatorname{Log}[F], 2]]/(2*d*\operatorname{Rt}[b*\operatorname{Log}[F], 2])), x] /; \operatorname{FreeQ}\{F, a, b, c, d, x\} \&\& \operatorname{PosQ}[b]$

Rule 2236

```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

Rule 5407

```
Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n)
, x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ
[n, 1]
```

Rule 5465

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbo
l] := Dist[1/(m + 1), Subst[Int[(a + b*Cosh[c + d*x^Simplify[n/(m + 1)]])^p
, x], x, x^(m + 1)], x] /; FreeQ[{a, b, c, d, m, n}, x] && IntegerQ[p] && N
eQ[m, -1] && IGtQ[Simplify[n/(m + 1)], 0] && !IntegerQ[n]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{2 \text{Subst}\left(\int \cosh(a + bx^2) dx, x, x^{n/2}\right)}{n} \\ &= \frac{\text{Subst}\left(\int e^{-a-bx^2} dx, x, x^{n/2}\right)}{n} + \frac{\text{Subst}\left(\int e^{a+bx^2} dx, x, x^{n/2}\right)}{n} \\ &= \frac{e^{-a}\sqrt{\pi}\text{erf}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{bn}} + \frac{e^a\sqrt{\pi}\text{erfi}\left(\sqrt{b}x^{n/2}\right)}{2\sqrt{bn}} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.80

$$\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx = \frac{e^{-a}\sqrt{\pi}\left(\text{erf}\left(\sqrt{b}x^{n/2}\right) + e^{2a}\text{erfi}\left(\sqrt{b}x^{n/2}\right)\right)}{2\sqrt{bn}}$$

```
[In] Integrate[x^(-1 + n/2)*Cosh[a + b*x^n],x]
```

```
[Out] (Sqrt[Pi]*(Erf[Sqrt[b]*x^(n/2)] + E^(2*a)*Erfi[Sqrt[b]*x^(n/2)]))/(2*Sqrt[b]
]*E^a*n)
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
risch	$\frac{e^{-a}\sqrt{\pi}\operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2n\sqrt{b}} + \frac{e^a\sqrt{\pi}\operatorname{erf}\left(\sqrt{-b}x^{\frac{n}{2}}\right)}{2n\sqrt{-b}}$
meijerg	$\frac{\sqrt{2}\sqrt{\pi}\left(\frac{\sqrt{ib}\sqrt{2}\operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}} + \frac{\sqrt{ib}\sqrt{2}\operatorname{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2\sqrt{b}}\right)\cosh(a)}{2\sqrt{ib}n} - \frac{i\sqrt{2}\sqrt{\pi}\left(-\frac{\sqrt{2}(ib)^{\frac{3}{2}}\operatorname{erf}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}} + \frac{\sqrt{2}(ib)^{\frac{3}{2}}\operatorname{erfi}\left(x^{\frac{n}{2}}\sqrt{b}\right)}{2b^{\frac{3}{2}}}\right)\sinh(a)}{2\sqrt{ib}n}$

```
[In] int(x^(-1+1/2*n)*cosh(a+b*x^n),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/n*exp(-a)*Pi^(1/2)/b^(1/2)*erf(x^(1/2*n)*b^(1/2))+1/2/n*exp(a)*Pi^(1/2)
/(-b)^(1/2)*erf((-b)^(1/2)*x^(1/2*n))
```

Fricas [A] (verification not implemented)

none

Time = 0.25 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.38

$$\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx = \frac{\sqrt{\pi}\sqrt{-b}(\cosh(a) + \sinh(a))\operatorname{erf}\left(\sqrt{-b}x\cosh\left(\frac{1}{2}(n-2)\log(x)\right) + \sqrt{-b}x\sinh\left(\frac{1}{2}(n-2)\log(x)\right)\right) - \sqrt{\pi}\sqrt{b}(\cosh(a) - \sinh(a))\operatorname{erf}\left(\sqrt{b}x\cosh\left(\frac{1}{2}(n-2)\log(x)\right) + \sqrt{b}x\sinh\left(\frac{1}{2}(n-2)\log(x)\right)\right)}{2bn}$$

```
[In] integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(pi)*sqrt(-b)*(cosh(a) + sinh(a))*erf(sqrt(-b)*x*cosh(1/2*(n - 2)
*log(x)) + sqrt(-b)*x*sinh(1/2*(n - 2)*log(x))) - sqrt(pi)*sqrt(b)*(cosh(a)
- sinh(a))*erf(sqrt(b)*x*cosh(1/2*(n - 2)*log(x)) + sqrt(b)*x*sinh(1/2*(n
- 2)*log(x))))/(b*n)
```

Sympy [F]

$$\int x^{-1+\frac{n}{2}} \cosh(a + bx^n) dx = \int x^{\frac{n}{2}-1} \cosh(a + bx^n) dx$$

```
[In] integrate(x**(-1+1/2*n)*cosh(a+b*x**n),x)
```

```
[Out] Integral(x**(n/2 - 1)*cosh(a + b*x**n), x)
```

Maxima [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97

$$\int x^{-1+\frac{n}{2}} \cosh(a+bx^n) dx = \frac{\sqrt{\pi}x^{\frac{1}{2}n}(\operatorname{erf}(\sqrt{bx^n})-1)e^{-a}}{2\sqrt{bx^n}n} + \frac{\sqrt{\pi}x^{\frac{1}{2}n}(\operatorname{erf}(\sqrt{-bx^n})-1)e^a}{2\sqrt{-bx^n}n}$$

[In] integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="maxima")

[Out] 1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(b*x^n))-1)*e^(-a)/(sqrt(b*x^n)*n) + 1/2*sqrt(pi)*x^(1/2*n)*(erf(sqrt(-b*x^n))-1)*e^a/(sqrt(-b*x^n)*n)

Giac [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.73

$$\int x^{-1+\frac{n}{2}} \cosh(a+bx^n) dx = -\frac{\frac{\sqrt{\pi} \operatorname{erf}(-\sqrt{b}\sqrt{x^n})e^{-a}}{\sqrt{b}} + \frac{\sqrt{\pi} \operatorname{erf}(-\sqrt{-b}\sqrt{x^n})e^a}{\sqrt{-b}}}{2n}$$

[In] integrate(x^(-1+1/2*n)*cosh(a+b*x^n),x, algorithm="giac")

[Out] -1/2*(sqrt(pi)*erf(-sqrt(b)*sqrt(x^n))*e^(-a)/sqrt(b) + sqrt(pi)*erf(-sqrt(-b)*sqrt(x^n))*e^a/sqrt(-b))/n

Mupad [F(-1)]

Timed out.

$$\int x^{-1+\frac{n}{2}} \cosh(a+bx^n) dx = \int x^{\frac{n}{2}-1} \cosh(a+bx^n) dx$$

[In] int(x^(n/2 - 1)*cosh(a + b*x^n),x)

[Out] int(x^(n/2 - 1)*cosh(a + b*x^n), x)

3.54 $\int x^2 \cosh((a + bx)^2) dx$

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Mathematica [A] (verified)	274
Maple [C] (verified)	274
Fricas [B] (verification not implemented)	274
Sympy [F]	275
Maxima [B] (verification not implemented)	275
Giac [C] (verification not implemented)	276
Mupad [F(-1)]	276

Optimal result

Integrand size = 12, antiderivative size = 113

$$\int x^2 \cosh((a + bx)^2) dx = \frac{\sqrt{\pi} \operatorname{erf}(a + bx)}{8b^3} + \frac{a^2 \sqrt{\pi} \operatorname{erf}(a + bx)}{4b^3} - \frac{\sqrt{\pi} \operatorname{erfi}(a + bx)}{8b^3} + \frac{a^2 \sqrt{\pi} \operatorname{erfi}(a + bx)}{4b^3} - \frac{a \sinh((a + bx)^2)}{b^3} + \frac{(a + bx) \sinh((a + bx)^2)}{2b^3}$$

[Out] $-a \sinh((b*x+a)^2)/b^3 + 1/2*(b*x+a)*\sinh((b*x+a)^2)/b^3 + 1/8*\operatorname{erf}(b*x+a)*\operatorname{Pi}^{(1/2)}/b^3 + 1/4*a^2*\operatorname{erf}(b*x+a)*\operatorname{Pi}^{(1/2)}/b^3 - 1/8*\operatorname{erfi}(b*x+a)*\operatorname{Pi}^{(1/2)}/b^3 + 1/4*a^2*\operatorname{erfi}(b*x+a)*\operatorname{Pi}^{(1/2)}/b^3$

Rubi [A] (verified)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {5473, 6874, 5407, 2235, 2236, 5429, 2717, 5433, 5406}

$$\int x^2 \cosh((a + bx)^2) dx = \frac{\sqrt{\pi} a^2 \operatorname{erf}(a + bx)}{4b^3} + \frac{\sqrt{\pi} a^2 \operatorname{erfi}(a + bx)}{4b^3} + \frac{\sqrt{\pi} \operatorname{erf}(a + bx)}{8b^3} - \frac{\sqrt{\pi} \operatorname{erfi}(a + bx)}{8b^3} - \frac{a \sinh((a + bx)^2)}{b^3} + \frac{(a + bx) \sinh((a + bx)^2)}{2b^3}$$

[In] $\operatorname{Int}[x^2 * \operatorname{Cosh}[(a + b*x)^2], x]$

[Out] $(\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[a + b*x]) / (8*b^3) + (a^2 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erf}[a + b*x]) / (4*b^3) - (\operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[a + b*x]) / (8*b^3) + (a^2 * \operatorname{Sqrt}[\operatorname{Pi}] * \operatorname{Erfi}[a + b*x]) / (4*b^3) - (a * \operatorname{Sinh}[(a + b*x)^2]) / b^3 + ((a + b*x) * \operatorname{Sinh}[(a + b*x)^2]) / (2*b^3)$

Rule 2235

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]

Rule 2236

Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 5406

Int[Sinh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] - Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ[n, 1]

Rule 5407

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ[n, 1]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 5433

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)]*((e_.)*(x_)^(m_.), x_Symbol] := Simp[e^(n - 1)*(e*x)^(m - n + 1)*(Sinh[c + d*x^n]/(d*n)), x] - Dist[e^n*((m - n + 1)/(d*n)), Int[(e*x)^(m - n)*Sinh[c + d*x^n], x], x] /; FreeQ[{c, d, e}, x] && IGtQ[n, 0] && LtQ[0, n, m + 1]

Rule 5473

Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,

$0])^m (a + b \cosh[c + d x^n])^p, x], x, u], x] /; \text{FreeQ}\{a, b, c, d, n, p\}, x] \&\& \text{LinearQ}[u, x] \&\& \text{NeQ}[u, x] \&\& \text{IntegerQ}[m]$

Rule 6874

$\text{Int}[u, x_Symbol] :> \text{With}\{v = \text{ExpandIntegrand}[u, x]\}, \text{Int}[v, x] /; \text{SumQ}[v]$
]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\text{Subst}\left(\int (-a+x)^2 \cosh(x^2) dx, x, a+bx\right)}{b^3} \\
 &= \frac{\text{Subst}\left(\int (a^2 \cosh(x^2) - 2ax \cosh(x^2) + x^2 \cosh(x^2)) dx, x, a+bx\right)}{b^3} \\
 &= \frac{\text{Subst}\left(\int x^2 \cosh(x^2) dx, x, a+bx\right)}{b^3} - \frac{(2a)\text{Subst}\left(\int x \cosh(x^2) dx, x, a+bx\right)}{b^3} \\
 &\quad + \frac{a^2 \text{Subst}\left(\int \cosh(x^2) dx, x, a+bx\right)}{b^3} \\
 &= \frac{(a+bx) \sinh((a+bx)^2)}{2b^3} - \frac{\text{Subst}\left(\int \sinh(x^2) dx, x, a+bx\right)}{2b^3} \\
 &\quad - \frac{a \text{Subst}\left(\int \cosh(x) dx, x, (a+bx)^2\right)}{b^3} \\
 &\quad + \frac{a^2 \text{Subst}\left(\int e^{-x^2} dx, x, a+bx\right)}{2b^3} + \frac{a^2 \text{Subst}\left(\int e^{x^2} dx, x, a+bx\right)}{2b^3} \\
 &= \frac{a^2 \sqrt{\pi} \text{erf}(a+bx)}{4b^3} + \frac{a^2 \sqrt{\pi} \text{erfi}(a+bx)}{4b^3} \\
 &\quad - \frac{a \sinh((a+bx)^2)}{b^3} + \frac{(a+bx) \sinh((a+bx)^2)}{2b^3} \\
 &\quad + \frac{\text{Subst}\left(\int e^{-x^2} dx, x, a+bx\right)}{4b^3} - \frac{\text{Subst}\left(\int e^{x^2} dx, x, a+bx\right)}{4b^3} \\
 &= \frac{\sqrt{\pi} \text{erf}(a+bx)}{8b^3} + \frac{a^2 \sqrt{\pi} \text{erf}(a+bx)}{4b^3} - \frac{\sqrt{\pi} \text{erfi}(a+bx)}{8b^3} \\
 &\quad + \frac{a^2 \sqrt{\pi} \text{erfi}(a+bx)}{4b^3} - \frac{a \sinh((a+bx)^2)}{b^3} + \frac{(a+bx) \sinh((a+bx)^2)}{2b^3}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.55

$$\int x^2 \cosh((a + bx)^2) dx = \frac{(1 + 2a^2) \sqrt{\pi} \operatorname{erf}(a + bx) + (-1 + 2a^2) \sqrt{\pi} \operatorname{erfi}(a + bx) - 4(a - bx) \sinh((a + bx)^2)}{8b^3}$$

[In] Integrate[x^2*Cosh[(a + b*x)^2],x]

[Out] ((1 + 2*a^2)*Sqrt[Pi]*Erf[a + b*x] + (-1 + 2*a^2)*Sqrt[Pi]*Erfi[a + b*x] - 4*(a - b*x)*Sinh[(a + b*x)^2])/(8*b^3)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.08 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.20

method	result
risch	$-\frac{x e^{-(bx+a)^2}}{4b^2} + \frac{a e^{-(bx+a)^2}}{4b^3} + \frac{a^2 \operatorname{erf}(bx+a)\sqrt{\pi}}{4b^3} + \frac{\operatorname{erf}(bx+a)\sqrt{\pi}}{8b^3} + \frac{x e^{(bx+a)^2}}{4b^2} - \frac{a e^{(bx+a)^2}}{4b^3} - \frac{ia^2 \sqrt{\pi} \operatorname{erf}(ixb+ia)}{4b^3} + \frac{i\sqrt{\pi}}{4b^3}$

[In] int(x^2*cosh((b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] -1/4/b^2*x*exp(-(b*x+a)^2)+1/4*a/b^3*exp(-(b*x+a)^2)+1/4*a^2*erf(b*x+a)*Pi^(1/2)/b^3+1/8*erf(b*x+a)*Pi^(1/2)/b^3+1/4/b^2*x*exp((b*x+a)^2)-1/4*a/b^3*exp((b*x+a)^2)-1/4*I*a^2/b^3*Pi^(1/2)*erf(I*x*b+I*a)+1/8*I/b^3*Pi^(1/2)*erf(I*x*b+I*a)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(95) = 190.

Time = 0.26 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.84

$$\int x^2 \cosh((a + bx)^2) dx = \frac{2b^2x - 2(b^2x - ab) \cosh(b^2x^2 + 2abx + a^2)^2 + \sqrt{\pi} \sqrt{-b^2} ((2a^2 - 1) \cosh(b^2x^2 + 2abx + a^2) + (2a^2 - 1) \sinh(b^2x^2 + 2abx + a^2))}{8b^3}$$

[In] integrate(x^2*cosh((b*x+a)^2),x, algorithm="fricas")

[Out] -1/8*(2*b^2*x - 2*(b^2*x - a*b)*cosh(b^2*x^2 + 2*a*b*x + a^2)^2 + sqrt(pi)*sqrt(-b^2)*((2*a^2 - 1)*cosh(b^2*x^2 + 2*a*b*x + a^2) + (2*a^2 - 1)*sinh(b^2*x^2 + 2*a*b*x + a^2)))/8*b^3

$$2x^2 + 2abx + a^2) \operatorname{erf}(\sqrt{-b^2}(bx + a)/b) - \sqrt{\pi} \sqrt{b^2} ((2a^2 + 1) \cosh(b^2x^2 + 2abx + a^2) + (2a^2 + 1) \sinh(b^2x^2 + 2abx + a^2)) \operatorname{erf}(\sqrt{b^2}(bx + a)/b) - 4(b^2x - ab) \cosh(b^2x^2 + 2abx + a^2) \sinh(b^2x^2 + 2abx + a^2) - 2(b^2x - ab) \sinh(b^2x^2 + 2abx + a^2)^2 - 2ab / (b^4 \cosh(b^2x^2 + 2abx + a^2) + b^4 \sinh(b^2x^2 + 2abx + a^2))$$

Sympy [F]

$$\int x^2 \cosh((a + bx)^2) dx = \int x^2 \cosh(a^2 + 2abx + b^2x^2) dx$$

[In] integrate(x**2*cosh((b*x+a)**2),x)

[Out] Integral(x**2*cosh(a**2 + 2*a*b*x + b**2*x**2), x)

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(95) = 190.

Time = 0.44 (sec) , antiderivative size = 818, normalized size of antiderivative = 7.24

$$\int x^2 \cosh((a + bx)^2) dx = \text{Too large to display}$$

[In] integrate(x^2*cosh((b*x+a)^2),x, algorithm="maxima")

[Out] $\frac{1}{3}x^3 \cosh((bx + a)^2) - \frac{1}{6}((\sqrt{\pi})(b^2x + ab)a^3b^4(\operatorname{erf}(\sqrt{(b^2x + ab)^2}/b) - 1)/(\sqrt{(b^2x + ab)^2}(-b^2)^{7/2}) - 3(b^2x + ab)^3ab^4\gamma(3/2, (b^2x + ab)^2/b^2)/(((b^2x + ab)^2)^{3/2}(-b^2)^{7/2}) + 3a^2b^4e^{-(b^2x + ab)^2/b^2}/(-b^2)^{7/2} + b^4\gamma(2, (b^2x + ab)^2/b^2)/(-b^2)^{7/2})a/\sqrt{-b^2} + (\sqrt{\pi})(b^2x + ab)a^4b^5(\operatorname{erf}(\sqrt{(b^2x + ab)^2}/b) - 1)/(\sqrt{(b^2x + ab)^2}(-b^2)^{9/2}) - 6(b^2x + ab)^3a^2b^5\gamma(3/2, (b^2x + ab)^2/b^2)/(((b^2x + ab)^2)^{3/2}(-b^2)^{9/2}) + 4a^3b^5e^{-(b^2x + ab)^2/b^2}/(-b^2)^{9/2} - (b^2x + ab)^5b^5\gamma(5/2, (b^2x + ab)^2/b^2)/(((b^2x + ab)^2)^{5/2}(-b^2)^{9/2}) + 4ab^5\gamma(2, (b^2x + ab)^2/b^2)/(-b^2)^{9/2})b/\sqrt{-b^2} - a(\sqrt{\pi})(b^2x + ab)a^3(\operatorname{erf}(\sqrt{-(b^2x + ab)^2/b^2}) - 1)/(b^4\sqrt{-(b^2x + ab)^2/b^2}) - 3a^2e^{((b^2x + ab)^2/b^2)}/b^3 + \gamma(2, -(b^2x + ab)^2/b^2)/b^3 - 3(b^2x + ab)^3a\gamma(3/2, -(b^2x + ab)^2/b^2)/(b^6(-(b^2x + ab)^2/b^2)^{3/2})/b + \sqrt{\pi}(b^2x + ab)a^4(\operatorname{erf}(\sqrt{-(b^2x + ab)^2/b^2}) - 1)/(b^5\sqrt{-(b^2x + ab)^2/b^2}) - 4a^3e^{((b^2x + ab)^2/b^2)}/b^4 + 4a\gamma(2, -(b^2x + ab)^2/b^2)/b^4 - 6(b^2x + ab)^3a^2\gamma(3/2, -(b^2x + ab)^2/b^2)/(b^7(-(b^2x + ab)^2/b^2)^{3/2}) - (b^2x + ab)^5\gamma(5/2, -(b^2x + ab)^2/b^2)/(b^9(-(b^2x + ab)^2/b^2)^{5/2}))b$

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21

$$\int x^2 \cosh((a + bx)^2) dx = -\frac{\frac{i\sqrt{\pi}(2a^2-1)\operatorname{erf}(ib(x+\frac{a}{b}))}{b} - \frac{2(b(x+\frac{a}{b})-2a)e^{(b^2x^2+2abx+a^2)}}{b}}{8b^2} - \frac{\frac{\sqrt{\pi}(2a^2+1)\operatorname{erf}(-b(x+\frac{a}{b}))}{b} + \frac{2(b(x+\frac{a}{b})-2a)e^{(-b^2x^2-2abx-a^2)}}{b}}{8b^2}$$

[In] integrate(x^2*cosh((b*x+a)^2),x, algorithm="giac")

[Out] -1/8*(I*sqrt(pi)*(2*a^2 - 1)*erf(I*b*(x + a/b))/b - 2*(b*(x + a/b) - 2*a)*e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b^2 - 1/8*(sqrt(pi)*(2*a^2 + 1)*erf(-b*(x + a/b))/b + 2*(b*(x + a/b) - 2*a)*e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b^2

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh((a + bx)^2) dx = \int x^2 \cosh((a + bx)^2) dx$$

[In] int(x^2*cosh((a + b*x)^2),x)

[Out] int(x^2*cosh((a + b*x)^2), x)

3.55 $\int x \cosh((a + bx)^2) dx$

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Optimal result

Integrand size = 10, antiderivative size = 54

$$\int x \cosh((a + bx)^2) dx = -\frac{a\sqrt{\pi}\operatorname{erf}(a + bx)}{4b^2} - \frac{a\sqrt{\pi}\operatorname{erfi}(a + bx)}{4b^2} + \frac{\sinh((a + bx)^2)}{2b^2}$$

[Out] $\frac{1}{2}\sinh((b*x+a)^2)/b^2 - \frac{1}{4}a*\operatorname{erf}(b*x+a)*\pi^{(1/2)}/b^2 - \frac{1}{4}a*\operatorname{erfi}(b*x+a)*\pi^{(1/2)}/b^2$

Rubi [A] (verified)

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {5473, 6874, 5407, 2235, 2236, 5429, 2717}

$$\int x \cosh((a + bx)^2) dx = -\frac{\sqrt{\pi}a\operatorname{erf}(a + bx)}{4b^2} - \frac{\sqrt{\pi}a\operatorname{erfi}(a + bx)}{4b^2} + \frac{\sinh((a + bx)^2)}{2b^2}$$

[In] `Int[x*Cosh[(a + b*x)^2],x]`

[Out] $-\frac{1}{4}*(a*\sqrt{\pi}*\operatorname{Erf}[a + b*x])/b^2 - (a*\sqrt{\pi}*\operatorname{Erfi}[a + b*x])/(4*b^2) + \sinh[(a + b*x)^2]/(2*b^2)$

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] := Simp[F^a*sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] := Simp[F^a*sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr`

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]

Rule 5407

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ[n, 1]

Rule 5429

Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*Cosh[c + d*x])^p, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]] && (EqQ[p, 1] || EqQ[m, n - 1] || (IntegerQ[p] && GtQ[Simplify[(m + 1)/n], 0]))

Rule 5473

Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]

Rule 6874

Int[u_, x_Symbol] := With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]]

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\text{Subst}\left(\int (-a + x) \cosh(x^2) dx, x, a + bx\right)}{b^2} \\
 &= \frac{\text{Subst}\left(\int (-a \cosh(x^2) + x \cosh(x^2)) dx, x, a + bx\right)}{b^2} \\
 &= \frac{\text{Subst}\left(\int x \cosh(x^2) dx, x, a + bx\right)}{b^2} - \frac{a \text{Subst}\left(\int \cosh(x^2) dx, x, a + bx\right)}{b^2} \\
 &= \frac{\text{Subst}\left(\int \cosh(x) dx, x, (a + bx)^2\right)}{2b^2} \\
 &\quad - \frac{a \text{Subst}\left(\int e^{-x^2} dx, x, a + bx\right)}{2b^2} - \frac{a \text{Subst}\left(\int e^{x^2} dx, x, a + bx\right)}{2b^2}
 \end{aligned}$$

$$= -\frac{a\sqrt{\pi}\operatorname{erf}(a+bx)}{4b^2} - \frac{a\sqrt{\pi}\operatorname{erfi}(a+bx)}{4b^2} + \frac{\sinh((a+bx)^2)}{2b^2}$$

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.72

$$\int x \cosh((a+bx)^2) dx = \frac{-a\sqrt{\pi}(\operatorname{erf}(a+bx) + \operatorname{erfi}(a+bx)) + 2\sinh((a+bx)^2)}{4b^2}$$

[In] Integrate[x*Cosh[(a + b*x)^2],x]

[Out] $(-(a*\sqrt{\pi}*(\operatorname{Erf}[a + b*x] + \operatorname{Erfi}[a + b*x])) + 2*\operatorname{Sinh}[(a + b*x)^2])/(4*b^2)$

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

method	result	size
risch	$-\frac{e^{-(bx+a)^2}}{4b^2} - \frac{a \operatorname{erf}(bx+a)\sqrt{\pi}}{4b^2} + \frac{e^{(bx+a)^2}}{4b^2} + \frac{ia\sqrt{\pi} \operatorname{erf}(ixb+ia)}{4b^2}$	66

[In] int(x*cosh((b*x+a)^2),x,method=_RETURNVERBOSE)

[Out] $-1/4/b^2*\exp(-(b*x+a)^2)-1/4*a*\operatorname{erf}(b*x+a)*\pi^{(1/2)}/b^2+1/4/b^2*\exp((b*x+a)^2)+1/4*I*a/b^2*\pi^{(1/2)}*\operatorname{erf}(I*x*b+I*a)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 261 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 261, normalized size of antiderivative = 4.83

$$\int x \cosh((a+bx)^2) dx$$

$$= \frac{b \cosh(b^2x^2 + 2abx + a^2)^2 + \sqrt{\pi}\sqrt{-b^2}(a \cosh(b^2x^2 + 2abx + a^2) + a \sinh(b^2x^2 + 2abx + a^2)) \operatorname{erf}\left(\frac{\sqrt{-b^2}(a+bx)}{b}\right)}{4b^2}$$

[In] integrate(x*cosh((b*x+a)^2),x, algorithm="fricas")

[Out] $1/4*(b*\cosh(b^2*x^2 + 2*a*b*x + a^2)^2 + \sqrt{\pi}*\sqrt{-b^2}*(a*\cosh(b^2*x^2 + 2*a*b*x + a^2) + a*\sinh(b^2*x^2 + 2*a*b*x + a^2))*\operatorname{erf}(\sqrt{-b^2}*(b*x + a)/b) - \sqrt{\pi}*\sqrt{b^2}*(a*\cosh(b^2*x^2 + 2*a*b*x + a^2) + a*\sinh(b^2*x^2 + 2*a*b*x + a^2))$

$(b^2x^2 + 2abx + a^2) \operatorname{erf}(\sqrt{b^2} \cdot (bx + a)/b) + 2b \cosh(b^2x^2 + 2abx + a^2) \sinh(b^2x^2 + 2abx + a^2) + b \sinh(b^2x^2 + 2abx + a^2)^2 - b) / (b^3 \cosh(b^2x^2 + 2abx + a^2) + b^3 \sinh(b^2x^2 + 2abx + a^2))$

Sympy [F]

$$\int x \cosh((a + bx)^2) dx = \int x \cosh(a^2 + 2abx + b^2x^2) dx$$

[In] `integrate(x*cosh((b*x+a)**2),x)`

[Out] `Integral(x*cosh(a**2 + 2*a*b*x + b**2*x**2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(44) = 88$.

Time = 0.40 (sec) , antiderivative size = 650, normalized size of antiderivative = 12.04

$$\int x \cosh((a + bx)^2) dx = \frac{1}{2} x^2 \cosh((bx + a)^2) - \frac{1}{4} \left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3 \left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right) - 1\right)}{\sqrt{(b^2x+ab)^2(-b^2)^{\frac{5}{2}}}} - \frac{(b^2x+ab)^3 b^3 \Gamma\left(\frac{3}{2}, \frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}} (-b^2)^{\frac{5}{2}}} + \frac{2ab^3 e^{-\frac{(b^2x+ab)^2}{b^2}}}{(-b^2)^{\frac{5}{2}}} \right) a}{\sqrt{-b^2}} + \frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a}{\sqrt{-b^2}} \right)}{\sqrt{-b^2}} \right)$$

[In] `integrate(x*cosh((b*x+a)^2),x, algorithm="maxima")`

[Out] $\frac{1}{2}x^2 \cosh((bx + a)^2) - \frac{1}{4} \left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3 \left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right) - 1\right)}{\sqrt{(b^2x+ab)^2(-b^2)^{\frac{5}{2}}}} - \frac{(b^2x+ab)^3 b^3 \Gamma\left(\frac{3}{2}, \frac{(b^2x+ab)^2}{b^2}\right)}{\left((b^2x+ab)^2\right)^{\frac{3}{2}} (-b^2)^{\frac{5}{2}}} + \frac{2ab^3 e^{-\frac{(b^2x+ab)^2}{b^2}}}{(-b^2)^{\frac{5}{2}}} \right) a}{\sqrt{-b^2}} + \frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a}{\sqrt{-b^2}} \right)}{\sqrt{-b^2}} \right)$

$2*x + a*b)^3*\text{gamma}(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-(b^2*x + a*b)^2/b^2)^{(3/2)})/b - \text{sqrt}(\text{pi})*(b^2*x + a*b)*a^3*(\text{erf}(\text{sqrt}(-(b^2*x + a*b)^2/b^2)) - 1)/(b^4*\text{sqrt}(-(b^2*x + a*b)^2/b^2)) + 3*a^2*e^{((b^2*x + a*b)^2/b^2)}/b^3 - \text{gamma}(2, -(b^2*x + a*b)^2/b^2)/b^3 + 3*(b^2*x + a*b)^3*a*\text{gamma}(3/2, -(b^2*x + a*b)^2/b^2)/(b^6*(-(b^2*x + a*b)^2/b^2)^{(3/2)})*b$

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.83

$$\int x \cosh((a + bx)^2) dx = -\frac{\frac{i\sqrt{\pi}a \operatorname{erf}(ib(x+\frac{a}{b}))}{b} - \frac{e^{(b^2x^2+2abx+a^2)}}{b}}{4b} + \frac{\frac{\sqrt{\pi}a \operatorname{erf}(-b(x+\frac{a}{b}))}{b} - \frac{e^{(-b^2x^2-2abx-a^2)}}{b}}{4b}$$

[In] integrate(x*cosh((b*x+a)^2),x, algorithm="giac")

[Out] -1/4*(-I*sqrt(pi)*a*erf(I*b*(x + a/b))/b - e^(b^2*x^2 + 2*a*b*x + a^2)/b)/b + 1/4*(sqrt(pi)*a*erf(-b*(x + a/b))/b - e^(-b^2*x^2 - 2*a*b*x - a^2)/b)/b

Mupad [F(-1)]

Timed out.

$$\int x \cosh((a + bx)^2) dx = \int x \cosh((a + bx)^2) dx$$

[In] int(x*cosh((a + b*x)^2),x)

[Out] int(x*cosh((a + b*x)^2), x)

3.56 $\int \cosh((a + bx)^2) dx$

Optimal result	282
Rubi [A] (verified)	282
Mathematica [A] (verified)	283
Maple [C] (verified)	283
Fricas [B] (verification not implemented)	284
Sympy [F]	284
Maxima [B] (verification not implemented)	284
Giac [C] (verification not implemented)	285
Mupad [F(-1)]	285

Optimal result

Integrand size = 8, antiderivative size = 37

$$\int \cosh((a + bx)^2) dx = \frac{\sqrt{\pi} \operatorname{erf}(a + bx)}{4b} + \frac{\sqrt{\pi} \operatorname{erfi}(a + bx)}{4b}$$

[Out] $1/4*\operatorname{erf}(b*x+a)*\operatorname{Pi}^{(1/2)}/b+1/4*\operatorname{erfi}(b*x+a)*\operatorname{Pi}^{(1/2)}/b$

Rubi [A] (verified)

Time = 0.01 (sec), antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {5419, 5407, 2235, 2236}

$$\int \cosh((a + bx)^2) dx = \frac{\sqrt{\pi} \operatorname{erf}(a + bx)}{4b} + \frac{\sqrt{\pi} \operatorname{erfi}(a + bx)}{4b}$$

[In] `Int[Cosh[(a + b*x)^2], x]`

[Out] `(Sqrt[Pi]*Erf[a + b*x])/(4*b) + (Sqrt[Pi]*Erfi[a + b*x])/(4*b)`

Rule 2235

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{F, a, b, c, d}, x] && PosQ[b]`

Rule 2236

`Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_)) ^2), x_Symbol] := Simp[F^a*Sqrt[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr`

eeQ[{F, a, b, c, d}, x] && NegQ[b]

Rule 5407

Int[Cosh[(c_.) + (d_.)*(x_)^(n_)], x_Symbol] := Dist[1/2, Int[E^(c + d*x^n), x], x] + Dist[1/2, Int[E^(-c - d*x^n), x], x] /; FreeQ[{c, d}, x] && IGtQ[n, 1]

Rule 5419

Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.), x_Symbol] := Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int \cosh(x^2) dx, x, a + bx\right)}{b} \\ &= \frac{\text{Subst}\left(\int e^{-x^2} dx, x, a + bx\right)}{2b} + \frac{\text{Subst}\left(\int e^{x^2} dx, x, a + bx\right)}{2b} \\ &= \frac{\sqrt{\pi}\text{erf}(a + bx)}{4b} + \frac{\sqrt{\pi}\text{erfi}(a + bx)}{4b} \end{aligned}$$

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \cosh((a + bx)^2) dx = \frac{\sqrt{\pi}(\text{erf}(a + bx) + \text{erfi}(a + bx))}{4b}$$

[In] Integrate[Cosh[(a + b*x)^2], x]

[Out] (Sqrt[Pi]*(Erf[a + b*x] + Erfi[a + b*x]))/(4*b)

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
risch	$\frac{\text{erf}(bx+a)\sqrt{\pi}}{4b} - \frac{i\sqrt{\pi}\text{erf}(ixb+ia)}{4b}$	36

[In] `int(cosh((b*x+a)^2),x,method=_RETURNVERBOSE)`

[Out] $1/4*\operatorname{erf}(b*x+a)*\pi^{1/2}/b-1/4*I*\pi^{1/2}/b*\operatorname{erf}(I*x*b+I*a)$

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. 2(29) = 58.

Time = 0.26 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \cosh((a+bx)^2) dx = -\frac{\sqrt{\pi}\sqrt{-b^2} \operatorname{erf}\left(\frac{\sqrt{-b^2}(bx+a)}{b}\right) - \sqrt{\pi}\sqrt{b^2} \operatorname{erf}\left(\frac{\sqrt{b^2}(bx+a)}{b}\right)}{4b^2}$$

[In] `integrate(cosh((b*x+a)^2),x, algorithm="fricas")`

[Out] $-1/4*(\sqrt{\pi}*\sqrt{-b^2}*\operatorname{erf}(\sqrt{-b^2}*(b*x+a)/b) - \sqrt{\pi}*\sqrt{b^2}*\operatorname{erf}(\sqrt{b^2}*(b*x+a)/b))/b^2$

Sympy [F]

$$\int \cosh((a+bx)^2) dx = \int \cosh((a+bx)^2) dx$$

[In] `integrate(cosh((b*x+a)**2),x)`

[Out] `Integral(cosh((a + b*x)**2), x)`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(29) = 58.

Time = 0.39 (sec) , antiderivative size = 478, normalized size of antiderivative = 12.92

$$\int \cosh((a+bx)^2) dx = -\frac{1}{2} \left(\frac{\left(\frac{\sqrt{\pi}(b^2x+ab)ab^2 \left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right) - 1\right)}{\sqrt{(b^2x+ab)^2(-b^2)^{\frac{3}{2}}}} + \frac{b^2 e^{-\frac{(b^2x+ab)^2}{b^2}}}{(-b^2)^{\frac{3}{2}}} \right) a}{\sqrt{-b^2}} + \frac{\left(\frac{\sqrt{\pi}(b^2x+ab)a^2b^3 \left(\operatorname{erf}\left(\frac{\sqrt{(b^2x+ab)^2}}{b}\right) - 1\right)}{\sqrt{(b^2x+ab)^2(-b^2)^{\frac{5}{2}}}} - \frac{(b^2x+ab)^2}{\sqrt{-b^2}} \right)}{\sqrt{-b^2}} \right) + x \cosh((bx+a)^2)$$

[In] integrate(cosh((b*x+a)^2),x, algorithm="maxima")

[Out]
$$-1/2*((\sqrt{\pi}*(b^2*x + a*b)*a*b^2*(\operatorname{erf}(\sqrt{(b^2*x + a*b)^2}/b) - 1)/(\sqrt{(b^2*x + a*b)^2})*(-b^2)^{(3/2)}) + b^2*e^{-(b^2*x + a*b)^2/b^2}/(-b^2)^{(3/2)}) * a/\sqrt{-b^2} + (\sqrt{\pi}*(b^2*x + a*b)*a^2*b^3*(\operatorname{erf}(\sqrt{(b^2*x + a*b)^2}/b) - 1)/(\sqrt{(b^2*x + a*b)^2})*(-b^2)^{(5/2)}) - (b^2*x + a*b)^3*b^3*\operatorname{gamma}(3/2, (b^2*x + a*b)^2/b^2)/(((b^2*x + a*b)^2)^{(3/2)}*(-b^2)^{(5/2)}) + 2*a*b^3*e^{-(b^2*x + a*b)^2/b^2}/(-b^2)^{(5/2)} * b/\sqrt{-b^2} - a*(\sqrt{\pi}*(b^2*x + a*b)*a*(\operatorname{erf}(\sqrt{-(b^2*x + a*b)^2/b^2}) - 1)/(b^2*\sqrt{-(b^2*x + a*b)^2/b^2})) - e^{((b^2*x + a*b)^2/b^2)/b}/b + \sqrt{\pi}*(b^2*x + a*b)*a^2*(\operatorname{erf}(\sqrt{-(b^2*x + a*b)^2/b^2}) - 1)/(b^3*\sqrt{-(b^2*x + a*b)^2/b^2}) - 2*a*e^{((b^2*x + a*b)^2/b^2)/b^2} - (b^2*x + a*b)^3*\operatorname{gamma}(3/2, -(b^2*x + a*b)^2/b^2)/(b^5*(-(b^2*x + a*b)^2/b^2)^{(3/2)})) * b + x*\cosh((b*x + a)^2)$$

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.05

$$\int \cosh((a + bx)^2) dx = -\frac{i\sqrt{\pi} \operatorname{erf}(ib(x + \frac{a}{b}))}{4b} - \frac{\sqrt{\pi} \operatorname{erf}(-b(x + \frac{a}{b}))}{4b}$$

[In] integrate(cosh((b*x+a)^2),x, algorithm="giac")

[Out] $-1/4*I*\sqrt{\pi}*\operatorname{erf}(I*b*(x + a/b))/b - 1/4*\sqrt{\pi}*\operatorname{erf}(-b*(x + a/b))/b$

Mupad [F(-1)]

Timed out.

$$\int \cosh((a + bx)^2) dx = \int \cosh((a + bx)^2) dx$$

[In] int(cosh((a + b*x)^2),x)

[Out] int(cosh((a + b*x)^2), x)

$$3.57 \quad \int \frac{\cosh((a+bx)^2)}{x} dx$$

Optimal result	286
Rubi [N/A]	286
Mathematica [N/A]	287
Maple [N/A] (verified)	287
Fricas [N/A]	287
Sympy [N/A]	287
Maxima [N/A]	288
Giac [N/A]	288
Mupad [N/A]	288

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh((a+bx)^2)}{x} dx = b \text{Int}\left(\frac{\cosh((a+bx)^2)}{bx}, x\right)$$

[Out] b*CannotIntegrate(cosh((b*x+a)^2)/b/x,x)

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh((a+bx)^2)}{x} dx = \int \frac{\cosh((a+bx)^2)}{x} dx$$

[In] Int[Cosh[(a + b*x)^2]/x,x]

[Out] Defer[Subst][Defer[Int][Cosh[x^2]/(-a + x), x], x, a + b*x]

Rubi steps

$$\text{integral} = \text{Subst}\left(\int \frac{\cosh(x^2)}{-a+x} dx, x, a+bx\right)$$

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a + bx)^2)}{x} dx = \int \frac{\cosh((a + bx)^2)}{x} dx$$

[In] Integrate[Cosh[(a + b*x)^2]/x,x]

[Out] Integrate[Cosh[(a + b*x)^2]/x, x]

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh((bx + a)^2)}{x} dx$$

[In] int(cosh((b*x+a)^2)/x,x)

[Out] int(cosh((b*x+a)^2)/x,x)

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cosh((a + bx)^2)}{x} dx = \int \frac{\cosh((bx + a)^2)}{x} dx$$

[In] integrate(cosh((b*x+a)^2)/x,x, algorithm="fricas")

[Out] integral(cosh(b^2*x^2 + 2*a*b*x + a^2)/x, x)

Sympy [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\cosh((a + bx)^2)}{x} dx = \int \frac{\cosh(a^2 + 2abx + b^2x^2)}{x} dx$$

[In] integrate(cosh((b*x+a)**2)/x,x)

[Out] Integral(cosh(a**2 + 2*a*b*x + b**2*x**2)/x, x)

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a + bx)^2)}{x} dx = \int \frac{\cosh((bx + a)^2)}{x} dx$$

[In] integrate(cosh((b*x+a)^2)/x,x, algorithm="maxima")

[Out] integrate(cosh((b*x + a)^2)/x, x)

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a + bx)^2)}{x} dx = \int \frac{\cosh((bx + a)^2)}{x} dx$$

[In] integrate(cosh((b*x+a)^2)/x,x, algorithm="giac")

[Out] integrate(cosh((b*x + a)^2)/x, x)

Mupad [N/A]

Not integrable

Time = 1.65 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a + bx)^2)}{x} dx = \int \frac{\cosh((a + bx)^2)}{x} dx$$

[In] int(cosh((a + b*x)^2)/x,x)

[Out] int(cosh((a + b*x)^2)/x, x)

3.58 $\int \frac{\cosh((a+bx)^2)}{x^2} dx$

Optimal result	289
Rubi [N/A]	289
Mathematica [N/A]	290
Maple [N/A] (verified)	290
Fricas [N/A]	290
Sympy [N/A]	290
Maxima [N/A]	291
Giac [N/A]	291
Mupad [N/A]	291

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int \frac{\cosh((a+bx)^2)}{x^2} dx = \text{Int}\left(\frac{\cosh((a+bx)^2)}{x^2}, x\right)$$

[Out] Unintegrable(cosh((b*x+a)^2)/x^2,x)

Rubi [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 0, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{\cosh((a+bx)^2)}{x^2} dx = \int \frac{\cosh((a+bx)^2)}{x^2} dx$$

[In] Int[Cosh[(a + b*x)^2]/x^2,x]

[Out] b*Defer[Subst][Defer[Int][Cosh[x^2]/(-a + x)^2, x], x, a + b*x]

Rubi steps

$$\text{integral} = b\text{Subst}\left(\int \frac{\cosh(x^2)}{(-a+x)^2} dx, x, a+bx\right)$$

Mathematica [N/A]

Not integrable

Time = 3.36 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a + bx)^2)}{x^2} dx = \int \frac{\cosh((a + bx)^2)}{x^2} dx$$

[In] Integrate[Cosh[(a + b*x)^2]/x^2,x]

[Out] Integrate[Cosh[(a + b*x)^2]/x^2, x]

Maple [N/A] (verified)

Not integrable

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\cosh((bx + a)^2)}{x^2} dx$$

[In] int(cosh((b*x+a)^2)/x^2,x)

[Out] int(cosh((b*x+a)^2)/x^2,x)

Fricas [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\cosh((a + bx)^2)}{x^2} dx = \int \frac{\cosh((bx + a)^2)}{x^2} dx$$

[In] integrate(cosh((b*x+a)^2)/x^2,x, algorithm="fricas")

[Out] integral(cosh(b^2*x^2 + 2*a*b*x + a^2)/x^2, x)

Sympy [N/A]

Not integrable

Time = 3.04 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\cosh((a + bx)^2)}{x^2} dx = \int \frac{\cosh(a^2 + 2abx + b^2x^2)}{x^2} dx$$

[In] integrate(cosh((b*x+a)**2)/x**2,x)

[Out] Integral(cosh(a**2 + 2*a*b*x + b**2*x**2)/x**2, x)

Maxima [N/A]

Not integrable

Time = 0.61 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a+bx)^2)}{x^2} dx = \int \frac{\cosh((bx+a)^2)}{x^2} dx$$

[In] integrate(cosh((b*x+a)^2)/x^2,x, algorithm="maxima")

[Out] integrate(cosh((b*x + a)^2)/x^2, x)

Giac [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a+bx)^2)}{x^2} dx = \int \frac{\cosh((bx+a)^2)}{x^2} dx$$

[In] integrate(cosh((b*x+a)^2)/x^2,x, algorithm="giac")

[Out] integrate(cosh((b*x + a)^2)/x^2, x)

Mupad [N/A]

Not integrable

Time = 1.57 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\cosh((a+bx)^2)}{x^2} dx = \int \frac{\cosh((a+bx)^2)}{x^2} dx$$

[In] int(cosh((a + b*x)^2)/x^2,x)

[Out] int(cosh((a + b*x)^2)/x^2, x)

3.59 $\int x^2 \cosh(a + b\sqrt{c + dx}) dx$

Optimal result	292
Rubi [A] (verified)	293
Mathematica [A] (verified)	297
Maple [B] (verified)	297
Fricas [A] (verification not implemented)	298
Sympy [A] (verification not implemented)	298
Maxima [A] (verification not implemented)	299
Giac [B] (verification not implemented)	299
Mupad [F(-1)]	300

Optimal result

Integrand size = 18, antiderivative size = 346

$$\begin{aligned}
 \int x^2 \cosh(a + b\sqrt{c + dx}) dx = & -\frac{240 \cosh(a + b\sqrt{c + dx})}{b^6 d^3} + \frac{24c \cosh(a + b\sqrt{c + dx})}{b^4 d^3} \\
 & - \frac{2c^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
 & - \frac{120(c + dx) \cosh(a + b\sqrt{c + dx})}{b^4 d^3} \\
 & + \frac{12c(c + dx) \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
 & - \frac{10(c + dx)^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
 & + \frac{240\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^5 d^3} \\
 & - \frac{24c\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} \\
 & + \frac{2c^2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd^3} \\
 & + \frac{40(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} \\
 & - \frac{4c(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{bd^3} \\
 & + \frac{2(c + dx)^{5/2} \sinh(a + b\sqrt{c + dx})}{bd^3}
 \end{aligned}$$

[Out] $-240*\cosh(a+b*(d*x+c)^{(1/2)})/b^6/d^3+24*c*\cosh(a+b*(d*x+c)^{(1/2)})/b^4/d^3-2*c^2*\cosh(a+b*(d*x+c)^{(1/2)})/b^2/d^3-120*(d*x+c)*\cosh(a+b*(d*x+c)^{(1/2)})/b^4/d^3+12*c*(d*x+c)*\cosh(a+b*(d*x+c)^{(1/2)})/b^2/d^3-10*(d*x+c)^2*\cosh(a+b*(d$

$*x+c)^{(1/2)}/b^2/d^3+40*(d*x+c)^{(3/2)}*\sinh(a+b*(d*x+c)^{(1/2)})/b^3/d^3-4*c*(d*x+c)^{(3/2)}*\sinh(a+b*(d*x+c)^{(1/2)})/b/d^3+2*(d*x+c)^{(5/2)}*\sinh(a+b*(d*x+c)^{(1/2)})/b/d^3+240*\sinh(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b^5/d^3-24*c*\sinh(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b^3/d^3+2*c^2*\sinh(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b/d^3$

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {5473, 5395, 3377, 2718}

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx = -\frac{240 \cosh(a + b\sqrt{c + dx})}{b^6 d^3} + \frac{240\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^5 d^3} - \frac{120(c + dx) \cosh(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{24c \cosh(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{40(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} - \frac{24c\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} - \frac{2c^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{10(c + dx)^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} + \frac{12c(c + dx) \cosh(a + b\sqrt{c + dx})}{b^2 d^3} + \frac{2c^2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd^3} + \frac{2(c + dx)^{5/2} \sinh(a + b\sqrt{c + dx})}{bd^3} - \frac{4c(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{bd^3}$$

[In] Int[x^2*Cosh[a + b*Sqrt[c + d*x]],x]

[Out] (-240*Cosh[a + b*Sqrt[c + d*x]]/(b^6*d^3) + (24*c*Cosh[a + b*Sqrt[c + d*x]])/(b^4*d^3) - (2*c^2*Cosh[a + b*Sqrt[c + d*x]]/(b^2*d^3) - (120*(c + d*x)*Cosh[a + b*Sqrt[c + d*x]]/(b^4*d^3) + (12*c*(c + d*x)*Cosh[a + b*Sqrt[c + d*x]]/(b^2*d^3) - (10*(c + d*x)^2*Cosh[a + b*Sqrt[c + d*x]]/(b^2*d^3) +

$$(240*\text{Sqrt}[c + d*x]*\text{Sinh}[a + b*\text{Sqrt}[c + d*x]])/(b^5*d^3) - (24*c*\text{Sqrt}[c + d*x]*\text{Sinh}[a + b*\text{Sqrt}[c + d*x]])/(b^3*d^3) + (2*c^2*\text{Sqrt}[c + d*x]*\text{Sinh}[a + b*\text{Sqrt}[c + d*x]])/(b*d^3) + (40*(c + d*x)^(3/2)*\text{Sinh}[a + b*\text{Sqrt}[c + d*x]])/(b^3*d^3) - (4*c*(c + d*x)^(3/2)*\text{Sinh}[a + b*\text{Sqrt}[c + d*x]])/(b*d^3) + (2*(c + d*x)^(5/2)*\text{Sinh}[a + b*\text{Sqrt}[c + d*x]])/(b*d^3)$$

Rule 2718

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5395

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \frac{\text{Subst}\left(\int (-c + x)^2 \cosh(a + b\sqrt{x}) dx, x, c + dx\right)}{d^3} \\ &= \frac{2\text{Subst}\left(\int x(c - x^2)^2 \cosh(a + bx) dx, x, \sqrt{c + dx}\right)}{d^3} \\ &= \frac{2\text{Subst}\left(\int (c^2x \cosh(a + bx) - 2cx^3 \cosh(a + bx) + x^5 \cosh(a + bx)) dx, x, \sqrt{c + dx}\right)}{d^3} \\ &= \frac{2\text{Subst}\left(\int x^5 \cosh(a + bx) dx, x, \sqrt{c + dx}\right)}{d^3} \\ &\quad - \frac{(4c)\text{Subst}\left(\int x^3 \cosh(a + bx) dx, x, \sqrt{c + dx}\right)}{d^3} \\ &\quad + \frac{(2c^2)\text{Subst}\left(\int x \cosh(a + bx) dx, x, \sqrt{c + dx}\right)}{d^3} \end{aligned}$$

$$\begin{aligned}
&= \frac{2c^2\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{bd^3} - \frac{4c(c+dx)^{3/2}\sinh(a+b\sqrt{c+dx})}{bd^3} \\
&+ \frac{2(c+dx)^{5/2}\sinh(a+b\sqrt{c+dx})}{bd^3} - \frac{10\text{Subst}\left(\int x^4\sinh(ax)dx, x, \sqrt{c+dx}\right)}{bd^3} \\
&+ \frac{(12c)\text{Subst}\left(\int x^2\sinh(ax)dx, x, \sqrt{c+dx}\right)}{bd^3} \\
&- \frac{(2c^2)\text{Subst}\left(\int \sinh(ax)dx, x, \sqrt{c+dx}\right)}{bd^3} \\
&= -\frac{2c^2\cosh(a+b\sqrt{c+dx})}{b^2d^3} + \frac{12c(c+dx)\cosh(a+b\sqrt{c+dx})}{b^2d^3} \\
&- \frac{10(c+dx)^2\cosh(a+b\sqrt{c+dx})}{b^2d^3} + \frac{2c^2\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{bd^3} \\
&- \frac{4c(c+dx)^{3/2}\sinh(a+b\sqrt{c+dx})}{bd^3} + \frac{2(c+dx)^{5/2}\sinh(a+b\sqrt{c+dx})}{bd^3} \\
&+ \frac{40\text{Subst}\left(\int x^3\cosh(ax)dx, x, \sqrt{c+dx}\right)}{b^2d^3} \\
&- \frac{(24c)\text{Subst}\left(\int x\cosh(ax)dx, x, \sqrt{c+dx}\right)}{b^2d^3} \\
&= -\frac{2c^2\cosh(a+b\sqrt{c+dx})}{b^2d^3} + \frac{12c(c+dx)\cosh(a+b\sqrt{c+dx})}{b^2d^3} \\
&- \frac{10(c+dx)^2\cosh(a+b\sqrt{c+dx})}{b^2d^3} - \frac{24c\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{b^3d^3} \\
&+ \frac{2c^2\sqrt{c+dx}\sinh(a+b\sqrt{c+dx})}{bd^3} + \frac{40(c+dx)^{3/2}\sinh(a+b\sqrt{c+dx})}{b^3d^3} \\
&- \frac{4c(c+dx)^{3/2}\sinh(a+b\sqrt{c+dx})}{bd^3} + \frac{2(c+dx)^{5/2}\sinh(a+b\sqrt{c+dx})}{bd^3} \\
&- \frac{120\text{Subst}\left(\int x^2\sinh(ax)dx, x, \sqrt{c+dx}\right)}{b^3d^3} \\
&+ \frac{(24c)\text{Subst}\left(\int \sinh(ax)dx, x, \sqrt{c+dx}\right)}{b^3d^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{24c \cosh(a + b\sqrt{c + dx})}{b^4 d^3} - \frac{2c^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
&\quad - \frac{120(c + dx) \cosh(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{12c(c + dx) \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
&\quad - \frac{10(c + dx)^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{24c\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} \\
&\quad + \frac{2c^2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd^3} + \frac{40(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} \\
&\quad - \frac{4c(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{bd^3} + \frac{2(c + dx)^{5/2} \sinh(a + b\sqrt{c + dx})}{bd^3} \\
&\quad + \frac{240 \text{Subst}(\int x \cosh(ax) dx, x, \sqrt{c + dx})}{b^4 d^3} \\
&= \frac{24c \cosh(a + b\sqrt{c + dx})}{b^4 d^3} - \frac{2c^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
&\quad - \frac{120(c + dx) \cosh(a + b\sqrt{c + dx})}{b^4 d^3} + \frac{12c(c + dx) \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
&\quad - \frac{10(c + dx)^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} + \frac{240\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^5 d^3} \\
&\quad - \frac{24c\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} + \frac{2c^2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd^3} \\
&\quad + \frac{40(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} - \frac{4c(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{bd^3} \\
&\quad + \frac{2(c + dx)^{5/2} \sinh(a + b\sqrt{c + dx})}{bd^3} - \frac{240 \text{Subst}(\int \sinh(ax) dx, x, \sqrt{c + dx})}{b^5 d^3} \\
&= -\frac{240 \cosh(a + b\sqrt{c + dx})}{b^6 d^3} + \frac{24c \cosh(a + b\sqrt{c + dx})}{b^4 d^3} \\
&\quad - \frac{2c^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{120(c + dx) \cosh(a + b\sqrt{c + dx})}{b^4 d^3} \\
&\quad + \frac{12c(c + dx) \cosh(a + b\sqrt{c + dx})}{b^2 d^3} - \frac{10(c + dx)^2 \cosh(a + b\sqrt{c + dx})}{b^2 d^3} \\
&\quad + \frac{240\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^5 d^3} - \frac{24c\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} \\
&\quad + \frac{2c^2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd^3} + \frac{40(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{b^3 d^3} \\
&\quad - \frac{4c(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{bd^3} + \frac{2(c + dx)^{5/2} \sinh(a + b\sqrt{c + dx})}{bd^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.60

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx$$

$$= \frac{e^{a+b\sqrt{c+dx}}(-120 + 120b\sqrt{c+dx} + b^5 d^2 x^2 \sqrt{c+dx} + 4b^3 \sqrt{c+dx}(2c + 5dx) - 12b^2(4c + 5dx) - b^4 dx(4c + 5dx))}{b^6 d^3}$$

[In] Integrate[x^2*Cosh[a + b*Sqrt[c + d*x]],x]

[Out] (E^(a + b*Sqrt[c + d*x])*(-120 + 120*b*Sqrt[c + d*x] + b^5*d^2*x^2*Sqrt[c + d*x] + 4*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) - 12*b^2*(4*c + 5*d*x) - b^4*d*x*(4*c + 5*d*x)) - E^(-a - b*Sqrt[c + d*x])*(120 + 120*b*Sqrt[c + d*x] + b^5*d^2*x^2*Sqrt[c + d*x] + 4*b^3*Sqrt[c + d*x]*(2*c + 5*d*x) + 12*b^2*(4*c + 5*d*x) + b^4*d*x*(4*c + 5*d*x)))/(b^6*d^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 830 vs. 2(310) = 620.

Time = 0.18 (sec) , antiderivative size = 831, normalized size of antiderivative = 2.40

method	result
derivativedivides	$\frac{10a^4((a+b\sqrt{dx+c})\sinh(a+b\sqrt{dx+c})-\cosh(a+b\sqrt{dx+c}))}{b^4} - \frac{2a^5\sinh(a+b\sqrt{dx+c})}{b^4} - \frac{20a^3((a+b\sqrt{dx+c})^2\sinh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx+c}))}{b^4}$
default	$\frac{10a^4((a+b\sqrt{dx+c})\sinh(a+b\sqrt{dx+c})-\cosh(a+b\sqrt{dx+c}))}{b^4} - \frac{2a^5\sinh(a+b\sqrt{dx+c})}{b^4} - \frac{20a^3((a+b\sqrt{dx+c})^2\sinh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx+c}))}{b^4}$
parts	$\frac{2x^2\sinh(a+b\sqrt{dx+c})\sqrt{dx+c}}{db} - \frac{2x^2\cosh(a+b\sqrt{dx+c})}{db^2} - \frac{8\left(\frac{6a^2((a+b\sqrt{dx+c})^2\cosh(a+b\sqrt{dx+c})-2(a+b\sqrt{dx+c}))\sinh(a+b\sqrt{dx+c})}{b^2}\right)}{db^2}$

[In] int(x^2*cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)

[Out] 2/d^3/b^2*(5/b^4*a^4*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2)))-1/b^4*a^5*sinh(a+b*(d*x+c)^(1/2))-10/b^4*a^3*((a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+2*sinh(a+b*(d*x+c)^(1/2)))+10/b^4*a^2*((a+b*(d*x+c)^(1/2))^3*sinh(a+b*(d*x+c)^(1/2))-3*(a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))+6*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-6*cosh(a+b*(d*x+c)^(1/2)))-6/b^2*a^2*c*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2)))+2/b^2*a^3*c*sinh(a+b*(d*x+c)^(1/2))-5/b^4*a*((a+b*(d*x+c)^(1/2))^4*sinh(a+b*(d*x+c)^(1/2))-4*(a+b*(d*x+c)^(1/2))^3*cosh(a+b*(d*x+c)^(1/2))+12*(a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-24*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+24*sinh(a+b*(d*x+c)^(1/2)))+6/b^2*a*c*((a+b*(d*x+c)^(1/2))^2*sinh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+2*sinh(a+b*(d*x+c)^(1/2)))-2*(a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))+2*sinh(a+b*(d*x+c)^(1/2))

$$\begin{aligned} &)^{(1/2)})+1/b^4*((a+b*(d*x+c))^{(1/2)})^5*\sinh(a+b*(d*x+c))^{(1/2)})-5*(a+b*(d*x+c))^{(1/2)})^4*\cosh(a+b*(d*x+c))^{(1/2)})+20*(a+b*(d*x+c))^{(1/2)})^3*\sinh(a+b*(d*x+c))^{(1/2)})-60*(a+b*(d*x+c))^{(1/2)})^2*\cosh(a+b*(d*x+c))^{(1/2)})+120*(a+b*(d*x+c))^{(1/2)})*\sinh(a+b*(d*x+c))^{(1/2)})-120*\cosh(a+b*(d*x+c))^{(1/2)})-2/b^2*c*((a+b*(d*x+c))^{(1/2)})^3*\sinh(a+b*(d*x+c))^{(1/2)})-3*(a+b*(d*x+c))^{(1/2)})^2*\cosh(a+b*(d*x+c))^{(1/2)})+6*(a+b*(d*x+c))^{(1/2)})*\sinh(a+b*(d*x+c))^{(1/2)})-6*\cosh(a+b*(d*x+c))^{(1/2)})+c^2*((a+b*(d*x+c))^{(1/2)})*\sinh(a+b*(d*x+c))^{(1/2)})-\cosh(a+b*(d*x+c))^{(1/2)})-c^2*a*\sinh(a+b*(d*x+c))^{(1/2)}) \end{aligned}$$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.30

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx$$

$$= \frac{2((b^5 d^2 x^2 + 20 b^3 dx + 8 b^3 c + 120 b)\sqrt{dx + c} \sinh(\sqrt{dx + c} b + a) - (5 b^4 d^2 x^2 + 48 b^2 c + 4(b^4 c + 15 b^2) dx + 120) \cosh(\sqrt{dx + c} b + a))}{b^6 d^3}$$

[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] 2*((b^5*d^2*x^2 + 20*b^3*d*x + 8*b^3*c + 120*b)*sqrt(d*x + c)*sinh(sqrt(d*x + c)*b + a) - (5*b^4*d^2*x^2 + 48*b^2*c + 4*(b^4*c + 15*b^2)*d*x + 120)*cosh(sqrt(d*x + c)*b + a))/(b^6*d^3)

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.78

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx$$

$$= \begin{cases} \frac{x^3 \cosh(a)}{3} \\ \frac{x^3 \cosh(a+b\sqrt{c})}{3} \\ \frac{2x^2\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd} - \frac{8cx \cosh(a+b\sqrt{c+dx})}{b^2 d^2} - \frac{10x^2 \cosh(a+b\sqrt{c+dx})}{b^2 d} + \frac{16c\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{b^3 d^3} + \frac{40x\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{b^3 d^2} \end{cases}$$

[In] integrate(x**2*cosh(a+b*(d*x+c)**(1/2)),x)

[Out] Piecewise((x**3*cosh(a)/3, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**3*cosh(a + b*sqrt(c))/3, Eq(d, 0)), (2*x**2*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b*d) - 8*c*x*cosh(a + b*sqrt(c + d*x))/(b**2*d**2) - 10*x**2*cosh(a + b*sqrt(c + d*x))/(b**2*d) + 16*c*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**3) + 40*x*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**2) - 96*c*cosh(a + b*sqrt(c + d*x))/(b**4*d**3) - 120*x*cosh(a + b*sqrt(c + d*x))/(b**4*d**2) + 240*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**5*d**3) - 240*cosh(a + b*sqrt(c + d*x))/(b**6*d**3), True))

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 486, normalized size of antiderivative = 1.40

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx$$

$$= \frac{2d^3x^3 \cosh(\sqrt{dx + cb} + a) + \left(\frac{c^3e^{(\sqrt{dx+cb+a})}}{b} + \frac{c^3e^{(-\sqrt{dx+cb-a})}}{b} - \frac{3((dx+c)b^2e^a - 2\sqrt{dx+cb}e^a + 2e^a)c^2e^{(\sqrt{dx+cb})}}{b^3} - \frac{3((dx+c)b^2e^{-a} - 2\sqrt{dx+cb}e^{-a} + 2e^{-a})c^2e^{(-\sqrt{dx+cb})}}{b^3} \right)}{d^3}$$

[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

```
[Out] 1/6*(2*d^3*x^3*cosh(sqrt(d*x + c)*b + a) + (c^3*e^(sqrt(d*x + c)*b + a)/b +
c^3*e^(-sqrt(d*x + c)*b - a)/b - 3*((d*x + c)*b^2*e^a - 2*sqrt(d*x + c)*b*
e^a + 2*e^a)*c^2*e^(sqrt(d*x + c)*b)/b^3 - 3*((d*x + c)*b^2 + 2*sqrt(d*x +
c)*b + 2)*c^2*e^(-sqrt(d*x + c)*b - a)/b^3 + 3*((d*x + c)^2*b^4*e^a - 4*(d*
x + c)^(3/2)*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*sqrt(d*x + c)*b*e^a + 24*e
^a)*c*e^(sqrt(d*x + c)*b)/b^5 + 3*((d*x + c)^2*b^4 + 4*(d*x + c)^(3/2)*b^3
+ 12*(d*x + c)*b^2 + 24*sqrt(d*x + c)*b + 24)*c*e^(-sqrt(d*x + c)*b - a)/b^
5 - ((d*x + c)^3*b^6*e^a - 6*(d*x + c)^(5/2)*b^5*e^a + 30*(d*x + c)^2*b^4*e
^a - 120*(d*x + c)^(3/2)*b^3*e^a + 360*(d*x + c)*b^2*e^a - 720*sqrt(d*x + c
)*b*e^a + 720*e^a)*e^(sqrt(d*x + c)*b)/b^7 - ((d*x + c)^3*b^6 + 6*(d*x + c)
^(5/2)*b^5 + 30*(d*x + c)^2*b^4 + 120*(d*x + c)^(3/2)*b^3 + 360*(d*x + c)*b
^2 + 720*sqrt(d*x + c)*b + 720)*e^(-sqrt(d*x + c)*b - a)/b^7)*b)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 915 vs. 2(310) = 620.

Time = 0.29 (sec) , antiderivative size = 915, normalized size of antiderivative = 2.64

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx$$

$$= \frac{((\sqrt{dx+cb+a})b^4c^2 - ab^4c^2 - 2(\sqrt{dx+cb+a})^3b^2c + 6(\sqrt{dx+cb+a})^2ab^2c - 6(\sqrt{dx+cb+a})a^2b^2c + 2a^3b^2c - b^4c^2 + (\sqrt{dx+cb+a})^5 - 5(\sqrt{dx+cb+a})^4a)}{d^3}$$

[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

```
[Out] (((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b + a)^3*b^2
*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*a^2*b^2*c
+ 2*a^3*b^2*c - b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x + c)*b + a)
^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^2*a^3 + 5*
(sqrt(d*x + c)*b + a)*a^4 - a^5 + 6*(sqrt(d*x + c)*b + a)^2*b^2*c - 12*(sqr
t(d*x + c)*b + a)*a*b^2*c + 6*a^2*b^2*c - 5*(sqrt(d*x + c)*b + a)^4 + 20*(s
```

```

qrt(d*x + c)*b + a)^3*a - 30*(sqrt(d*x + c)*b + a)^2*a^2 + 20*(sqrt(d*x + c)
)*b + a)*a^3 - 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*c + 20*(sq
rt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*x + c)*b +
a)*a^2 - 20*a^3 + 12*b^2*c - 60*(sqrt(d*x + c)*b + a)^2 + 120*(sqrt(d*x +
c)*b + a)*a - 60*a^2 + 120*sqrt(d*x + c)*b - 120)*e^(sqrt(d*x + c)*b + a)/(
b^5*d^2) - ((sqrt(d*x + c)*b + a)*b^4*c^2 - a*b^4*c^2 - 2*(sqrt(d*x + c)*b
+ a)^3*b^2*c + 6*(sqrt(d*x + c)*b + a)^2*a*b^2*c - 6*(sqrt(d*x + c)*b + a)*
a^2*b^2*c + 2*a^3*b^2*c + b^4*c^2 + (sqrt(d*x + c)*b + a)^5 - 5*(sqrt(d*x +
c)*b + a)^4*a + 10*(sqrt(d*x + c)*b + a)^3*a^2 - 10*(sqrt(d*x + c)*b + a)^
2*a^3 + 5*(sqrt(d*x + c)*b + a)*a^4 - a^5 - 6*(sqrt(d*x + c)*b + a)^2*b^2*c
+ 12*(sqrt(d*x + c)*b + a)*a*b^2*c - 6*a^2*b^2*c + 5*(sqrt(d*x + c)*b + a)
^4 - 20*(sqrt(d*x + c)*b + a)^3*a + 30*(sqrt(d*x + c)*b + a)^2*a^2 - 20*(sq
rt(d*x + c)*b + a)*a^3 + 5*a^4 - 12*(sqrt(d*x + c)*b + a)*b^2*c + 12*a*b^2*
c + 20*(sqrt(d*x + c)*b + a)^3 - 60*(sqrt(d*x + c)*b + a)^2*a + 60*(sqrt(d*
x + c)*b + a)*a^2 - 20*a^3 - 12*b^2*c + 60*(sqrt(d*x + c)*b + a)^2 - 120*(s
qrt(d*x + c)*b + a)*a + 60*a^2 + 120*sqrt(d*x + c)*b + 120)*e^(-sqrt(d*x +
c)*b - a)/(b^5*d^2))/(b*d)

```

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh(a + b\sqrt{c + dx}) dx = \int x^2 \cosh(a + b\sqrt{c + dx}) dx$$

```
[In] int(x^2*cosh(a + b*(c + d*x)^(1/2)),x)
```

```
[Out] int(x^2*cosh(a + b*(c + d*x)^(1/2)), x)
```

3.60 $\int x \cosh (a + b\sqrt{c + dx}) dx$

Optimal result	301
Rubi [A] (verified)	301
Mathematica [A] (verified)	304
Maple [B] (verified)	304
Fricas [A] (verification not implemented)	305
Sympy [A] (verification not implemented)	305
Maxima [A] (verification not implemented)	305
Giac [B] (verification not implemented)	306
Mupad [F(-1)]	306

Optimal result

Integrand size = 16, antiderivative size = 167

$$\int x \cosh (a + b\sqrt{c + dx}) dx = -\frac{12 \cosh (a + b\sqrt{c + dx})}{b^4 d^2} + \frac{2c \cosh (a + b\sqrt{c + dx})}{b^2 d^2} - \frac{6(c + dx) \cosh (a + b\sqrt{c + dx})}{b^2 d^2} + \frac{12\sqrt{c + dx} \sinh (a + b\sqrt{c + dx})}{b^3 d^2} - \frac{2c\sqrt{c + dx} \sinh (a + b\sqrt{c + dx})}{bd^2} + \frac{2(c + dx)^{3/2} \sinh (a + b\sqrt{c + dx})}{bd^2}$$

[Out] $-12*\cosh(a+b*(d*x+c)^{(1/2)})/b^4/d^2+2*c*\cosh(a+b*(d*x+c)^{(1/2)})/b^2/d^2-6*(d*x+c)*\cosh(a+b*(d*x+c)^{(1/2)})/b^2/d^2+2*(d*x+c)^{(3/2)}*\sinh(a+b*(d*x+c)^{(1/2)})/b/d^2+12*\sinh(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b^3/d^2-2*c*\sinh(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b/d^2$

Rubi [A] (verified)

Time = 0.14 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used

= {5473, 5395, 3377, 2718}

$$\int x \cosh(a + b\sqrt{c + dx}) dx = -\frac{12 \cosh(a + b\sqrt{c + dx})}{b^4 d^2} + \frac{12\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{b^3 d^2} - \frac{6(c + dx) \cosh(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{2c \cosh(a + b\sqrt{c + dx})}{b^2 d^2} + \frac{2(c + dx)^{3/2} \sinh(a + b\sqrt{c + dx})}{bd^2} - \frac{2c\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd^2}$$

[In] Int[x*Cosh[a + b*Sqrt[c + d*x]],x]

[Out] (-12*Cosh[a + b*Sqrt[c + d*x]])/(b^4*d^2) + (2*c*Cosh[a + b*Sqrt[c + d*x]])/(b^2*d^2) - (6*(c + d*x)*Cosh[a + b*Sqrt[c + d*x]])/(b^2*d^2) + (12*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])/(b^3*d^2) - (2*c*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]])/(b*d^2) + (2*(c + d*x)^(3/2)*Sinh[a + b*Sqrt[c + d*x]])/(b*d^2)

Rule 2718

Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5395

Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rule 5473

Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.)^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (-c+x) \cosh(a+b\sqrt{x}) dx, x, c+dx\right)}{d^2} \\
&= \frac{2\text{Subst}\left(\int x(-c+x^2) \cosh(a+bx) dx, x, \sqrt{c+dx}\right)}{d^2} \\
&= \frac{2\text{Subst}\left(\int (-cx \cosh(a+bx) + x^3 \cosh(a+bx)) dx, x, \sqrt{c+dx}\right)}{d^2} \\
&= \frac{2\text{Subst}\left(\int x^3 \cosh(a+bx) dx, x, \sqrt{c+dx}\right)}{d^2} - \frac{(2c)\text{Subst}\left(\int x \cosh(a+bx) dx, x, \sqrt{c+dx}\right)}{d^2} \\
&= -\frac{2c\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd^2} + \frac{2(c+dx)^{3/2} \sinh(a+b\sqrt{c+dx})}{bd^2} \\
&\quad - \frac{6\text{Subst}\left(\int x^2 \sinh(a+bx) dx, x, \sqrt{c+dx}\right)}{bd^2} \\
&\quad + \frac{(2c)\text{Subst}\left(\int \sinh(a+bx) dx, x, \sqrt{c+dx}\right)}{bd^2} \\
&= \frac{2c \cosh(a+b\sqrt{c+dx})}{b^2 d^2} - \frac{6(c+dx) \cosh(a+b\sqrt{c+dx})}{b^2 d^2} - \frac{2c\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd^2} \\
&\quad + \frac{2(c+dx)^{3/2} \sinh(a+b\sqrt{c+dx})}{bd^2} + \frac{12\text{Subst}\left(\int x \cosh(a+bx) dx, x, \sqrt{c+dx}\right)}{b^2 d^2} \\
&= \frac{2c \cosh(a+b\sqrt{c+dx})}{b^2 d^2} - \frac{6(c+dx) \cosh(a+b\sqrt{c+dx})}{b^2 d^2} \\
&\quad + \frac{12\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{b^3 d^2} - \frac{2c\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd^2} \\
&\quad + \frac{2(c+dx)^{3/2} \sinh(a+b\sqrt{c+dx})}{bd^2} - \frac{12\text{Subst}\left(\int \sinh(a+bx) dx, x, \sqrt{c+dx}\right)}{b^3 d^2} \\
&= -\frac{12 \cosh(a+b\sqrt{c+dx})}{b^4 d^2} + \frac{2c \cosh(a+b\sqrt{c+dx})}{b^2 d^2} \\
&\quad - \frac{6(c+dx) \cosh(a+b\sqrt{c+dx})}{b^2 d^2} + \frac{12\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{b^3 d^2} \\
&\quad - \frac{2c\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd^2} + \frac{2(c+dx)^{3/2} \sinh(a+b\sqrt{c+dx})}{bd^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.43

$$\int x \cosh(a + b\sqrt{c + dx}) dx$$

$$= \frac{-2(6 + b^2(2c + 3dx)) \cosh(a + b\sqrt{c + dx}) + 2b\sqrt{c + dx}(6 + b^2dx) \sinh(a + b\sqrt{c + dx})}{b^4 d^2}$$

[In] Integrate[x*Cosh[a + b*Sqrt[c + d*x]],x]

[Out] (-2*(6 + b^2*(2*c + 3*d*x))*Cosh[a + b*Sqrt[c + d*x]] + 2*b*Sqrt[c + d*x]*(6 + b^2*d*x)*Sinh[a + b*Sqrt[c + d*x]])/(b^4*d^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(149) = 298.

Time = 0.12 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.80

method	result
parts	$\frac{2x\sqrt{dx+c} \sinh(a+b\sqrt{dx+c})}{db} - \frac{2x \cosh(a+b\sqrt{dx+c})}{db^2} - 2 \left(\frac{2(a+b\sqrt{dx+c})^2 \cosh(a+b\sqrt{dx+c}) - 4(a+b\sqrt{dx+c}) \sinh(a+b\sqrt{dx+c})}{b^2} \right)$
derivativedivides	$\frac{6a^2((a+b\sqrt{dx+c}) \sinh(a+b\sqrt{dx+c}) - \cosh(a+b\sqrt{dx+c}))}{b^2} - \frac{2a^3 \sinh(a+b\sqrt{dx+c})}{b^2} - \frac{6a((a+b\sqrt{dx+c})^2 \sinh(a+b\sqrt{dx+c}) - 2(a+b\sqrt{dx+c}))}{b^2}$
default	$\frac{6a^2((a+b\sqrt{dx+c}) \sinh(a+b\sqrt{dx+c}) - \cosh(a+b\sqrt{dx+c}))}{b^2} - \frac{2a^3 \sinh(a+b\sqrt{dx+c})}{b^2} - \frac{6a((a+b\sqrt{dx+c})^2 \sinh(a+b\sqrt{dx+c}) - 2(a+b\sqrt{dx+c}))}{b^2}$

[In] int(x*cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)

[Out] 2/d/b*x*(d*x+c)^(1/2)*sinh(a+b*(d*x+c)^(1/2))-2/d/b^2*x*cosh(a+b*(d*x+c)^(1/2))-2/d/b^2*(2/d/b^2*((a+b*(d*x+c)^(1/2))^2*cosh(a+b*(d*x+c)^(1/2))-2*(a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))+2*cosh(a+b*(d*x+c)^(1/2))-a*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2))))-2*a/d/b^2*((a+b*(d*x+c)^(1/2))*cosh(a+b*(d*x+c)^(1/2))-sinh(a+b*(d*x+c)^(1/2)))-a*cosh(a+b*(d*x+c)^(1/2))-2/d/b^2*((a+b*(d*x+c)^(1/2))*sinh(a+b*(d*x+c)^(1/2))-cosh(a+b*(d*x+c)^(1/2))-a*sinh(a+b*(d*x+c)^(1/2)))

Fricas [A] (verification not implemented)

none

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.41

$$\int x \cosh \left(a + b\sqrt{c + dx} \right) dx$$

$$= \frac{2 \left((b^3 dx + 6b)\sqrt{dx + c} \sinh(\sqrt{dx + cb} + a) - (3b^2 dx + 2b^2 c + 6) \cosh(\sqrt{dx + cb} + a) \right)}{b^4 d^2}$$

[In] integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")

[Out] 2*((b^3*d*x + 6*b)*sqrt(d*x + c)*sinh(sqrt(d*x + c)*b + a) - (3*b^2*d*x + 2*b^2*c + 6)*cosh(sqrt(d*x + c)*b + a))/(b^4*d^2)

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.90

$$\int x \cosh \left(a + b\sqrt{c + dx} \right) dx$$

$$= \begin{cases} \frac{x^2 \cosh(a)}{2} \\ \frac{x^2 \cosh(a+b\sqrt{c})}{2} \\ \frac{2x\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd} - \frac{4c \cosh(a+b\sqrt{c+dx})}{b^2 d^2} - \frac{6x \cosh(a+b\sqrt{c+dx})}{b^2 d} + \frac{12\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{b^3 d^2} - \frac{12 \cosh(a+b\sqrt{c+dx})}{b^4 d^2} \end{cases}$$

[In] integrate(x*cosh(a+b*(d*x+c)**(1/2)),x)

[Out] Piecewise((x**2*cosh(a)/2, Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x**2*cosh(a + b*sqrt(c))/2, Eq(d, 0)), (2*x*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b*d) - 4*c*cosh(a + b*sqrt(c + d*x))/(b**2*d**2) - 6*x*cosh(a + b*sqrt(c + d*x))/(b**2*d) + 12*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b**3*d**2) - 12*cosh(a + b*sqrt(c + d*x))/(b**4*d**2), True))

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.74

$$\int x \cosh \left(a + b\sqrt{c + dx} \right) dx$$

$$= \frac{2d^2 x^2 \cosh(\sqrt{dx + cb} + a) - \left(\frac{c^2 e^{(\sqrt{dx+cb}+a)}}{b} + \frac{c^2 e^{(-\sqrt{dx+cb}-a)}}{b} - \frac{2((dx+c)b^2 e^a - 2\sqrt{dx+cb} e^a + 2e^a) c e^{(\sqrt{dx+cb})}}{b^3} - \frac{2((dx+c)b^2 e^{-a} - 2\sqrt{dx+cb} e^{-a} + 2e^{-a}) c e^{(-\sqrt{dx+cb})}}{b^3} \right)}{2d^2}$$

[In] integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] $\frac{1}{4}*(2*d^2*x^2*\cosh(\sqrt{d*x + c})*b + a) - (c^2*e^{(\sqrt{d*x + c})*b + a})/b + c^2*e^{(-\sqrt{d*x + c})*b - a}/b - 2*((d*x + c)*b^2*e^a - 2*\sqrt{d*x + c}*b*e^a + 2*e^a)*c*e^{(\sqrt{d*x + c})*b}/b^3 - 2*((d*x + c)*b^2 + 2*\sqrt{d*x + c}*b + 2)*c*e^{(-\sqrt{d*x + c})*b - a}/b^3 + ((d*x + c)^2*b^4*e^a - 4*(d*x + c)^{(3/2)}*b^3*e^a + 12*(d*x + c)*b^2*e^a - 24*\sqrt{d*x + c}*b*e^a + 24*e^a)*e^{(\sqrt{d*x + c})*b}/b^5 + ((d*x + c)^2*b^4 + 4*(d*x + c)^{(3/2)}*b^3 + 12*(d*x + c)*b^2 + 24*\sqrt{d*x + c}*b + 24)*e^{(-\sqrt{d*x + c})*b - a}/b^5*b)/d^2$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. 2(149) = 298.

Time = 0.26 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.80

$$\int x \cosh\left(a + b\sqrt{c + dx}\right) dx =$$

$$\frac{((\sqrt{dx+cb+a})b^2c-ab^2c-(\sqrt{dx+cb+a})^3+3(\sqrt{dx+cb+a})^2a-3(\sqrt{dx+cb+a})a^2+a^3-b^2c+3(\sqrt{dx+cb+a})^2-6(\sqrt{dx+cb+a})a+3a^2-6\sqrt{dx+cb+a})}{b^3d}$$

[In] integrate(x*cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] $-(((\sqrt{d*x + c})*b + a)*b^2*c - a*b^2*c - (\sqrt{d*x + c})*b + a)^3 + 3*(\sqrt{d*x + c})*b + a)^2*a - 3*(\sqrt{d*x + c})*b + a)*a^2 + a^3 - b^2*c + 3*(\sqrt{d*x + c})*b + a)^2 - 6*(\sqrt{d*x + c})*b + a)*a + 3*a^2 - 6*\sqrt{d*x + c}*b + 6)*e^{(\sqrt{d*x + c})*b + a}/(b^3*d) - ((\sqrt{d*x + c})*b + a)*b^2*c - a*b^2*c - (\sqrt{d*x + c})*b + a)^3 + 3*(\sqrt{d*x + c})*b + a)^2*a - 3*(\sqrt{d*x + c})*b + a)*a^2 + a^3 + b^2*c - 3*(\sqrt{d*x + c})*b + a)^2 + 6*(\sqrt{d*x + c})*b + a)*a - 3*a^2 - 6*\sqrt{d*x + c}*b - 6)*e^{(-\sqrt{d*x + c})*b - a}/(b^3*d))/(b*d)$

Mupad [F(-1)]

Timed out.

$$\int x \cosh\left(a + b\sqrt{c + dx}\right) dx = \int x \cosh\left(a + b\sqrt{c + dx}\right) dx$$

[In] int(x*cosh(a + b*(c + d*x)^(1/2)),x)

[Out] int(x*cosh(a + b*(c + d*x)^(1/2)), x)

3.61 $\int \cosh(a + b\sqrt{c + dx}) dx$

Optimal result	307
Rubi [A] (verified)	307
Mathematica [A] (verified)	308
Maple [A] (verified)	309
Fricas [A] (verification not implemented)	309
Sympy [A] (verification not implemented)	309
Maxima [B] (verification not implemented)	310
Giac [A] (verification not implemented)	310
Mupad [B] (verification not implemented)	310

Optimal result

Integrand size = 14, antiderivative size = 54

$$\int \cosh(a + b\sqrt{c + dx}) dx = -\frac{2 \cosh(a + b\sqrt{c + dx})}{b^2 d} + \frac{2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd}$$

[Out] $-2*\cosh(a+b*(d*x+c)^{(1/2)})/b^2/d+2*\sinh(a+b*(d*x+c)^{(1/2)})*(d*x+c)^{(1/2)}/b/d$

Rubi [A] (verified)

Time = 0.03 (sec), antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5419, 5413, 3377, 2718}

$$\int \cosh(a + b\sqrt{c + dx}) dx = \frac{2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd} - \frac{2 \cosh(a + b\sqrt{c + dx})}{b^2 d}$$

[In] Int[Cosh[a + b*Sqrt[c + d*x]],x]

[Out] $(-2*\cosh[a + b*\sqrt{c + d*x}])/(b^2*d) + (2*\sqrt{c + d*x}*\sinh[a + b*\sqrt{c + d*x}])/(b*d)$

Rule 2718

Int[sin[(c_.) + (d_.)*(x_.)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_.)*(x_.))^(m_.)*sin[(e_.) + (f_.)*(x_.)], x_Symbol] :> Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co

`s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 5413

`Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.), x_Symbol] :> Module[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Cosh[c + d*x^(k*n)])]^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && IntegerQ[p]`

Rule 5419

`Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.), x_Symbol] :> Dist[1/Coefficient[u, x, 1], Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\text{Subst}\left(\int \cosh(a + b\sqrt{x}) dx, x, c + dx\right)}{d} \\
 &= \frac{2\text{Subst}\left(\int x \cosh(a + bx) dx, x, \sqrt{c + dx}\right)}{d} \\
 &= \frac{2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd} - \frac{2\text{Subst}\left(\int \sinh(a + bx) dx, x, \sqrt{c + dx}\right)}{bd} \\
 &= -\frac{2 \cosh(a + b\sqrt{c + dx})}{b^2d} + \frac{2\sqrt{c + dx} \sinh(a + b\sqrt{c + dx})}{bd}
 \end{aligned}$$

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \cosh(a + b\sqrt{c + dx}) dx = \frac{2(-\cosh(a + b\sqrt{c + dx}) + b\sqrt{c + dx} \sinh(a + b\sqrt{c + dx}))}{b^2d}$$

`[In] Integrate[Cosh[a + b*Sqrt[c + d*x]],x]`

`[Out] (2*(-Cosh[a + b*Sqrt[c + d*x]] + b*Sqrt[c + d*x]*Sinh[a + b*Sqrt[c + d*x]]))/(b^2*d)`

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.17

method	result	size
derivativedivides	$\frac{2(a+b\sqrt{dx+c}) \sinh(a+b\sqrt{dx+c}) - 2 \cosh(a+b\sqrt{dx+c}) - 2a \sinh(a+b\sqrt{dx+c})}{b^2 d}$	63
default	$\frac{2(a+b\sqrt{dx+c}) \sinh(a+b\sqrt{dx+c}) - 2 \cosh(a+b\sqrt{dx+c}) - 2a \sinh(a+b\sqrt{dx+c})}{b^2 d}$	63

[In] `int(cosh(a+b*(d*x+c)^(1/2)),x,method=_RETURNVERBOSE)`[Out] $2/d/b^2*((a+b*(d*x+c)^(1/2))*\sinh(a+b*(d*x+c)^(1/2))-\cosh(a+b*(d*x+c)^(1/2))-a*\sinh(a+b*(d*x+c)^(1/2)))$ **Fricas [A] (verification not implemented)**

none

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cosh(a + b\sqrt{c + dx}) dx = \frac{2(\sqrt{dx + cb} \sinh(\sqrt{dx + cb} + a) - \cosh(\sqrt{dx + cb} + a))}{b^2 d}$$

[In] `integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="fricas")`[Out] $2*(\sqrt{d*x + c}*b*\sinh(\sqrt{d*x + c}*b + a) - \cosh(\sqrt{d*x + c}*b + a))/(b^2*d)$ **Sympy [A] (verification not implemented)**

Time = 0.24 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \cosh(a + b\sqrt{c + dx}) dx = \begin{cases} x \cosh(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x \cosh(a + b\sqrt{c}) & \text{for } d = 0 \\ \frac{2\sqrt{c+dx} \sinh(a+b\sqrt{c+dx})}{bd} - \frac{2 \cosh(a+b\sqrt{c+dx})}{b^2 d} & \text{otherwise} \end{cases}$$

[In] `integrate(cosh(a+b*(d*x+c)**(1/2)),x)`[Out] `Piecewise((x*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cosh(a + b*sqrt(c)), Eq(d, 0)), (2*sqrt(c + d*x)*sinh(a + b*sqrt(c + d*x))/(b*d) - 2*cosh(a + b*sqrt(c + d*x))/(b**2*d), True))`

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 110 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.04

$$\int \cosh\left(a + b\sqrt{c + dx}\right) dx = \frac{b\left(\frac{((dx+c)b^2e^a - 2\sqrt{dx+cb}e^a + 2e^a)e^{\sqrt{dx+cb}}}{b^3} + \frac{((dx+c)b^2 + 2\sqrt{dx+cb} + 2)e^{-\sqrt{dx+cb}-a}}{b^3}\right) - 2(dx+c)\cosh(\sqrt{dx+cb}+a)}{2d}$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="maxima")

[Out] $-1/2*(b*(((d*x + c)*b^2*e^a - 2*\sqrt{d*x + c}*b*e^a + 2*e^a)*e^{\sqrt{d*x + c}*b}/b^3 + ((d*x + c)*b^2 + 2*\sqrt{d*x + c}*b + 2)*e^{-\sqrt{d*x + c}*b - a})/b^3 - 2*(d*x + c)*\cosh(\sqrt{d*x + c}*b + a))/d$

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \cosh\left(a + b\sqrt{c + dx}\right) dx = \frac{(\sqrt{dx+cb}-1)e^{\sqrt{dx+cb}+a}}{b^2d} - \frac{(\sqrt{dx+cb}+1)e^{-\sqrt{dx+cb}-a}}{b^2d}$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2)),x, algorithm="giac")

[Out] $(\sqrt{d*x + c}*b - 1)*e^{\sqrt{d*x + c}*b + a}/(b^2*d) - (\sqrt{d*x + c}*b + 1)*e^{-\sqrt{d*x + c}*b - a}/(b^2*d)$

Mupad [B] (verification not implemented)

Time = 1.61 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.80

$$\int \cosh\left(a + b\sqrt{c + dx}\right) dx = -\frac{2\left(\cosh(a + b\sqrt{c + dx}) - b\sinh(a + b\sqrt{c + dx})\sqrt{c + dx}\right)}{b^2d}$$

[In] int(cosh(a + b*(c + d*x)^(1/2)),x)

[Out] $-(2*(\cosh(a + b*(c + d*x)^(1/2)) - b*\sinh(a + b*(c + d*x)^(1/2))*(c + d*x)^(1/2)))/(b^2*d)$

3.62 $\int \frac{\cosh(a+b\sqrt{c+dx})}{x} dx$

Optimal result	311
Rubi [A] (verified)	311
Mathematica [A] (verified)	313
Maple [F]	314
Fricas [B] (verification not implemented)	314
Sympy [F]	314
Maxima [F]	315
Giac [F]	315
Mupad [F(-1)]	315

Optimal result

Integrand size = 18, antiderivative size = 124

$$\int \frac{\cosh(a+b\sqrt{c+dx})}{x} dx = \cosh(a+b\sqrt{c}) \operatorname{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) \\ + \cosh(a-b\sqrt{c}) \operatorname{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) \\ - \sinh(a+b\sqrt{c}) \operatorname{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) \\ + \sinh(a-b\sqrt{c}) \operatorname{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)$$

[Out] Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*cosh(a-b*c^(1/2))+Chi(b*(c^(1/2)-(d*x+c)^(1/2)))*cosh(a+b*c^(1/2))+Shi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))-Shi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5473, 5401, 3384, 3379, 3382}

$$\int \frac{\cosh(a+b\sqrt{c+dx})}{x} dx = \cosh(a+b\sqrt{c}) \operatorname{Chi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) \\ + \cosh(a-b\sqrt{c}) \operatorname{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) \\ - \sinh(a+b\sqrt{c}) \operatorname{Shi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right) \\ + \sinh(a-b\sqrt{c}) \operatorname{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)$$

[In] Int[Cosh[a + b*Sqrt[c + d*x]]/x,x]

```
[Out] Cosh[a + b*Sqrt[c]]*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Cosh[a - b*
Sqrt[c]]*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])] - Sinh[a + b*Sqrt[c]]*Si
nhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])] + Sinh[a - b*Sqrt[c]]*SinhIntegral[
b*(Sqrt[c] + Sqrt[c + d*x])]
```

Rule 3379

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Dist[Cos[(d*
e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f
)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] &&
NeQ[d*e - c*f, 0]
```

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= \text{Subst} \left(\int \frac{\cosh(a + b\sqrt{x})}{-c + x} dx, x, c + dx \right) \\ &= 2\text{Subst} \left(\int \frac{x \cosh(a + bx)}{-c + x^2} dx, x, \sqrt{c + dx} \right) \end{aligned}$$

$$\begin{aligned}
&= 2\text{Subst}\left(\int\left(-\frac{\cosh(a+bx)}{2(\sqrt{c}-x)}+\frac{\cosh(a+bx)}{2(\sqrt{c}+x)}\right)dx,x,\sqrt{c+dx}\right) \\
&= -\text{Subst}\left(\int\frac{\cosh(a+bx)}{\sqrt{c}-x}dx,x,\sqrt{c+dx}\right)+\text{Subst}\left(\int\frac{\cosh(a+bx)}{\sqrt{c}+x}dx,x,\sqrt{c+dx}\right) \\
&= \cosh(a-b\sqrt{c})\text{Subst}\left(\int\frac{\cosh(b\sqrt{c}+bx)}{\sqrt{c}+x}dx,x,\sqrt{c+dx}\right) \\
&\quad -\cosh(a+b\sqrt{c})\text{Subst}\left(\int\frac{\cosh(b\sqrt{c}-bx)}{\sqrt{c}-x}dx,x,\sqrt{c+dx}\right) \\
&\quad +\sinh(a-b\sqrt{c})\text{Subst}\left(\int\frac{\sinh(b\sqrt{c}+bx)}{\sqrt{c}+x}dx,x,\sqrt{c+dx}\right) \\
&\quad +\sinh(a+b\sqrt{c})\text{Subst}\left(\int\frac{\sinh(b\sqrt{c}-bx)}{\sqrt{c}-x}dx,x,\sqrt{c+dx}\right) \\
&= \cosh(a-b\sqrt{c})\text{Chi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)+\cosh(a+b\sqrt{c})\text{Chi}\left(b\sqrt{c}-b\sqrt{c+dx}\right) \\
&\quad +\sinh(a-b\sqrt{c})\text{Shi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)-\sinh(a+b\sqrt{c})\text{Shi}\left(b\sqrt{c}-b\sqrt{c+dx}\right)
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.02

$$\begin{aligned}
\int\frac{\cosh(a+b\sqrt{c+dx})}{x}dx &= \frac{1}{2}e^{-a-b\sqrt{c}}\left(\text{ExpIntegralEi}\left(b\left(\sqrt{c}-\sqrt{c+dx}\right)\right)\right) \\
&\quad +e^{2(a+b\sqrt{c})}\text{ExpIntegralEi}\left(b\left(-\sqrt{c}+\sqrt{c+dx}\right)\right) \\
&\quad +e^{2b\sqrt{c}}\text{ExpIntegralEi}\left(-b\left(\sqrt{c}+\sqrt{c+dx}\right)\right) \\
&\quad +e^{2a}\text{ExpIntegralEi}\left(b\left(\sqrt{c}+\sqrt{c+dx}\right)\right)
\end{aligned}$$

[In] Integrate[Cosh[a + b*Sqrt[c + d*x]]/x,x]

[Out] (E^(-a - b*Sqrt[c])*(ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])] + E^(2*(a + b*Sqrt[c]))*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] + E^(2*b*Sqrt[c])*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + E^(2*a)*ExpIntegralEi[b*(Sqrt[c] + Sqrt[c + d*x])]))/2

Maple [F]

$$\int \frac{\cosh(a + b\sqrt{dx + c})}{x} dx$$

[In] int(cosh(a+b*(d*x+c)^(1/2))/x,x)

[Out] int(cosh(a+b*(d*x+c)^(1/2))/x,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. 2(102) = 204.

Time = 0.27 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.75

$$\begin{aligned} & \int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx \\ &= \frac{1}{2} \left(\operatorname{Ei}(\sqrt{dx + cb} - \sqrt{b^2c}) + \operatorname{Ei}(-\sqrt{dx + cb} + \sqrt{b^2c}) \right) \cosh(a + \sqrt{b^2c}) \\ &+ \frac{1}{2} \left(\operatorname{Ei}(\sqrt{dx + cb} + \sqrt{b^2c}) + \operatorname{Ei}(-\sqrt{dx + cb} - \sqrt{b^2c}) \right) \cosh(-a + \sqrt{b^2c}) \\ &+ \frac{1}{2} \left(\operatorname{Ei}(\sqrt{dx + cb} - \sqrt{b^2c}) - \operatorname{Ei}(-\sqrt{dx + cb} + \sqrt{b^2c}) \right) \sinh(a + \sqrt{b^2c}) \\ &- \frac{1}{2} \left(\operatorname{Ei}(\sqrt{dx + cb} + \sqrt{b^2c}) - \operatorname{Ei}(-\sqrt{dx + cb} - \sqrt{b^2c}) \right) \sinh(-a + \sqrt{b^2c}) \end{aligned}$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="fricas")

[Out] 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) + 1/2*(Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) - 1/2*(Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*sinh(-a + sqrt(b^2*c))

Sympy [F]

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx = \int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx$$

[In] integrate(cosh(a+b*(d*x+c)**(1/2))/x,x)

[Out] Integral(cosh(a + b*sqrt(c + d*x))/x, x)

Maxima [F]

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx = \int \frac{\cosh(\sqrt{dx + cb} + a)}{x} dx$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="maxima")

[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x, x)

Giac [F]

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx = \int \frac{\cosh(\sqrt{dx + cb} + a)}{x} dx$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x,x, algorithm="giac")

[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx = \int \frac{\cosh(a + b\sqrt{c + dx})}{x} dx$$

[In] int(cosh(a + b*(c + d*x)^(1/2))/x,x)

[Out] int(cosh(a + b*(c + d*x)^(1/2))/x, x)

3.63 $\int \frac{\cosh(a+b\sqrt{c+dx})}{x^2} dx$

Optimal result	316
Rubi [A] (verified)	316
Mathematica [A] (verified)	319
Maple [F]	319
Fricas [B] (verification not implemented)	320
Sympy [F]	320
Maxima [F]	320
Giac [F]	321
Mupad [F(-1)]	321

Optimal result

Integrand size = 18, antiderivative size = 182

$$\int \frac{\cosh(a+b\sqrt{c+dx})}{x^2} dx = -\frac{\cosh(a+b\sqrt{c+dx})}{x} - \frac{bd\text{Chi}(b(\sqrt{c}+\sqrt{c+dx}))\sinh(a-b\sqrt{c})}{2\sqrt{c}} + \frac{bd\text{Chi}(b(\sqrt{c}-\sqrt{c+dx}))\sinh(a+b\sqrt{c})}{2\sqrt{c}} - \frac{bd\cosh(a+b\sqrt{c})\text{Shi}(b(\sqrt{c}-\sqrt{c+dx}))}{2\sqrt{c}} - \frac{bd\cosh(a-b\sqrt{c})\text{Shi}(b(\sqrt{c}+\sqrt{c+dx}))}{2\sqrt{c}}$$

```
[Out] -cosh(a+b*(d*x+c)^(1/2))/x-1/2*b*d*cosh(a+b*c^(1/2))*Shi(b*(c^(1/2)-(d*x+c)^(1/2)))/c^(1/2)-1/2*b*d*cosh(a-b*c^(1/2))*Shi(b*(c^(1/2)+(d*x+c)^(1/2)))/c^(1/2)-1/2*b*d*Chi(b*(c^(1/2)+(d*x+c)^(1/2)))*sinh(a-b*c^(1/2))/c^(1/2)+1/2*b*d*Chi(b*(c^(1/2)-(d*x+c)^(1/2)))*sinh(a+b*c^(1/2))/c^(1/2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {5473, 5397, 5388, 3384, 3379, 3382}

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = -\frac{bd \sinh(a - b\sqrt{c}) \operatorname{Chi}(b(\sqrt{c} + \sqrt{c + dx}))}{2\sqrt{c}} + \frac{bd \sinh(a + b\sqrt{c}) \operatorname{Chi}(b(\sqrt{c} - \sqrt{c + dx}))}{2\sqrt{c}} - \frac{bd \cosh(a + b\sqrt{c}) \operatorname{Shi}(b(\sqrt{c} - \sqrt{c + dx}))}{2\sqrt{c}} - \frac{bd \cosh(a - b\sqrt{c}) \operatorname{Shi}(b(\sqrt{c} + \sqrt{c + dx}))}{2\sqrt{c}} - \frac{\cosh(a + b\sqrt{c + dx})}{x}$$

[In] Int[Cosh[a + b*Sqrt[c + d*x]]/x^2,x]

[Out] -(Cosh[a + b*Sqrt[c + d*x]]/x) - (b*d*CoshIntegral[b*(Sqrt[c] + Sqrt[c + d*x])*Sinh[a - b*Sqrt[c]]]/(2*Sqrt[c])) + (b*d*CoshIntegral[b*(Sqrt[c] - Sqrt[c + d*x])*Sinh[a + b*Sqrt[c]]]/(2*Sqrt[c])) - (b*d*Cosh[a + b*Sqrt[c]]*SinhIntegral[b*(Sqrt[c] - Sqrt[c + d*x])]/(2*Sqrt[c])) - (b*d*Cosh[a - b*Sqrt[c]]*SinhIntegral[b*(Sqrt[c] + Sqrt[c + d*x])]/(2*Sqrt[c]))

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5388

Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] := Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_.)]*((e_.)*(x_.))^(m_.)*((a_.) + (b_.)*(x_.)^(n_.))^(p_.), x_Symbol] := Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] - Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= d\text{Subst}\left(\int \frac{\cosh(a + b\sqrt{x})}{(-c + x)^2} dx, x, c + dx\right) \\
&= (2d)\text{Subst}\left(\int \frac{x \cosh(a + bx)}{(c - x^2)^2} dx, x, \sqrt{c + dx}\right) \\
&= -\frac{\cosh(a + b\sqrt{c + dx})}{x} - (bd)\text{Subst}\left(\int \frac{\sinh(a + bx)}{c - x^2} dx, x, \sqrt{c + dx}\right) \\
&= -\frac{\cosh(a + b\sqrt{c + dx})}{x} - (bd)\text{Subst}\left(\int \left(\frac{\sinh(a + bx)}{2\sqrt{c}(\sqrt{c} - x)} + \frac{\sinh(a + bx)}{2\sqrt{c}(\sqrt{c} + x)}\right) dx, x, \sqrt{c + dx}\right) \\
&= -\frac{\cosh(a + b\sqrt{c + dx})}{x} - \frac{(bd)\text{Subst}\left(\int \frac{\sinh(a + bx)}{\sqrt{c} - x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&\quad - \frac{(bd)\text{Subst}\left(\int \frac{\sinh(a + bx)}{\sqrt{c} + x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&= -\frac{\cosh(a + b\sqrt{c + dx})}{x} - \frac{(bd \cosh(a - b\sqrt{c})) \text{Subst}\left(\int \frac{\sinh(b\sqrt{c} + bx)}{\sqrt{c} + x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&\quad + \frac{(bd \cosh(a + b\sqrt{c})) \text{Subst}\left(\int \frac{\sinh(b\sqrt{c} - bx)}{\sqrt{c} - x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&\quad - \frac{(bd \sinh(a - b\sqrt{c})) \text{Subst}\left(\int \frac{\cosh(b\sqrt{c} + bx)}{\sqrt{c} + x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}} \\
&\quad - \frac{(bd \sinh(a + b\sqrt{c})) \text{Subst}\left(\int \frac{\cosh(b\sqrt{c} - bx)}{\sqrt{c} - x} dx, x, \sqrt{c + dx}\right)}{2\sqrt{c}}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\cosh(a + b\sqrt{c + dx})}{x} - \frac{bd\text{Chi}(b(\sqrt{c} + \sqrt{c + dx})) \sinh(a - b\sqrt{c})}{2\sqrt{c}} \\
&\quad + \frac{bd\text{Chi}(b\sqrt{c} - b\sqrt{c + dx}) \sinh(a + b\sqrt{c})}{2\sqrt{c}} \\
&\quad - \frac{bd \cosh(a - b\sqrt{c}) \text{Shi}(b(\sqrt{c} + \sqrt{c + dx}))}{2\sqrt{c}} \\
&\quad - \frac{bd \cosh(a + b\sqrt{c}) \text{Shi}(b\sqrt{c} - b\sqrt{c + dx})}{2\sqrt{c}}
\end{aligned}$$

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = \frac{e^{-a} \left(2\sqrt{c} e^{-b\sqrt{c+dx}} + 2\sqrt{c} e^{2a+b\sqrt{c+dx}} + b d e^{-b\sqrt{c}} x \text{ExpIntegralEi}(b(\sqrt{c} - \sqrt{c+dx})) - b d e^{2a+b\sqrt{c}} x \text{ExpIntegralEi}(b(\sqrt{c} + \sqrt{c+dx})) \right)}{x^2}$$

[In] Integrate[Cosh[a + b*Sqrt[c + d*x]]/x^2,x]

[Out] -1/4*((2*Sqrt[c])/E^(b*Sqrt[c + d*x]) + 2*Sqrt[c]*E^(2*a + b*Sqrt[c + d*x]) + (b*d*x*ExpIntegralEi[b*(Sqrt[c] - Sqrt[c + d*x])])/E^(b*Sqrt[c]) - b*d*E^(2*a + b*Sqrt[c])*x*ExpIntegralEi[b*(-Sqrt[c] + Sqrt[c + d*x])] - b*d*E^(b*Sqrt[c])*x*ExpIntegralEi[-(b*(Sqrt[c] + Sqrt[c + d*x]))] + b*d*E^(2*a - b*Sqrt[c])*x*ExpIntegralEi[b*(Sqrt[c] + Sqrt[c + d*x])])/(Sqrt[c]*E^a*x)

Maple [F]

$$\int \frac{\cosh(a + b\sqrt{dx + c})}{x^2} dx$$

[In] int(cosh(a+b*(d*x+c)^(1/2))/x^2,x)

[Out] int(cosh(a+b*(d*x+c)^(1/2))/x^2,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 317 vs. 2(142) = 284.

Time = 0.26 (sec) , antiderivative size = 317, normalized size of antiderivative = 1.74

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = \frac{4c \cosh(\sqrt{dx + cb} + a) - (\sqrt{b^2cdx} \operatorname{Ei}(\sqrt{dx + cb} - \sqrt{b^2c}) - \sqrt{b^2cdx} \operatorname{Ei}(-\sqrt{dx + cb} + \sqrt{b^2c})) \cosh(a - \dots}{\dots}$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="fricas")

[Out] -1/4*(4*c*cosh(sqrt(d*x + c)*b + a) - (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*cosh(a + sqrt(b^2*c)) + (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) - sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*cosh(-a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b - sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b + sqrt(b^2*c)))*sinh(a + sqrt(b^2*c)) - (sqrt(b^2*c)*d*x*Ei(sqrt(d*x + c)*b + sqrt(b^2*c)) + sqrt(b^2*c)*d*x*Ei(-sqrt(d*x + c)*b - sqrt(b^2*c)))*sinh(-a + sqrt(b^2*c)))/(c*x)

Sympy [F]

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = \int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx$$

[In] integrate(cosh(a+b*(d*x+c)**(1/2))/x**2,x)

[Out] Integral(cosh(a + b*sqrt(c + d*x))/x**2, x)

Maxima [F]

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = \int \frac{\cosh(\sqrt{dx + cb} + a)}{x^2} dx$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="maxima")

[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x^2, x)

Giac [F]

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = \int \frac{\cosh(\sqrt{dx + cb} + a)}{x^2} dx$$

[In] integrate(cosh(a+b*(d*x+c)^(1/2))/x^2,x, algorithm="giac")

[Out] integrate(cosh(sqrt(d*x + c)*b + a)/x^2, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx = \int \frac{\cosh(a + b\sqrt{c + dx})}{x^2} dx$$

[In] int(cosh(a + b*(c + d*x)^(1/2))/x^2,x)

[Out] int(cosh(a + b*(c + d*x)^(1/2))/x^2, x)

3.64 $\int x^2 \cosh (a + b\sqrt[3]{c + dx}) dx$

Optimal result	323
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Optimal result

Integrand size = 18, antiderivative size = 537

$$\begin{aligned}
 \int x^2 \cosh \left(a + b\sqrt[3]{c + dx} \right) dx = & \frac{720c \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^6 d^3} \\
 & - \frac{120960 \sqrt[3]{c + dx} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^8 d^3} \\
 & - \frac{6c^2 \sqrt[3]{c + dx} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^3} \\
 & + \frac{360c(c + dx)^{2/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^4 d^3} \\
 & - \frac{20160(c + dx) \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^6 d^3} \\
 & + \frac{30c(c + dx)^{4/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^3} \\
 & - \frac{1008(c + dx)^{5/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^4 d^3} \\
 & - \frac{24(c + dx)^{7/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^3} \\
 & + \frac{120960 \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^9 d^3} + \frac{6c^2 \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^3} \\
 & - \frac{720c \sqrt[3]{c + dx} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^5 d^3} \\
 & + \frac{60480(c + dx)^{2/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^7 d^3} \\
 & + \frac{3c^2(c + dx)^{2/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{bd^3} \\
 & - \frac{120c(c + dx) \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^3} \\
 & + \frac{5040(c + dx)^{4/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^5 d^3} \\
 & - \frac{6c(c + dx)^{5/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{bd^3} \\
 & + \frac{168(c + dx)^2 \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^3} \\
 & + \frac{3(c + dx)^{8/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{bd^3}
 \end{aligned}$$

[Out] $720*c*cosh(a+b*(d*x+c)^{(1/3)})/b^6/d^3-120960*(d*x+c)^{(1/3)}*cosh(a+b*(d*x+c)^{(1/3)})/b^8/d^3-6*c^2*(d*x+c)^{(1/3)}*cosh(a+b*(d*x+c)^{(1/3)})/b^2/d^3+360*c*(d*x+c)^{(2/3)}*cosh(a+b*(d*x+c)^{(1/3)})/b^4/d^3-20160*(d*x+c)*cosh(a+b*(d*x+c)^{(1/3)})/b^6/d^3+30*c*(d*x+c)^{(4/3)}*cosh(a+b*(d*x+c)^{(1/3)})/b^2/d^3-1008*(d*x+c)^{(5/3)}*cosh(a+b*(d*x+c)^{(1/3)})/b^4/d^3-24*(d*x+c)^{(7/3)}*cosh(a+b*(d*x+c)^{(1/3)})/b^2/d^3+120960*sinh(a+b*(d*x+c)^{(1/3)})/b^9/d^3+6*c^2*sinh(a+b*(d*x+c)^{(1/3)})/b^3/d^3-720*c*(d*x+c)^{(1/3)}*sinh(a+b*(d*x+c)^{(1/3)})/b^5/d^3+60480*(d*x+c)^{(2/3)}*sinh(a+b*(d*x+c)^{(1/3)})/b^7/d^3+3*c^2*(d*x+c)^{(2/3)}*sinh(a+b*(d*x+c)^{(1/3)})/b/d^3-120*c*(d*x+c)*sinh(a+b*(d*x+c)^{(1/3)})/b^3/d^3+5040*(d*x+c)^{(4/3)}*sinh(a+b*(d*x+c)^{(1/3)})/b^5/d^3-6*c*(d*x+c)^{(5/3)}*sinh(a+b*(d*x+c)^{(1/3)})/b/d^3+168*(d*x+c)^2*sinh(a+b*(d*x+c)^{(1/3)})/b^3/d^3+3*(d*x+c)^{(8/3)}*sinh(a+b*(d*x+c)^{(1/3)})/b/d^3$

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 537, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used

= {5473, 1607, 5395, 3377, 2717, 2718}

$$\begin{aligned}
 \int x^2 \cosh(a + b\sqrt[3]{c + dx}) dx = & \frac{120960 \sinh(a + b\sqrt[3]{c + dx})}{b^9 d^3} \\
 & - \frac{120960 \sqrt[3]{c + dx} \cosh(a + b\sqrt[3]{c + dx})}{b^8 d^3} \\
 & + \frac{60480 (c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{b^7 d^3} \\
 & - \frac{20160 (c + dx) \cosh(a + b\sqrt[3]{c + dx})}{b^6 d^3} \\
 & + \frac{720c \cosh(a + b\sqrt[3]{c + dx})}{b^6 d^3} \\
 & + \frac{5040 (c + dx)^{4/3} \sinh(a + b\sqrt[3]{c + dx})}{b^5 d^3} \\
 & - \frac{720c \sqrt[3]{c + dx} \sinh(a + b\sqrt[3]{c + dx})}{b^5 d^3} \\
 & - \frac{1008 (c + dx)^{5/3} \cosh(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
 & + \frac{360c (c + dx)^{2/3} \cosh(a + b\sqrt[3]{c + dx})}{b^4 d^3} \\
 & + \frac{6c^2 \sinh(a + b\sqrt[3]{c + dx})}{b^3 d^3} \\
 & + \frac{168 (c + dx)^2 \sinh(a + b\sqrt[3]{c + dx})}{b^3 d^3} \\
 & - \frac{120c (c + dx) \sinh(a + b\sqrt[3]{c + dx})}{b^3 d^3} \\
 & - \frac{6c^2 \sqrt[3]{c + dx} \cosh(a + b\sqrt[3]{c + dx})}{b^2 d^3} \\
 & - \frac{24 (c + dx)^{7/3} \cosh(a + b\sqrt[3]{c + dx})}{b^2 d^3} \\
 & + \frac{30c (c + dx)^{4/3} \cosh(a + b\sqrt[3]{c + dx})}{b^2 d^3} \\
 & + \frac{3c^2 (c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd^3} \\
 & + \frac{3 (c + dx)^{8/3} \sinh(a + b\sqrt[3]{c + dx})}{bd^3} \\
 & - \frac{6c (c + dx)^{5/3} \sinh(a + b\sqrt[3]{c + dx})}{bd^3}
 \end{aligned}$$

[In] Int[x^2*Cosh[a + b*(c + d*x)^(1/3)],x]

[Out] (720*c*Cosh[a + b*(c + d*x)^(1/3)]/(b^6*d^3) - (120960*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)]/(b^8*d^3) - (6*c^2*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)]/(b^2*d^3) + (360*c*(c + d*x)^(2/3)*Cosh[a + b*(c + d*x)^(1/3)]/(b^4*d^3) - (20160*(c + d*x)*Cosh[a + b*(c + d*x)^(1/3)]/(b^6*d^3) + (30*c*(c + d*x)^(4/3)*Cosh[a + b*(c + d*x)^(1/3)]/(b^2*d^3) - (1008*(c + d*x)^(5/3)*Cosh[a + b*(c + d*x)^(1/3)]/(b^4*d^3) - (24*(c + d*x)^(7/3)*Cosh[a + b*(c + d*x)^(1/3)]/(b^2*d^3) + (120960*Sinh[a + b*(c + d*x)^(1/3)]/(b^9*d^3) + (6*c^2*Sinh[a + b*(c + d*x)^(1/3)]/(b^3*d^3) - (720*c*(c + d*x)^(1/3)*Sinh[a + b*(c + d*x)^(1/3)]/(b^5*d^3) + (60480*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)]/(b^7*d^3) + (3*c^2*(c + d*x)^(2/3)*Sinh[a + b*(c + d*x)^(1/3)]/(b*d^3) - (120*c*(c + d*x)*Sinh[a + b*(c + d*x)^(1/3)]/(b^3*d^3) + (5040*(c + d*x)^(4/3)*Sinh[a + b*(c + d*x)^(1/3)]/(b^5*d^3) - (6*c*(c + d*x)^(5/3)*Sinh[a + b*(c + d*x)^(1/3)]/(b*d^3) + (168*(c + d*x)^2*Sinh[a + b*(c + d*x)^(1/3)]/(b^3*d^3) + (3*(c + d*x)^(8/3)*Sinh[a + b*(c + d*x)^(1/3)]/(b*d^3))

Rule 1607

Int[(u_)*((a_)*(x_)^(p_) + (b_)*(x_)^(q_))^(n_), x_Symbol] := Int[u*x^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] && PosQ[q - p]

Rule 2717

Int[sin[Pi/2 + (c_.) + (d_)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 2718

Int[sin[(c_.) + (d_)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /; FreeQ[{c, d}, x]

Rule 3377

Int[((c_.) + (d_)*(x_))^(m_)*sin[(e_.) + (f_)*(x_)], x_Symbol] := Simp[(-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]

Rule 5395

Int[Cosh[(c_.) + (d_)*(x_)]*((e_)*(x_))^(m_)*((a_.) + (b_)*(x_)^(n_))^(p_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:= Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
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Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int (-c + x)^2 \cosh(a + b\sqrt[3]{x}) dx, x, c + dx\right)}{d^3} \\
&= \frac{3\text{Subst}\left(\int (-cx + x^4)^2 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&= \frac{3\text{Subst}\left(\int x^2(-c + x^3)^2 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&= \frac{3\text{Subst}\left(\int (c^2x^2 \cosh(a + bx) - 2cx^5 \cosh(a + bx) + x^8 \cosh(a + bx)) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&= \frac{3\text{Subst}\left(\int x^8 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&\quad - \frac{(6c)\text{Subst}\left(\int x^5 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&\quad + \frac{(3c^2)\text{Subst}\left(\int x^2 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^3} \\
&= \frac{3c^2(c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd^3} - \frac{6c(c + dx)^{5/3} \sinh(a + b\sqrt[3]{c + dx})}{bd^3} \\
&\quad + \frac{3(c + dx)^{8/3} \sinh(a + b\sqrt[3]{c + dx})}{bd^3} - \frac{24\text{Subst}\left(\int x^7 \sinh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{bd^3} \\
&\quad + \frac{(30c)\text{Subst}\left(\int x^4 \sinh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{bd^3} \\
&\quad - \frac{(6c^2)\text{Subst}\left(\int x \sinh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{bd^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad -\frac{24(c+dx)^{7/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{3c^2(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad -\frac{6c(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} + \frac{3(c+dx)^{8/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad +\frac{168\text{Subst}\left(\int x^6\cosh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad -\frac{(120c)\text{Subst}\left(\int x^3\cosh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad +\frac{(6c^2)\text{Subst}\left(\int\cosh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^2d^3} \\
&= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&\quad -\frac{24(c+dx)^{7/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{6c^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&\quad +\frac{3c^2(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad -\frac{120c(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad +\frac{168(c+dx)^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&\quad -\frac{1008\text{Subst}\left(\int x^5\sinh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3} \\
&\quad +\frac{(360c)\text{Subst}\left(\int x^2\sinh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^3d^3}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{1008(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&+ \frac{6c^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3c^2(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&- \frac{120c(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{6c(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{168(c+dx)^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{5040\text{Subst}\left(\int x^4\cosh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3} \\
&- \frac{(720c)\text{Subst}\left(\int x\cosh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^4d^3} \\
&= -\frac{6c^2\sqrt[3]{c+dx}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} + \frac{360c(c+dx)^{2/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} \\
&+ \frac{30c(c+dx)^{4/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&- \frac{1008(c+dx)^{5/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^3} - \frac{24(c+dx)^{7/3}\cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^3} \\
&+ \frac{6c^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} - \frac{720c\sqrt[3]{c+dx}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} \\
&+ \frac{3c^2(c+dx)^{2/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} - \frac{120c(c+dx)\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} \\
&+ \frac{5040(c+dx)^{4/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^3} - \frac{6c(c+dx)^{5/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&+ \frac{168(c+dx)^2\sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^3} + \frac{3(c+dx)^{8/3}\sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^3} \\
&- \frac{20160\text{Subst}\left(\int x^3\sinh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3} \\
&+ \frac{(720c)\text{Subst}\left(\int \sinh(ax)dx, x, \sqrt[3]{c+dx}\right)}{b^5d^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{720c \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} - \frac{6c^2 \sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&+ \frac{360c(c + dx)^{2/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} \\
&- \frac{20160(c + dx) \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} + \frac{30c(c + dx)^{4/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&- \frac{1008(c + dx)^{5/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} - \frac{24(c + dx)^{7/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&+ \frac{6c^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} - \frac{720c\sqrt[3]{c + dx} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} \\
&+ \frac{3c^2(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} - \frac{120c(c + dx) \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} \\
&+ \frac{5040(c + dx)^{4/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} - \frac{6c(c + dx)^{5/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&+ \frac{168(c + dx)^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} + \frac{3(c + dx)^{8/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&+ \frac{60480 \text{Subst}\left(\int x^2 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{b^6 d^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{720c \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} - \frac{6c^2 \sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&+ \frac{360c(c + dx)^{2/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} - \frac{20160(c + dx) \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} \\
&+ \frac{30c(c + dx)^{4/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} - \frac{1008(c + dx)^{5/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} \\
&- \frac{24(c + dx)^{7/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} + \frac{6c^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} \\
&- \frac{720c \sqrt[3]{c + dx} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} + \frac{60480(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^7 d^3} \\
&+ \frac{3c^2(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} - \frac{120c(c + dx) \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} \\
&+ \frac{5040(c + dx)^{4/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} - \frac{6c(c + dx)^{5/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&+ \frac{168(c + dx)^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} + \frac{3(c + dx)^{8/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&- \frac{120960 \text{Subst}\left(\int x \sinh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{b^7 d^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{720c \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} - \frac{120960\sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^8 d^3} \\
&- \frac{6c^2\sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} + \frac{360c(c + dx)^{2/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} \\
&- \frac{20160(c + dx) \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} + \frac{30c(c + dx)^{4/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&- \frac{1008(c + dx)^{5/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} - \frac{24(c + dx)^{7/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&+ \frac{6c^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} - \frac{720c\sqrt[3]{c + dx} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} \\
&+ \frac{60480(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^7 d^3} \\
&+ \frac{3c^2(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} - \frac{120c(c + dx) \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} \\
&+ \frac{5040(c + dx)^{4/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} - \frac{6c(c + dx)^{5/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&+ \frac{168(c + dx)^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} + \frac{3(c + dx)^{8/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&+ \frac{120960 \text{Subst}\left(\int \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{b^8 d^3}
\end{aligned}$$

$$\begin{aligned}
&= \frac{720c \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} - \frac{120960\sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^8 d^3} \\
&- \frac{6c^2\sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} + \frac{360c(c + dx)^{2/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} \\
&- \frac{20160(c + dx) \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^6 d^3} + \frac{30c(c + dx)^{4/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&- \frac{1008(c + dx)^{5/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^4 d^3} - \frac{24(c + dx)^{7/3} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2 d^3} \\
&+ \frac{120960 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^9 d^3} + \frac{6c^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} \\
&- \frac{720c\sqrt[3]{c + dx} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} + \frac{60480(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^7 d^3} \\
&+ \frac{3c^2(c + dx)^{2/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} - \frac{120c(c + dx) \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} \\
&+ \frac{5040(c + dx)^{4/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^5 d^3} - \frac{6c(c + dx)^{5/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3} \\
&+ \frac{168(c + dx)^2 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3 d^3} + \frac{3(c + dx)^{8/3} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd^3}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.66

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

$$\frac{e^{-a-b\sqrt[3]{c+dx}} \left(120960 \left(-1 + e^{2(a+b\sqrt[3]{c+dx})} \right) \right) - 120960b \left(1 + e^{2(a+b\sqrt[3]{c+dx})} \right) \sqrt[3]{c+dx} + 60480b^2 \left(-1 + e^{2(a+b\sqrt[3]{c+dx})} \right)}{b^9 d^3}$$

[In] Integrate[x^2*Cosh[a + b*(c + d*x)^(1/3)],x]

[Out] (E^(-a - b*(c + d*x)^(1/3))*(120960*(-1 + E^(2*(a + b*(c + d*x)^(1/3)))) - 120960*b*(1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3) + 60480*b^2*(-1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3) + 3*b^8*d^2*(-1 + E^(2*(a + b*(c + d*x)^(1/3))))*x^2*(c + d*x)^(2/3) - 6*b^7*d*(1 + E^(2*(a + b*(c + d*x)^(1/3))))*x*(c + d*x)^(1/3)*(3*c + 4*d*x) + 720*b^4*(-1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(1/3)*(6*c + 7*d*x) - 72*b^5*(1 + E^(2*(a + b*(c + d*x)^(1/3))))*(c + d*x)^(2/3)*(9*c + 14*d*x) - 720*b^3*(1 + E^(2*(a + b*(c + d*x)^(1/3))))*(27*c + 28*d*x) + 6*b^6*(-1 + E^(2*(a + b*(c + d*x)^(1/3))))*(9*c^2 + 36*c*d*x + 28*d^2*x^2))/(2*b^9*d^3)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1814 vs. $2(477) = 954$.

Time = 0.21 (sec) , antiderivative size = 1815, normalized size of antiderivative = 3.38

method	result	size
derivativedivides	Expression too large to display	1815
default	Expression too large to display	1815
parts	Expression too large to display	2938

[In] `int(x^2*cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)`

[Out]
$$\begin{aligned} & 3/d^3/b^3*(1/b^6*((a+b*(d*x+c)^{(1/3)})^8*\sinh(a+b*(d*x+c)^{(1/3)})-8*(a+b*(d*x+c)^{(1/3)})^7*\cosh(a+b*(d*x+c)^{(1/3)})+56*(a+b*(d*x+c)^{(1/3)})^6*\sinh(a+b*(d*x+c)^{(1/3)})-336*(a+b*(d*x+c)^{(1/3)})^5*\cosh(a+b*(d*x+c)^{(1/3)})+1680*(a+b*(d*x+c)^{(1/3)})^4*\sinh(a+b*(d*x+c)^{(1/3)})-6720*(a+b*(d*x+c)^{(1/3)})^3*\cosh(a+b*(d*x+c)^{(1/3)})+20160*(a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-40320*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+40320*\sinh(a+b*(d*x+c)^{(1/3)}))+1/b^6*a^8*\sinh(a+b*(d*x+c)^{(1/3)})+c^2*a^2*\sinh(a+b*(d*x+c)^{(1/3)})+c^2*((a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-2*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+2*\sinh(a+b*(d*x+c)^{(1/3)}))-56/b^6*a^5*((a+b*(d*x+c)^{(1/3)})^3*\sinh(a+b*(d*x+c)^{(1/3)})-3*(a+b*(d*x+c)^{(1/3)})^2*\cosh(a+b*(d*x+c)^{(1/3)})+6*(a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-6*\cosh(a+b*(d*x+c)^{(1/3)}))+2/b^3*a^5*c*\sinh(a+b*(d*x+c)^{(1/3)})+70/b^6*a^4*((a+b*(d*x+c)^{(1/3)})^4*\sinh(a+b*(d*x+c)^{(1/3)})-4*(a+b*(d*x+c)^{(1/3)})^3*\cosh(a+b*(d*x+c)^{(1/3)})+12*(a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-24*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+24*\sinh(a+b*(d*x+c)^{(1/3)}))-56/b^6*a^3*((a+b*(d*x+c)^{(1/3)})^5*\sinh(a+b*(d*x+c)^{(1/3)})-5*(a+b*(d*x+c)^{(1/3)})^4*\cosh(a+b*(d*x+c)^{(1/3)})+20*(a+b*(d*x+c)^{(1/3)})^3*\sinh(a+b*(d*x+c)^{(1/3)})-60*(a+b*(d*x+c)^{(1/3)})^2*\cosh(a+b*(d*x+c)^{(1/3)})+120*(a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-120*\cosh(a+b*(d*x+c)^{(1/3)}))-2/b^3*c*((a+b*(d*x+c)^{(1/3)})^5*\sinh(a+b*(d*x+c)^{(1/3)})-5*(a+b*(d*x+c)^{(1/3)})^4*\cosh(a+b*(d*x+c)^{(1/3)})+20*(a+b*(d*x+c)^{(1/3)})^3*\sinh(a+b*(d*x+c)^{(1/3)})-60*(a+b*(d*x+c)^{(1/3)})^2*\cosh(a+b*(d*x+c)^{(1/3)})+120*(a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-120*\cosh(a+b*(d*x+c)^{(1/3)}))+28/b^6*a^2*((a+b*(d*x+c)^{(1/3)})^6*\sinh(a+b*(d*x+c)^{(1/3)})-6*(a+b*(d*x+c)^{(1/3)})^5*\cosh(a+b*(d*x+c)^{(1/3)})+30*(a+b*(d*x+c)^{(1/3)})^4*\sinh(a+b*(d*x+c)^{(1/3)})-120*(a+b*(d*x+c)^{(1/3)})^3*\cosh(a+b*(d*x+c)^{(1/3)})+360*(a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-720*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+720*\sinh(a+b*(d*x+c)^{(1/3)}))-8/b^6*a*((a+b*(d*x+c)^{(1/3)})^7*\sinh(a+b*(d*x+c)^{(1/3)})-7*(a+b*(d*x+c)^{(1/3)})^6*\cosh(a+b*(d*x+c)^{(1/3)})+42*(a+b*(d*x+c)^{(1/3)})^5*\sinh(a+b*(d*x+c)^{(1/3)})-210*(a+b*(d*x+c)^{(1/3)})^4*\cosh(a+b*(d*x+c)^{(1/3)})+840*(a+b*(d*x+c)^{(1/3)})^3*\sinh(a+b*(d*x+c)^{(1/3)})-2520*(a+b*(d*x+c)^{(1/3)})^2*\cosh(a+b*(d*x+c)^{(1/3)})+5040*(a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-5040*\cosh(a+b*(d*x+c)^{(1/3)}))-8/b^6*a^7*((a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-\cosh(a+b*(d*x+c)^{(1/3)}))+28/b^6*a^6*((a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-2*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+2*\sinh(a+b*(d*x+c)^{(1/3)})) \end{aligned}$$

$$\begin{aligned}
& b*(d*x+c)^{(1/3)}-2*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+2*\sinh(a+b*(d*x+c)^{(1/3)}) \\
& -2*c^2*a*((a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-\cosh(a+b*(d*x+c)^{(1/3)})) \\
& -20/b^3*c*a^2*((a+b*(d*x+c)^{(1/3)})^3*\sinh(a+b*(d*x+c)^{(1/3)}))-3*(a+b*(d*x+c)^{(1/3)})^2*\cosh(a+b*(d*x+c)^{(1/3)})+6*(a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)}) \\
& -6*\cosh(a+b*(d*x+c)^{(1/3)}))+10/b^3*c*a*((a+b*(d*x+c)^{(1/3)})^4*\sinh(a+b*(d*x+c)^{(1/3)})-4*(a+b*(d*x+c)^{(1/3)})^3*\cosh(a+b*(d*x+c)^{(1/3)}))+12*(a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-24*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+24*\sinh(a+b*(d*x+c)^{(1/3)}))-10/b^3*a^4*c*((a+b*(d*x+c)^{(1/3)})*\sinh(a+b*(d*x+c)^{(1/3)})-\cosh(a+b*(d*x+c)^{(1/3)}))+20/b^3*a^3*c*((a+b*(d*x+c)^{(1/3)})^2*\sinh(a+b*(d*x+c)^{(1/3)})-2*(a+b*(d*x+c)^{(1/3)})*\cosh(a+b*(d*x+c)^{(1/3)})+2*\sinh(a+b*(d*x+c)^{(1/3)}))
\end{aligned}$$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.34

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \frac{3\left(2\left(3360b^3dx + 3240b^3c + 12(14b^5dx + 9b^5c)(dx + c)^{\frac{2}{3}} + (4b^7d^2x^2 + 3b^7cdx + 20160b)(dx + c)^{\frac{1}{3}}\right)\right)}{b^9d^3}$$

[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")

[Out] -3*(2*(3360*b^3*d*x + 3240*b^3*c + 12*(14*b^5*d*x + 9*b^5*c)*(d*x + c)^(2/3) + (4*b^7*d^2*x^2 + 3*b^7*c*d*x + 20160*b)*(d*x + c)^(1/3))*cosh((d*x + c)^(1/3)*b + a) - (56*b^6*d^2*x^2 + 72*b^6*c*d*x + 18*b^6*c^2 + (b^8*d^2*x^2 + 20160*b^2)*(d*x + c)^(2/3) + 240*(7*b^4*d*x + 6*b^4*c)*(d*x + c)^(1/3) + 40320)*sinh((d*x + c)^(1/3)*b + a))/(b^9*d^3)

Sympy [F]

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

[In] integrate(x**2*cosh(a+b*(d*x+c)**(1/3)),x)

[Out] Integral(x**2*cosh(a + b*(c + d*x)**(1/3)), x)

Maxima [A] (verification not implemented)

none

Time = 0.21 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.20

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

$$2d^3x^3 \cosh\left((dx + c)^{\frac{1}{3}}b + a\right) + \left(\frac{c^3e^{\left((dx+c)^{\frac{1}{3}}b+a\right)}}{b} + \frac{c^3e^{\left(-\left(dx+c\right)^{\frac{1}{3}}b-a\right)}}{b} - \frac{3\left((dx+c)b^3e^a - 3(dx+c)^{\frac{2}{3}}b^2e^a + 6(dx+c)^{\frac{1}{3}}be^a - 6e^a\right)}{b^4}\right)$$

[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")

```
[Out] 1/6*(2*d^3*x^3*cosh((d*x + c)^(1/3)*b + a) + (c^3*e^((d*x + c)^(1/3)*b + a)
/b + c^3*e^(-(d*x + c)^(1/3)*b - a)/b - 3*((d*x + c)*b^3*e^a - 3*(d*x + c)^(
2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*c^2*e^((d*x + c)^(1/3)*b)/
b^4 - 3*((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*c
^2*e^(-(d*x + c)^(1/3)*b - a)/b^4 + 3*((d*x + c)^2*b^6*e^a - 6*(d*x + c)^(5
/3)*b^5*e^a + 30*(d*x + c)^(4/3)*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x
+ c)^(2/3)*b^2*e^a - 720*(d*x + c)^(1/3)*b*e^a + 720*e^a)*c*e^((d*x + c)^(
1/3)*b)/b^7 + 3*((d*x + c)^2*b^6 + 6*(d*x + c)^(5/3)*b^5 + 30*(d*x + c)^(4/
3)*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^(2/3)*b^2 + 720*(d*x + c)^(1/3)*
b + 720)*c*e^(-(d*x + c)^(1/3)*b - a)/b^7 - ((d*x + c)^3*b^9*e^a - 9*(d*x +
c)^(8/3)*b^8*e^a + 72*(d*x + c)^(7/3)*b^7*e^a - 504*(d*x + c)^2*b^6*e^a +
3024*(d*x + c)^(5/3)*b^5*e^a - 15120*(d*x + c)^(4/3)*b^4*e^a + 60480*(d*x +
c)*b^3*e^a - 181440*(d*x + c)^(2/3)*b^2*e^a + 362880*(d*x + c)^(1/3)*b*e^a
- 362880*e^a)*e^((d*x + c)^(1/3)*b)/b^10 - ((d*x + c)^3*b^9 + 9*(d*x + c)^(
8/3)*b^8 + 72*(d*x + c)^(7/3)*b^7 + 504*(d*x + c)^2*b^6 + 3024*(d*x + c)^(
5/3)*b^5 + 15120*(d*x + c)^(4/3)*b^4 + 60480*(d*x + c)*b^3 + 181440*(d*x +
c)^(2/3)*b^2 + 362880*(d*x + c)^(1/3)*b + 362880)*e^(-(d*x + c)^(1/3)*b - a)
/b^10)*b)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2163 vs. 2(477) = 954.

Time = 0.36 (sec) , antiderivative size = 2163, normalized size of antiderivative = 4.03

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \text{Too large to display}$$

[In] integrate(x^2*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")

```
[Out] 3/2*(((d*x + c)^(1/3)*b + a)^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)*a*b^6*c
^2 + a^2*b^6*c^2 - 2*((d*x + c)^(1/3)*b + a)^5*b^3*c + 10*((d*x + c)^(1/3)*
```


$$\begin{aligned}
& b + a)^4 a^3 b^3 c - 20((d*x + c)^{(1/3)} * b + a)^3 a^2 b^3 c + 20((d*x + c)^{(1/3)} * b + a)^2 a^3 b^3 c - 10((d*x + c)^{(1/3)} * b + a) a^4 b^3 c + 2 a^5 b^3 c \\
& c - 2((d*x + c)^{(1/3)} * b + a) b^6 c^2 + 2 a b^6 c^2 + ((d*x + c)^{(1/3)} * b + a)^8 - 8((d*x + c)^{(1/3)} * b + a)^7 a + 28((d*x + c)^{(1/3)} * b + a)^6 a^2 - 5 \\
& 6((d*x + c)^{(1/3)} * b + a)^5 a^3 + 70((d*x + c)^{(1/3)} * b + a)^4 a^4 - 56((d*x + c)^{(1/3)} * b + a)^3 a^5 + 28((d*x + c)^{(1/3)} * b + a)^2 a^6 - 8((d*x + c)^{(1/3)} * b + a) \\
& a^7 + a^8 + 10((d*x + c)^{(1/3)} * b + a)^4 b^3 c - 40((d*x + c)^{(1/3)} * b + a)^3 a^2 b^3 c + 60((d*x + c)^{(1/3)} * b + a)^2 a^2 b^3 c - 40((d*x + c)^{(1/3)} * b + a) a^3 b^3 c \\
& + 10 a^4 b^3 c + 2 b^6 c^2 - 8((d*x + c)^{(1/3)} * b + a)^7 + 56((d*x + c)^{(1/3)} * b + a)^6 a - 168((d*x + c)^{(1/3)} * b + a)^5 a^2 + 280((d*x + c)^{(1/3)} * b + a)^4 a^3 \\
& - 280((d*x + c)^{(1/3)} * b + a)^3 a^4 + 168((d*x + c)^{(1/3)} * b + a)^2 a^5 - 56((d*x + c)^{(1/3)} * b + a) a^6 + 8 a^7 - 40((d*x + c)^{(1/3)} * b + a)^3 b^3 c + 120((d*x + c)^{(1/3)} * b + a)^2 \\
& a^2 b^3 c - 120((d*x + c)^{(1/3)} * b + a) a^2 b^3 c + 40 a^3 b^3 c + 56((d*x + c)^{(1/3)} * b + a)^6 - 336((d*x + c)^{(1/3)} * b + a)^5 a + 840((d*x + c)^{(1/3)} * b + a)^4 a^2 \\
& - 1120((d*x + c)^{(1/3)} * b + a)^3 a^3 + 840((d*x + c)^{(1/3)} * b + a)^2 a^4 - 336((d*x + c)^{(1/3)} * b + a) a^5 + 56 a^6 + 120((d*x + c)^{(1/3)} * b + a)^2 b^3 c \\
& - 240((d*x + c)^{(1/3)} * b + a) a^2 b^3 c + 120 a^2 b^3 c - 336((d*x + c)^{(1/3)} * b + a)^5 + 1680((d*x + c)^{(1/3)} * b + a)^4 a - 3360((d*x + c)^{(1/3)} * b + a)^3 a^2 \\
& + 3360((d*x + c)^{(1/3)} * b + a)^2 a^3 - 1680((d*x + c)^{(1/3)} * b + a) a^4 + 336 a^5 - 240((d*x + c)^{(1/3)} * b + a) b^3 c + 240 a^2 b^3 c + 1680((d*x + c)^{(1/3)} * b + a)^4 \\
& - 6720((d*x + c)^{(1/3)} * b + a)^3 a + 10080((d*x + c)^{(1/3)} * b + a)^2 a^2 - 6720((d*x + c)^{(1/3)} * b + a) a^3 + 1680 a^4 + 240 b^3 c - 6720((d*x + c)^{(1/3)} * b + a)^3 \\
& + 20160((d*x + c)^{(1/3)} * b + a)^2 a - 20160((d*x + c)^{(1/3)} * b + a) a^2 + 6720 a^3 + 20160((d*x + c)^{(1/3)} * b + a)^2 - 40320((d*x + c)^{(1/3)} * b + a) a + 20160 a^2 - 40320 \\
& * (d*x + c)^{(1/3)} * b + 40320 * e^{((d*x + c)^{(1/3)} * b + a) / (b^8 d^2)} - ((d*x + c)^{(1/3)} * b + a)^2 b^6 c^2 - 2((d*x + c)^{(1/3)} * b + a) a^2 b^6 c^2 + a^2 b^6 c^2 \\
& ^2 - 2((d*x + c)^{(1/3)} * b + a)^5 b^3 c + 10((d*x + c)^{(1/3)} * b + a)^4 a^2 b^3 c - 20((d*x + c)^{(1/3)} * b + a)^3 a^2 b^3 c + 20((d*x + c)^{(1/3)} * b + a)^2 a^3 b^3 c \\
& - 10((d*x + c)^{(1/3)} * b + a) a^4 b^3 c + 2 a^5 b^3 c + 2((d*x + c)^{(1/3)} * b + a) b^6 c^2 - 2 a b^6 c^2 + ((d*x + c)^{(1/3)} * b + a)^8 - 8((d*x + c)^{(1/3)} * b + a)^7 a \\
& + 28((d*x + c)^{(1/3)} * b + a)^6 a^2 - 56((d*x + c)^{(1/3)} * b + a)^5 a^3 + 70((d*x + c)^{(1/3)} * b + a)^4 a^4 - 56((d*x + c)^{(1/3)} * b + a)^3 a^5 + 28((d*x + c)^{(1/3)} * b + a)^2 a^6 \\
& - 8((d*x + c)^{(1/3)} * b + a) a^7 + a^8 - 10((d*x + c)^{(1/3)} * b + a)^4 b^3 c + 40((d*x + c)^{(1/3)} * b + a)^3 a^2 b^3 c - 60((d*x + c)^{(1/3)} * b + a)^2 a^2 b^3 c + 40((d*x + c)^{(1/3)} * b + a) \\
& a^3 b^3 c - 10 a^4 b^3 c + 2 b^6 c^2 + 8((d*x + c)^{(1/3)} * b + a)^7 - 56((d*x + c)^{(1/3)} * b + a)^6 a + 168((d*x + c)^{(1/3)} * b + a)^5 a^2 - 280((d*x + c)^{(1/3)} * b + a)^4 a^3 \\
& + 280((d*x + c)^{(1/3)} * b + a)^3 a^4 - 168((d*x + c)^{(1/3)} * b + a)^2 a^5 + 56((d*x + c)^{(1/3)} * b + a) a^6 - 8 a^7 - 40((d*x + c)^{(1/3)} * b + a)^3 b^3 c + 120((d*x + c)^{(1/3)} * b + a)^2 \\
& a^2 b^3 c - 120((d*x + c)^{(1/3)} * b + a) a^2 b^3 c + 40 a^3 b^3 c + 56((d*x + c)^{(1/3)} * b + a)^6 - 336((d*x + c)^{(1/3)} * b + a)^5 a + 840((d*x + c)^{(1/3)} * b + a)^4 a^2 \\
& - 1120((d*x + c)^{(1/3)} * b + a)^3 a^3 + 840((d*x + c)^{(1/3)} * b + a)^2 a^4 -
\end{aligned}$$

$$\begin{aligned}
& 336*((d*x + c)^{(1/3)}*b + a)*a^5 + 56*a^6 - 120*((d*x + c)^{(1/3)}*b + a)^2*b^3*c \\
& + 240*((d*x + c)^{(1/3)}*b + a)*a*b^3*c - 120*a^2*b^3*c + 336*((d*x + c)^{(1/3)}*b + a)^5 \\
& - 1680*((d*x + c)^{(1/3)}*b + a)^4*a + 3360*((d*x + c)^{(1/3)}*b + a)^3*a^2 - 3360*((d*x + c)^{(1/3)}*b + a)^2*a^3 \\
& + 1680*((d*x + c)^{(1/3)}*b + a)*a^4 - 336*a^5 - 240*((d*x + c)^{(1/3)}*b + a)*b^3*c + 240*a*b^3*c + 1680 \\
& *((d*x + c)^{(1/3)}*b + a)^4 - 6720*((d*x + c)^{(1/3)}*b + a)^3*a + 10080*((d*x + c)^{(1/3)}*b + a)^2*a^2 \\
& - 6720*((d*x + c)^{(1/3)}*b + a)*a^3 + 1680*a^4 - 240*b^3*c + 6720*((d*x + c)^{(1/3)}*b + a)^3 \\
& - 20160*((d*x + c)^{(1/3)}*b + a)^2*a + 20160*((d*x + c)^{(1/3)}*b + a)*a^2 - 6720*a^3 + 20160*((d*x + c)^{(1/3)}*b + a)^2 \\
& - 40320*((d*x + c)^{(1/3)}*b + a)*a + 20160*a^2 + 40320*(d*x + c)^{(1/3)}*b + 40320*e^{-(d*x + c)^{(1/3)}*b - a}/(b^8*d^2))/(b*d)
\end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x^2 \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \int x^2 \cosh\left(a + b(c + dx)^{1/3}\right) dx$$

[In] int(x^2*cosh(a + b*(c + d*x)^(1/3)),x)

[Out] int(x^2*cosh(a + b*(c + d*x)^(1/3)), x)

3.65 $\int x \cosh \left(a + b\sqrt[3]{c + dx} \right) dx$

Optimal result	339
Rubi [A] (verified)	340
Mathematica [A] (verified)	343
Maple [B] (verified)	344
Fricas [A] (verification not implemented)	344
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Maxima [A] (verification not implemented)	345
Giac [B] (verification not implemented)	346
Mupad [F(-1)]	346

Optimal result

Integrand size = 16, antiderivative size = 261

$$\begin{aligned}
 \int x \cosh \left(a + b\sqrt[3]{c + dx} \right) dx = & -\frac{360 \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^6 d^2} \\
 & + \frac{6c\sqrt[3]{c + dx} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} \\
 & - \frac{180(c + dx)^{2/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^4 d^2} \\
 & - \frac{15(c + dx)^{4/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} \\
 & - \frac{6c \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^2} \\
 & + \frac{360\sqrt[3]{c + dx} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^5 d^2} \\
 & - \frac{3c(c + dx)^{2/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{bd^2} \\
 & + \frac{60(c + dx) \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^2} \\
 & + \frac{3(c + dx)^{5/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{bd^2}
 \end{aligned}$$

[Out] $-360*\cosh(a+b*(d*x+c)^(1/3))/b^6/d^2+6*c*(d*x+c)^(1/3)*\cosh(a+b*(d*x+c)^(1/3))/b^2/d^2-180*(d*x+c)^(2/3)*\cosh(a+b*(d*x+c)^(1/3))/b^4/d^2-15*(d*x+c)^(4/3)*\cosh(a+b*(d*x+c)^(1/3))/b^2/d^2-6*c*\sinh(a+b*(d*x+c)^(1/3))/b^3/d^2+360$

$*(d*x+c)^{(1/3)}*\sinh(a+b*(d*x+c)^{(1/3)})/b^5/d^2-3*c*(d*x+c)^{(2/3)}*\sinh(a+b*(d*x+c)^{(1/3)})/b/d^2+60*(d*x+c)*\sinh(a+b*(d*x+c)^{(1/3)})/b^3/d^2+3*(d*x+c)^{(5/3)}*\sinh(a+b*(d*x+c)^{(1/3)})/b/d^2$

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5473, 5395, 3377, 2717, 2718}

$$\int x \cosh \left(a + b\sqrt[3]{c + dx} \right) dx = -\frac{360 \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^6 d^2} + \frac{360 \sqrt[3]{c + dx} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^5 d^2} - \frac{180(c + dx)^{2/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^4 d^2} + \frac{60(c + dx) \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^2} - \frac{6c \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b^3 d^2} - \frac{15(c + dx)^{4/3} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} + \frac{6c \sqrt[3]{c + dx} \cosh \left(a + b\sqrt[3]{c + dx} \right)}{b^2 d^2} + \frac{3(c + dx)^{5/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b d^2} - \frac{3c(c + dx)^{2/3} \sinh \left(a + b\sqrt[3]{c + dx} \right)}{b d^2}$$

[In] Int[x*Cosh[a + b*(c + d*x)^(1/3)],x]

[Out] $(-360*\text{Cosh}[a + b*(c + d*x)^{(1/3)}])/(b^6*d^2) + (6*c*(c + d*x)^{(1/3)}*\text{Cosh}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^2) - (180*(c + d*x)^{(2/3)}*\text{Cosh}[a + b*(c + d*x)^{(1/3)}])/(b^4*d^2) - (15*(c + d*x)^{(4/3)}*\text{Cosh}[a + b*(c + d*x)^{(1/3)}])/(b^2*d^2) - (6*c*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^2) + (360*(c + d*x)^{(1/3)}*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b^5*d^2) - (3*c*(c + d*x)^{(2/3)}*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b*d^2) + (60*(c + d*x)*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b^3*d^2) + (3*(c + d*x)^{(5/3)}*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b*d^2)$

Rule 2717

`Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 2718

`Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] := Simp[-Cos[c + d*x]/d, x] /;`
`FreeQ[{c, d}, x]`

Rule 3377

`Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(-`
`(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co`
`s[e + f*x], x], x] /;`
`FreeQ[{c, d, e, f}, x] && GtQ[m, 0]`

Rule 5395

`Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^(n_))^(p`
`_.), x_Symbol] := Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,`
`x], x] /;`
`FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]`

Rule 5473

`Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbo`
`l] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,`
`0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /;`
`FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]`

Rubi steps

$$\begin{aligned}
 \text{integral} &= \frac{\text{Subst}\left(\int (-c + x) \cosh(a + b\sqrt[3]{x}) dx, x, c + dx\right)}{d^2} \\
 &= \frac{3\text{Subst}\left(\int x^2(-c + x^3) \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^2} \\
 &= \frac{3\text{Subst}\left(\int (-cx^2 \cosh(a + bx) + x^5 \cosh(a + bx)) dx, x, \sqrt[3]{c + dx}\right)}{d^2} \\
 &= \frac{3\text{Subst}\left(\int x^5 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^2} - \frac{(3c)\text{Subst}\left(\int x^2 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d^2}
 \end{aligned}$$

$$\begin{aligned}
&= -\frac{3c(c+dx)^{2/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} + \frac{3(c+dx)^{5/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} \\
&\quad - \frac{15 \text{Subst}\left(\int x^4 \sinh(a+bx) dx, x, \sqrt[3]{c+dx}\right)}{bd^2} \\
&\quad + \frac{(6c) \text{Subst}\left(\int x \sinh(a+bx) dx, x, \sqrt[3]{c+dx}\right)}{bd^2} \\
&= \frac{6c\sqrt[3]{c+dx} \cosh(a+b\sqrt[3]{c+dx})}{b^2d^2} - \frac{15(c+dx)^{4/3} \cosh(a+b\sqrt[3]{c+dx})}{b^2d^2} \\
&\quad - \frac{3c(c+dx)^{2/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} + \frac{3(c+dx)^{5/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} \\
&\quad + \frac{60 \text{Subst}\left(\int x^3 \cosh(a+bx) dx, x, \sqrt[3]{c+dx}\right)}{b^2d^2} \\
&\quad - \frac{(6c) \text{Subst}\left(\int \cosh(a+bx) dx, x, \sqrt[3]{c+dx}\right)}{b^2d^2} \\
&= \frac{6c\sqrt[3]{c+dx} \cosh(a+b\sqrt[3]{c+dx})}{b^2d^2} - \frac{15(c+dx)^{4/3} \cosh(a+b\sqrt[3]{c+dx})}{b^2d^2} \\
&\quad - \frac{6c \sinh(a+b\sqrt[3]{c+dx})}{b^3d^2} - \frac{3c(c+dx)^{2/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} \\
&\quad + \frac{60(c+dx) \sinh(a+b\sqrt[3]{c+dx})}{b^3d^2} + \frac{3(c+dx)^{5/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} \\
&\quad - \frac{180 \text{Subst}\left(\int x^2 \sinh(a+bx) dx, x, \sqrt[3]{c+dx}\right)}{b^3d^2} \\
&= \frac{6c\sqrt[3]{c+dx} \cosh(a+b\sqrt[3]{c+dx})}{b^2d^2} - \frac{180(c+dx)^{2/3} \cosh(a+b\sqrt[3]{c+dx})}{b^4d^2} \\
&\quad - \frac{15(c+dx)^{4/3} \cosh(a+b\sqrt[3]{c+dx})}{b^2d^2} - \frac{6c \sinh(a+b\sqrt[3]{c+dx})}{b^3d^2} \\
&\quad - \frac{3c(c+dx)^{2/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} + \frac{60(c+dx) \sinh(a+b\sqrt[3]{c+dx})}{b^3d^2} \\
&\quad + \frac{3(c+dx)^{5/3} \sinh(a+b\sqrt[3]{c+dx})}{bd^2} + \frac{360 \text{Subst}\left(\int x \cosh(a+bx) dx, x, \sqrt[3]{c+dx}\right)}{b^4d^2}
\end{aligned}$$

$$\begin{aligned}
&= \frac{6c\sqrt[3]{c+dx} \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{180(c+dx)^{2/3} \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\
&\quad - \frac{15(c+dx)^{4/3} \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{6c \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\
&\quad + \frac{360\sqrt[3]{c+dx} \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3} \sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\
&\quad + \frac{60(c+dx) \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3} \sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\
&\quad - \frac{360 \operatorname{Subst}\left(\int \sinh(ax) dx, x, \sqrt[3]{c+dx}\right)}{b^5d^2} \\
&= -\frac{360 \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2} + \frac{6c\sqrt[3]{c+dx} \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} \\
&\quad - \frac{180(c+dx)^{2/3} \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^4d^2} \\
&\quad - \frac{15(c+dx)^{4/3} \cosh\left(a+b\sqrt[3]{c+dx}\right)}{b^2d^2} - \frac{6c \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} \\
&\quad + \frac{360\sqrt[3]{c+dx} \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^5d^2} - \frac{3c(c+dx)^{2/3} \sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2} \\
&\quad + \frac{60(c+dx) \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^3d^2} + \frac{3(c+dx)^{5/3} \sinh\left(a+b\sqrt[3]{c+dx}\right)}{bd^2}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.45

$$\begin{aligned}
&\int x \cosh\left(a+b\sqrt[3]{c+dx}\right) dx \\
&= \frac{-3\left(120+60b^2(c+dx)^{2/3}+b^4\sqrt[3]{c+dx}(3c+5dx)\right) \cosh\left(a+b\sqrt[3]{c+dx}\right)+3b\left(120\sqrt[3]{c+dx}+b^4dx(c+dx)\right) \sinh\left(a+b\sqrt[3]{c+dx}\right)}{b^6d^2}
\end{aligned}$$

[In] Integrate[x*Cosh[a + b*(c + d*x)^(1/3)],x]

[Out] (-3*(120 + 60*b^2*(c + d*x)^(2/3) + b^4*(c + d*x)^(1/3)*(3*c + 5*d*x))*Cosh[a + b*(c + d*x)^(1/3)] + 3*b*(120*(c + d*x)^(1/3) + b^4*d*x*(c + d*x)^(2/3) + 2*b^2*(9*c + 10*d*x))*Sinh[a + b*(c + d*x)^(1/3)]/(b^6*d^2)

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 658 vs. $2(231) = 462$.

Time = 0.19 (sec) , antiderivative size = 659, normalized size of antiderivative = 2.52

method	result
derivativedivides	$-\frac{3 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) a^5}{b^3} + \frac{15 a^4 \left(\left(a+b(dx+c)^{\frac{1}{3}}\right) \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - \cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3} - \frac{30 a^3 \left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - \cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3}$
default	$-\frac{3 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) a^5}{b^3} + \frac{15 a^4 \left(\left(a+b(dx+c)^{\frac{1}{3}}\right) \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - \cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3} - \frac{30 a^3 \left(\left(a+b(dx+c)^{\frac{1}{3}}\right)^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - \cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right)}{b^3}$
parts	Expression too large to display

[In] `int(x*cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)`

[Out]
$$\frac{3}{d^2 b^3} \left(-\sinh(a+b(dx+c)^{1/3}) a^5/b^3 + 5/b^3 a^4 \left((a+b(dx+c)^{1/3}) \sinh(a+b(dx+c)^{1/3}) - \cosh(a+b(dx+c)^{1/3}) \right) - 10/b^3 a^3 \left((a+b(dx+c)^{1/3})^2 \sinh(a+b(dx+c)^{1/3}) - \cosh(a+b(dx+c)^{1/3}) \right) + 2 \sinh(a+b(dx+c)^{1/3}) \right) + 10/b^3 a^2 \left((a+b(dx+c)^{1/3})^3 \sinh(a+b(dx+c)^{1/3}) - 3 \cosh(a+b(dx+c)^{1/3}) \right) + 6 \sinh(a+b(dx+c)^{1/3}) - 6 \cosh(a+b(dx+c)^{1/3}) - 5/b^3 a \left((a+b(dx+c)^{1/3})^4 \sinh(a+b(dx+c)^{1/3}) - 4 \cosh(a+b(dx+c)^{1/3}) \right) + 12 \sinh(a+b(dx+c)^{1/3}) - 24 \cosh(a+b(dx+c)^{1/3}) + 24 \sinh(a+b(dx+c)^{1/3}) + 1/b^3 \left((a+b(dx+c)^{1/3})^5 \sinh(a+b(dx+c)^{1/3}) - 5 \cosh(a+b(dx+c)^{1/3}) \right) + 20 \sinh(a+b(dx+c)^{1/3}) - 60 \cosh(a+b(dx+c)^{1/3}) + 120 \sinh(a+b(dx+c)^{1/3}) - 120 \cosh(a+b(dx+c)^{1/3}) - c a^2 \sinh(a+b(dx+c)^{1/3}) + 2 c a \left((a+b(dx+c)^{1/3}) \sinh(a+b(dx+c)^{1/3}) - \cosh(a+b(dx+c)^{1/3}) \right) - c \left((a+b(dx+c)^{1/3})^2 \sinh(a+b(dx+c)^{1/3}) - 2 \cosh(a+b(dx+c)^{1/3}) \right) + 2 \sinh(a+b(dx+c)^{1/3}) \right)$$

Fricas [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.42

$$\int x \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \frac{3 \left(\left(60(dx+c)^{\frac{2}{3}} b^2 + (5b^4 dx + 3b^4 c)(dx+c)^{\frac{1}{3}} + 120 \right) \cosh\left(\frac{1}{3} b \sqrt[3]{c + dx} + a\right) - \left((dx+c)^{\frac{2}{3}} b^5 dx + 20b^3 \right) \right)}{b^6 d^2}$$

[In] `integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")`

[Out] $-3*((60*(d*x + c)^{(2/3)}*b^2 + (5*b^4*d*x + 3*b^4*c)*(d*x + c)^{(1/3)} + 120)*\cosh((d*x + c)^{(1/3)}*b + a) - ((d*x + c)^{(2/3)}*b^5*d*x + 20*b^3*d*x + 18*b^3*c + 120*(d*x + c)^{(1/3)}*b)*\sinh((d*x + c)^{(1/3)}*b + a))/(b^6*d^2)$

Sympy [F]

$$\int x \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \int x \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

[In] `integrate(x*cosh(a+b*(d*x+c)**(1/3)),x)`

[Out] `Integral(x*cosh(a + b*(c + d*x)**(1/3)), x)`

Maxima [A] (verification not implemented)

none

Time = 0.19 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.41

$$\int x \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

$$2 d^2 x^2 \cosh\left((dx + c)^{\frac{1}{3}} b + a\right) - \left(\frac{c^2 e^{\left(\frac{dx+c}{3} b+a\right)}}{b} + \frac{c^2 e^{\left(-\frac{dx+c}{3} b-a\right)}}{b} - \frac{2 \left((dx+c) b^3 e^a - 3(dx+c)^{\frac{2}{3}} b^2 e^a + 6(dx+c)^{\frac{1}{3}} b e^a - 6e^a \right)}{b^4} \right)$$

[In] `integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")`

[Out] $\frac{1}{4}*(2*d^2*x^2*\cosh((d*x + c)^{(1/3)}*b + a) - (c^2*e^{((d*x + c)^{(1/3)}*b + a)}/b + c^2*e^{-(d*x + c)^{(1/3)}*b - a}/b - 2*((d*x + c)*b^3*e^a - 3*(d*x + c)^{(2/3)}*b^2*e^a + 6*(d*x + c)^{(1/3)}*b*e^a - 6*e^a)*c*e^{((d*x + c)^{(1/3)}*b)/b^4} - 2*((d*x + c)*b^3 + 3*(d*x + c)^{(2/3)}*b^2 + 6*(d*x + c)^{(1/3)}*b + 6)*c*e^{-(d*x + c)^{(1/3)}*b - a}/b^4 + ((d*x + c)^2*b^6*e^a - 6*(d*x + c)^{(5/3)}*b^5*e^a + 30*(d*x + c)^{(4/3)}*b^4*e^a - 120*(d*x + c)*b^3*e^a + 360*(d*x + c)^{(2/3)}*b^2*e^a - 720*(d*x + c)^{(1/3)}*b*e^a + 720*e^a)*e^{((d*x + c)^{(1/3)}*b)/b^7} + ((d*x + c)^2*b^6 + 6*(d*x + c)^{(5/3)}*b^5 + 30*(d*x + c)^{(4/3)}*b^4 + 120*(d*x + c)*b^3 + 360*(d*x + c)^{(2/3)}*b^2 + 720*(d*x + c)^{(1/3)}*b + 720)*e^{-(d*x + c)^{(1/3)}*b - a}/b^7)*b)/d^2$

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. 2(231) = 462.

Time = 0.28 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.71

$$\int x \cosh\left(a + b\sqrt[3]{c + dx}\right) dx =$$

$$3 \frac{\left(\left((dx+c)^{\frac{1}{3}}b+a \right)^2 b^3 c - 2 \left((dx+c)^{\frac{1}{3}}b+a \right) a b^3 c + a^2 b^3 c - \left((dx+c)^{\frac{1}{3}}b+a \right)^5 + 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 a - 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a^2 + 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^3 - 5 \left((dx+c)^{\frac{1}{3}}b+a \right) a^4 + a^5 - 2 \left((dx+c)^{\frac{1}{3}}b+a \right) b^3 c + 2 a b^3 c + 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 - 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a + 30 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^2 - 20 \left((dx+c)^{\frac{1}{3}}b+a \right) a^3 + 5 a^4 + 2 b^3 c - 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 + 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a - 60 \left((dx+c)^{\frac{1}{3}}b+a \right) a^2 + 20 a^3 + 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 - 120 \left((dx+c)^{\frac{1}{3}}b+a \right) a + 60 a^2 - 120 \left((dx+c)^{\frac{1}{3}}b+a \right) e^{\left((dx+c)^{\frac{1}{3}}b+a \right)} / (b^5 d) - \left(\left((dx+c)^{\frac{1}{3}}b+a \right)^2 b^3 c - 2 \left((dx+c)^{\frac{1}{3}}b+a \right) a b^3 c + a^2 b^3 c - \left((dx+c)^{\frac{1}{3}}b+a \right)^5 + 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 a - 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a^2 + 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^3 - 5 \left((dx+c)^{\frac{1}{3}}b+a \right) a^4 + a^5 + 2 \left((dx+c)^{\frac{1}{3}}b+a \right) b^3 c - 2 a b^3 c - 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 + 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a - 30 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^2 + 20 \left((dx+c)^{\frac{1}{3}}b+a \right) a^3 - 5 a^4 + 2 b^3 c - 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 + 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a - 60 \left((dx+c)^{\frac{1}{3}}b+a \right) a^2 + 20 a^3 - 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 + 120 \left((dx+c)^{\frac{1}{3}}b+a \right) a - 60 a^2 - 120 \left((dx+c)^{\frac{1}{3}}b+a \right) e^{-\left((dx+c)^{\frac{1}{3}}b+a \right)} / (b^5 d) \right) / (b*d)$$

[In] integrate(x*cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")

[Out]
$$\begin{aligned} & -3/2 * \left(\left((dx+c)^{\frac{1}{3}}b+a \right)^2 b^3 c - 2 \left((dx+c)^{\frac{1}{3}}b+a \right) a b^3 c + a^2 b^3 c - \left((dx+c)^{\frac{1}{3}}b+a \right)^5 + 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 a - 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a^2 + 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^3 - 5 \left((dx+c)^{\frac{1}{3}}b+a \right) a^4 + a^5 - 2 \left((dx+c)^{\frac{1}{3}}b+a \right) b^3 c + 2 a b^3 c + 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 - 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a + 30 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^2 - 20 \left((dx+c)^{\frac{1}{3}}b+a \right) a^3 + 5 a^4 + 2 b^3 c - 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 + 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a - 60 \left((dx+c)^{\frac{1}{3}}b+a \right) a^2 + 20 a^3 + 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 - 120 \left((dx+c)^{\frac{1}{3}}b+a \right) a + 60 a^2 - 120 \left((dx+c)^{\frac{1}{3}}b+a \right) e^{\left((dx+c)^{\frac{1}{3}}b+a \right)} / (b^5 d) - \left(\left((dx+c)^{\frac{1}{3}}b+a \right)^2 b^3 c - 2 \left((dx+c)^{\frac{1}{3}}b+a \right) a b^3 c + a^2 b^3 c - \left((dx+c)^{\frac{1}{3}}b+a \right)^5 + 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 a - 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a^2 + 10 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^3 - 5 \left((dx+c)^{\frac{1}{3}}b+a \right) a^4 + a^5 + 2 \left((dx+c)^{\frac{1}{3}}b+a \right) b^3 c - 2 a b^3 c - 5 \left((dx+c)^{\frac{1}{3}}b+a \right)^4 + 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 a - 30 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a^2 + 20 \left((dx+c)^{\frac{1}{3}}b+a \right) a^3 - 5 a^4 + 2 b^3 c - 20 \left((dx+c)^{\frac{1}{3}}b+a \right)^3 + 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 a - 60 \left((dx+c)^{\frac{1}{3}}b+a \right) a^2 + 20 a^3 - 60 \left((dx+c)^{\frac{1}{3}}b+a \right)^2 + 120 \left((dx+c)^{\frac{1}{3}}b+a \right) a - 60 a^2 - 120 \left((dx+c)^{\frac{1}{3}}b+a \right) e^{-\left((dx+c)^{\frac{1}{3}}b+a \right)} / (b^5 d) \right) / (b*d) \end{aligned}$$

Mupad [F(-1)]

Timed out.

$$\int x \cosh\left(a + b\sqrt[3]{c + dx}\right) dx = \int x \cosh\left(a + b(c + dx)^{1/3}\right) dx$$

[In] int(x*cosh(a + b*(c + d*x)^(1/3)),x)

[Out] int(x*cosh(a + b*(c + d*x)^(1/3)), x)

3.66 $\int \cosh(a + b\sqrt[3]{c + dx}) dx$

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Optimal result

Integrand size = 14, antiderivative size = 85

$$\int \cosh(a + b\sqrt[3]{c + dx}) dx = -\frac{6\sqrt[3]{c + dx} \cosh(a + b\sqrt[3]{c + dx})}{b^2 d} + \frac{6 \sinh(a + b\sqrt[3]{c + dx})}{b^3 d} + \frac{3(c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd}$$

[Out] $-6*(d*x+c)^{(1/3)}*\cosh(a+b*(d*x+c)^{(1/3)})/b^2/d+6*\sinh(a+b*(d*x+c)^{(1/3)})/b^3/d+3*(d*x+c)^{(2/3)}*\sinh(a+b*(d*x+c)^{(1/3)})/b/d$

Rubi [A] (verified)

Time = 0.06 (sec), antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {5419, 5413, 3377, 2717}

$$\int \cosh(a + b\sqrt[3]{c + dx}) dx = \frac{6 \sinh(a + b\sqrt[3]{c + dx})}{b^3 d} - \frac{6\sqrt[3]{c + dx} \cosh(a + b\sqrt[3]{c + dx})}{b^2 d} + \frac{3(c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd}$$

[In] $\text{Int}[\text{Cosh}[a + b*(c + d*x)^{(1/3)}], x]$

[Out] $(-6*(c + d*x)^{(1/3)}*\text{Cosh}[a + b*(c + d*x)^{(1/3)}])/(b^2*d) + (6*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b^3*d) + (3*(c + d*x)^{(2/3)}*\text{Sinh}[a + b*(c + d*x)^{(1/3)}])/(b*d)$

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] := Simp[Sin[c + d*x]/d, x] /;
FreeQ[{c, d}, x]
```

Rule 3377

```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] := Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Dist[d*(m/f), Int[(c + d*x)^(m - 1)*Co
s[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Rule 5413

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(x_)^(n_)])*(b_.))^(p_.), x_Symbol] := Modul
e[{k = Denominator[n]}, Dist[k, Subst[Int[x^(k - 1)*(a + b*Cosh[c + d*x^(k*
n)])^p, x], x, x^(1/k)], x]] /; FreeQ[{a, b, c, d}, x] && FractionQ[n] && I
ntegerQ[p]
```

Rule 5419

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.), x_Symbol] := Dist[
1/Coefficient[u, x, 1], Subst[Int[(a + b*Cosh[c + d*x^n])^p, x], x, u], x]
/; FreeQ[{a, b, c, d, n}, x] && IntegerQ[p] && LinearQ[u, x] && NeQ[u, x]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \frac{\text{Subst}\left(\int \cosh(a + b\sqrt[3]{x}) dx, x, c + dx\right)}{d} \\
&= \frac{3\text{Subst}\left(\int x^2 \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{d} \\
&= \frac{3(c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd} - \frac{6\text{Subst}\left(\int x \sinh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{bd} \\
&= -\frac{6\sqrt[3]{c + dx} \cosh(a + b\sqrt[3]{c + dx})}{b^2d} + \frac{3(c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd} \\
&\quad + \frac{6\text{Subst}\left(\int \cosh(a + bx) dx, x, \sqrt[3]{c + dx}\right)}{b^2d} \\
&= -\frac{6\sqrt[3]{c + dx} \cosh(a + b\sqrt[3]{c + dx})}{b^2d} + \frac{6 \sinh(a + b\sqrt[3]{c + dx})}{b^3d} \\
&\quad + \frac{3(c + dx)^{2/3} \sinh(a + b\sqrt[3]{c + dx})}{bd}
\end{aligned}$$

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

$$= \frac{-6b\sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right) + 3(2 + b^2(c + dx)^{2/3}) \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3d}$$

`[In] Integrate[Cosh[a + b*(c + d*x)^(1/3)],x]`

```
[Out] (-6*b*(c + d*x)^(1/3)*Cosh[a + b*(c + d*x)^(1/3)] + 3*(2 + b^2*(c + d*x)^(2/3))*Sinh[a + b*(c + d*x)^(1/3)]/(b^3*d)
```

Maple [A] (verified)

Time = 0.01 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{3a^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - 6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right) \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - \cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right) + 3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{db^3}$
default	$\frac{3a^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - 6a\left(\left(a+b(dx+c)^{\frac{1}{3}}\right) \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right) - \cosh\left(a+b(dx+c)^{\frac{1}{3}}\right)\right) + 3\left(a+b(dx+c)^{\frac{1}{3}}\right)^2 \sinh\left(a+b(dx+c)^{\frac{1}{3}}\right)}{db^3}$

`[In] int(cosh(a+b*(d*x+c)^(1/3)),x,method=_RETURNVERBOSE)`

```
[Out] 3/d/b^3*(a^2*sinh(a+b*(d*x+c)^(1/3))-2*a*((a+b*(d*x+c)^(1/3))*sinh(a+b*(d*x+c)^(1/3))-cosh(a+b*(d*x+c)^(1/3)))+(a+b*(d*x+c)^(1/3))^2*sinh(a+b*(d*x+c)^(1/3))-2*(a+b*(d*x+c)^(1/3))*cosh(a+b*(d*x+c)^(1/3))+2*sinh(a+b*(d*x+c)^(1/3)))
```

Fricas [A] (verification not implemented)

none

Time = 0.27 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.68

$$\int \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

$$= -\frac{3\left(2(dx+c)^{\frac{1}{3}}b \cosh\left((dx+c)^{\frac{1}{3}}b + a\right) - \left((dx+c)^{\frac{2}{3}}b^2 + 2\right) \sinh\left((dx+c)^{\frac{1}{3}}b + a\right)\right)}{b^3d}$$

`[In] integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="fricas")`

```
[Out] -3*(2*(d*x + c)^(1/3)*b*cosh((d*x + c)^(1/3)*b + a) - ((d*x + c)^(2/3)*b^2 + 2)*sinh((d*x + c)^(1/3)*b + a))/(b^3*d)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11

$$\int \cosh\left(a + b\sqrt[3]{c + dx}\right) dx$$

$$= \begin{cases} x \cosh(a) & \text{for } b = 0 \wedge (b = 0 \vee d = 0) \\ x \cosh\left(a + b\sqrt[3]{c}\right) & \text{for } d = 0 \\ \frac{3(c+dx)^{\frac{2}{3}} \sinh\left(a + b\sqrt[3]{c + dx}\right)}{bd} - \frac{6\sqrt[3]{c + dx} \cosh\left(a + b\sqrt[3]{c + dx}\right)}{b^2d} + \frac{6 \sinh\left(a + b\sqrt[3]{c + dx}\right)}{b^3d} & \text{otherwise} \end{cases}$$

```
[In] integrate(cosh(a+b*(d*x+c)**(1/3)),x)
```

```
[Out] Piecewise((x*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(d, 0))), (x*cosh(a + b*c**(1/3)), Eq(d, 0)), (3*(c + d*x)**(2/3)*sinh(a + b*(c + d*x)**(1/3))/(b*d) - 6*(c + d*x)**(1/3)*cosh(a + b*(c + d*x)**(1/3))/(b**2*d) + 6*sinh(a + b*(c + d*x)**(1/3))/(b**3*d), True))
```

Maxima [A] (verification not implemented)

none

Time = 0.20 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int \cosh\left(a + b\sqrt[3]{c + dx}\right) dx =$$

$$\frac{b \left(\frac{\left((dx+c)b^3e^a - 3(dx+c)^{\frac{2}{3}}b^2e^a + 6(dx+c)^{\frac{1}{3}}be^a - 6e^a \right) e^{\left((dx+c)^{\frac{1}{3}}b \right)}}{b^4} + \frac{\left((dx+c)b^3 + 3(dx+c)^{\frac{2}{3}}b^2 + 6(dx+c)^{\frac{1}{3}}b + 6 \right) e^{\left(-(dx+c)^{\frac{1}{3}}b - a \right)}}{b^4} \right)}{2d}$$

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="maxima")
```

```
[Out] -1/2*(b*(((d*x + c)*b^3*e^a - 3*(d*x + c)^(2/3)*b^2*e^a + 6*(d*x + c)^(1/3)*b*e^a - 6*e^a)*e^(((d*x + c)^(1/3)*b)/b^4 + ((d*x + c)*b^3 + 3*(d*x + c)^(2/3)*b^2 + 6*(d*x + c)^(1/3)*b + 6)*e^(-(d*x + c)^(1/3)*b - a)/b^4) - 2*(d*x + c)*cosh((d*x + c)^(1/3)*b + a))/d
```

Giac [A] (verification not implemented)

none

Time = 0.26 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.51

$$\int \cosh \left(a + b\sqrt[3]{c + dx} \right) dx$$

$$= \frac{3 \left(\left((dx + c)^{\frac{1}{3}} b + a \right)^2 - 2 \left((dx + c)^{\frac{1}{3}} b + a \right) a + a^2 - 2 (dx + c)^{\frac{1}{3}} b + 2 \right) e^{\left((dx + c)^{\frac{1}{3}} b + a \right)}}{2 b^3 d}$$

$$- \frac{3 \left(\left((dx + c)^{\frac{1}{3}} b + a \right)^2 - 2 \left((dx + c)^{\frac{1}{3}} b + a \right) a + a^2 + 2 (dx + c)^{\frac{1}{3}} b + 2 \right) e^{\left(-(dx + c)^{\frac{1}{3}} b - a \right)}}{2 b^3 d}$$

[In] integrate(cosh(a+b*(d*x+c)^(1/3)),x, algorithm="giac")

```
[Out] 3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 - 2*(d*x + c)^(1/3)*b + 2)*e^((d*x + c)^(1/3)*b + a)/(b^3*d) - 3/2*(((d*x + c)^(1/3)*b + a)^2 - 2*((d*x + c)^(1/3)*b + a)*a + a^2 + 2*(d*x + c)^(1/3)*b + 2)*e^(-(d*x + c)^(1/3)*b - a)/(b^3*d)
```

Mupad [B] (verification not implemented)

Time = 1.64 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.88

$$\int \cosh \left(a + b\sqrt[3]{c + dx} \right) dx = \frac{6 \sinh \left(a + b(c + dx)^{1/3} \right)}{b^3 d}$$

$$- \frac{6 \cosh \left(a + b(c + dx)^{1/3} \right) (c + dx)^{1/3}}{b^2 d}$$

$$+ \frac{3 \sinh \left(a + b(c + dx)^{1/3} \right) (c + dx)^{2/3}}{b d}$$

[In] int(cosh(a + b*(c + d*x)^(1/3)),x)

```
[Out] (6*sinh(a + b*(c + d*x)^(1/3)))/(b^3*d) - (6*cosh(a + b*(c + d*x)^(1/3))*(c + d*x)^(1/3))/(b^2*d) + (3*sinh(a + b*(c + d*x)^(1/3))*(c + d*x)^(2/3))/(b*d)
```

$$3.67 \quad \int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} dx$$

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Optimal result

Integrand size = 18, antiderivative size = 232

$$\begin{aligned} \int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} dx &= \cosh\left(a+b\sqrt[3]{c}\right) \operatorname{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) \\ &+ \cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right) \operatorname{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) \\ &+ \cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right) \operatorname{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right) \\ &- \sinh\left(a+b\sqrt[3]{c}\right) \operatorname{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) \\ &- \sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right) \operatorname{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right) + \sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right) \operatorname{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right) \end{aligned}$$

```
[Out] Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*cosh(a+b*c^(1/3))+Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*cosh(a-(-1)^(1/3)*b*c^(1/3))+Chi(-b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*cosh(a+(-1)^(2/3)*b*c^(1/3))-Shi(b*(c^(1/3)-(d*x+c)^(1/3)))*sinh(a+b*c^(1/3))+Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)*b*c^(1/3))-Shi(b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))*sinh(a+(-1)^(2/3)*b*c^(1/3))
```


Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {5473, 5401, 3384, 3379, 3382}

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = \cosh\left(a + b\sqrt[3]{c}\right) \text{Chi}\left(b\left(\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right) \\ + \cosh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right) \\ + \cosh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c} + \sqrt[3]{c + dx}\right)\right) \\ - \sinh\left(a + b\sqrt[3]{c}\right) \text{Shi}\left(b\left(\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right) \\ - \sinh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \text{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right) + \sinh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \text{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c} + \sqrt[3]{c + dx}\right)\right)$$

[In] Int[Cosh[a + b*(c + d*x)^(1/3)]/x,x]

[Out] Cosh[a + b*c^(1/3)]*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))] + Cosh[a + (-1)^(2/3)*b*c^(1/3)]*CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))] + Cosh[a - (-1)^(1/3)*b*c^(1/3)]*CoshIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))] - Sinh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))] - Sinh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3))] + Sinh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3382

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]

Rule 3384

Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]

Rule 5401

```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] := Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbo
l] := Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x,
0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /; FreeQ[{a, b, c, d, n, p}
, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\text{integral} &= \text{Subst} \left(\int \frac{\cosh(a + b\sqrt[3]{x})}{-c + x} dx, x, c + dx \right) \\
&= 3 \text{Subst} \left(\int \frac{x^2 \cosh(a + bx)}{-c + x^3} dx, x, \sqrt[3]{c + dx} \right) \\
&= 3 \text{Subst} \left(\int \left(-\frac{\cosh(a + bx)}{3(\sqrt[3]{c} - x)} - \frac{\cosh(a + bx)}{3(-\sqrt[3]{-1}\sqrt[3]{c} - x)} - \frac{\cosh(a + bx)}{3((-1)^{2/3}\sqrt[3]{c} - x)} \right) dx, x, \sqrt[3]{c + dx} \right) \\
&= -\text{Subst} \left(\int \frac{\cosh(a + bx)}{\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx} \right) \\
&\quad - \text{Subst} \left(\int \frac{\cosh(a + bx)}{-\sqrt[3]{-1}\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx} \right) \\
&\quad - \text{Subst} \left(\int \frac{\cosh(a + bx)}{(-1)^{2/3}\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx} \right) \\
&= - \left(\cosh(a + b\sqrt[3]{c}) \text{Subst} \left(\int \frac{\cosh(b\sqrt[3]{c} - bx)}{\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx} \right) \right) \\
&\quad - \cosh(a - \sqrt[3]{-1}b\sqrt[3]{c}) \text{Subst} \left(\int \frac{\cos((-1)^{5/6}b\sqrt[3]{c} + ibx)}{-\sqrt[3]{-1}\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx} \right) - \cosh(a \\
&\quad + (-1)^{2/3}b\sqrt[3]{c}) \text{Subst} \left(\int \frac{\cos(\sqrt[3]{-1}b\sqrt[3]{c} + ibx)}{(-1)^{2/3}\sqrt[3]{c} - x} dx, x, \sqrt[3]{c + dx} \right) + \sinh(a + b\sqrt[3]{c}) \text{Subst} \left(\int \frac{\sinh(b\sqrt[3]{x})}{\sqrt[3]{x}} dx, x, \sqrt[3]{c + dx} \right)
\end{aligned}$$

$$\begin{aligned}
&= \cosh(a + b\sqrt[3]{c}) \operatorname{Chi}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) \\
&\quad + \cosh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \operatorname{Chi}\left(\sqrt[3]{-1}b\sqrt[3]{c} + b\sqrt[3]{c + dx}\right) \\
&\quad + \cosh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \operatorname{Chi}\left(-(-1)^{2/3}b\sqrt[3]{c}\right. \\
&\quad\quad\quad \left.+ b\sqrt[3]{c + dx}\right) - \sinh(a + b\sqrt[3]{c}) \operatorname{Shi}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) \\
&\quad - \sinh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \operatorname{Shi}\left(-(-1)^{2/3}b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) + \sinh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \operatorname{Shi}\left(\sqrt[3]{-1}b\sqrt[3]{c} + b\sqrt[3]{c + dx}\right)
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00

$$\begin{aligned}
\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx &= \frac{1}{2} \left(\operatorname{RootSum}\left[c - \#1^3 \&, \cosh(a + b\#1) \operatorname{Chi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right)\right.\right. \\
&\quad - \operatorname{Chi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right) \sinh(a + b\#1) \\
&\quad - \cosh(a + b\#1) \operatorname{Shi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right) \\
&\quad \left. + \sinh(a + b\#1) \operatorname{Shi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right) \& \right] \\
&\quad + \operatorname{RootSum}\left[c - \#1^3 \&, \cosh(a + b\#1) \operatorname{Chi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right)\right.\right. \\
&\quad + \operatorname{Chi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right) \sinh(a + b\#1) \\
&\quad + \cosh(a + b\#1) \operatorname{Shi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right) \\
&\quad \left. + \sinh(a + b\#1) \operatorname{Shi}\left(b\left(\sqrt[3]{c + dx} - \#1\right)\right) \& \right]
\end{aligned}$$

[In] Integrate[Cosh[a + b*(c + d*x)^(1/3)]/x,x]

[Out] (RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)] - CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] - Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] &] + RootSum[c - #1^3 & , Cosh[a + b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1)] + CoshIntegral[b*((c + d*x)^(1/3) - #1)]*Sinh[a + b*#1] + Cosh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] + Sinh[a + b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1)] &])/2

Maple [F]

$$\int \frac{\cosh\left(a + b(dx + c)^{\frac{1}{3}}\right)}{x} dx$$

[In] int(cosh(a+b*(d*x+c)^(1/3))/x,x)

[Out] int(cosh(a+b*(d*x+c)^(1/3))/x,x)

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 503 vs. 2(182) = 364.

Time = 0.27 (sec) , antiderivative size = 503, normalized size of antiderivative = 2.17

$$\begin{aligned}
\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = & \frac{1}{2} \operatorname{Ei}\left(-\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. - \frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right)\right) \cosh\left(\frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right) - a\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. - \frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right)\right) \cosh\left(\frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right) + a\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(-\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. + \frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right)\right) \cosh\left(\frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right) + a\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. + \frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right)\right) \cosh\left(\frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right) - a\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(-\left(dx + c\right)^{\frac{1}{3}} b + \left(b^3 c\right)^{\frac{1}{3}}\right) \cosh\left(a + \left(b^3 c\right)^{\frac{1}{3}}\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(\left(dx + c\right)^{\frac{1}{3}} b + \left(-b^3 c\right)^{\frac{1}{3}}\right) \cosh\left(-a + \left(-b^3 c\right)^{\frac{1}{3}}\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(-\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. - \frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right)\right) \sinh\left(\frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right) - a\right) \\
& + \frac{1}{2} \operatorname{Ei}\left(\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. - \frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right)\right) \sinh\left(\frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} + 1\right) + a\right) \\
& - \frac{1}{2} \operatorname{Ei}\left(-\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. + \frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right)\right) \sinh\left(\frac{1}{2} \left(b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right) + a\right) \\
& - \frac{1}{2} \operatorname{Ei}\left(\left(dx + c\right)^{\frac{1}{3}} b\right. \\
& \left. + \frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right)\right) \sinh\left(\frac{1}{2} \left(-b^3 c\right)^{\frac{1}{3}} \left(\sqrt{-3} - 1\right) - a\right) \\
& - \frac{1}{2} \operatorname{Ei}\left(-\left(dx + c\right)^{\frac{1}{3}} b + \left(b^3 c\right)^{\frac{1}{3}}\right) \sinh\left(a + \left(b^3 c\right)^{\frac{1}{3}}\right) \\
& - \frac{1}{2} \operatorname{Ei}\left(\left(dx + c\right)^{\frac{1}{3}} b + \left(-b^3 c\right)^{\frac{1}{3}}\right) \sinh\left(-a + \left(-b^3 c\right)^{\frac{1}{3}}\right)
\end{aligned}$$

[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="fricas")

[Out] $\frac{1}{2} \operatorname{Ei}(- (d*x + c)^{1/3} * b - \frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} + 1)) * \cosh(\frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} + 1) - a) + \frac{1}{2} \operatorname{Ei}((d*x + c)^{1/3} * b - \frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} + 1)) * \cosh(\frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} + 1) + a) + \frac{1}{2} \operatorname{Ei}(- (d*x + c)^{1/3} * b + \frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} - 1)) * \cosh(\frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} - 1) + a) + \frac{1}{2} \operatorname{Ei}((d*x + c)^{1/3} * b + \frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} - 1)) * \cosh(\frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} - 1) - a) + \frac{1}{2} \operatorname{Ei}(- (d*x + c)^{1/3} * b + (b^3 * c)^{1/3}) * \cosh(a + (b^3 * c)^{1/3}) + \frac{1}{2} \operatorname{Ei}((d*x + c)^{1/3} * b + (-b^3 * c)^{1/3}) * \cosh(-a + (-b^3 * c)^{1/3}) + \frac{1}{2} \operatorname{Ei}(- (d*x + c)^{1/3} * b - \frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} + 1)) * \sinh(\frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} + 1) - a) + \frac{1}{2} \operatorname{Ei}((d*x + c)^{1/3} * b - \frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} + 1)) * \sinh(\frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} + 1) + a) - \frac{1}{2} \operatorname{Ei}(- (d*x + c)^{1/3} * b + \frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} - 1)) * \sinh(\frac{1}{2} * (b^3 * c)^{1/3} * (\sqrt{-3} - 1) + a) - \frac{1}{2} \operatorname{Ei}((d*x + c)^{1/3} * b + \frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} - 1)) * \sinh(\frac{1}{2} * (-b^3 * c)^{1/3} * (\sqrt{-3} - 1) - a) - \frac{1}{2} \operatorname{Ei}(- (d*x + c)^{1/3} * b + (b^3 * c)^{1/3}) * \sinh(a + (b^3 * c)^{1/3}) - \frac{1}{2} \operatorname{Ei}((d*x + c)^{1/3} * b + (-b^3 * c)^{1/3}) * \sinh(-a + (-b^3 * c)^{1/3})$

Sympy [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = \int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx$$

[In] integrate(cosh(a+b*(d*x+c)**(1/3))/x,x)

[Out] Integral(cosh(a + b*(c + d*x)**(1/3))/x, x)

Maxima [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = \int \frac{\cosh\left(\left(dx + c\right)^{\frac{1}{3}} b + a\right)}{x} dx$$

[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="maxima")

[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x, x)

Giac [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = \int \frac{\cosh\left(\frac{(dx + c)^{\frac{1}{3}}b + a}{x}\right)}{x} dx$$

[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x,x, algorithm="giac")

[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x, x)

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} dx = \int \frac{\cosh\left(a + b(c + dx)^{1/3}\right)}{x} dx$$

[In] int(cosh(a + b*(c + d*x)^(1/3))/x,x)

[Out] int(cosh(a + b*(c + d*x)^(1/3))/x, x)

$$3.68 \quad \int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2} dx$$

Optimal result	360
Rubi [A] (verified)	361
Mathematica [C] (verified)	364
Maple [F]	365
Fricas [B] (verification not implemented)	365
Sympy [F]	366
Maxima [F]	366
Giac [F]	366
Mupad [F(-1)]	366

Optimal result

Integrand size = 18, antiderivative size = 329

$$\begin{aligned} & \int \frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x^2} dx \\ &= -\frac{\cosh\left(a+b\sqrt[3]{c+dx}\right)}{x} + \frac{bd\text{Chi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+b\sqrt[3]{c}\right)}{3c^{2/3}} \\ & \quad - \frac{\sqrt[3]{-1}bd\text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)\sinh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)}{3c^{2/3}} \\ & \quad + \frac{(-1)^{2/3}bd\text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)\sinh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)}{3c^{2/3}} \\ & \quad - \frac{bd\cosh\left(a+b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ & \quad - \frac{(-1)^{2/3}bd\cosh\left(a+(-1)^{2/3}b\sqrt[3]{c}\right)\text{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c}-\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \\ & \quad - \frac{\sqrt[3]{-1}bd\cosh\left(a-\sqrt[3]{-1}b\sqrt[3]{c}\right)\text{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c}+\sqrt[3]{c+dx}\right)\right)}{3c^{2/3}} \end{aligned}$$

```
[Out] -cosh(a+b*(d*x+c)^(1/3))/x-1/3*b*d*cosh(a+b*c^(1/3))*Shi(b*(c^(1/3)-(d*x+c)^(1/3)))/c^(2/3)-1/3*(-1)^(2/3)*b*d*cosh(a+(-1)^(2/3)*b*c^(1/3))*Shi(b*((-1)^(2/3)*c^(1/3)-(d*x+c)^(1/3)))/c^(2/3)-1/3*(-1)^(1/3)*b*d*cosh(a-(-1)^(1/3)*b*c^(1/3))*Shi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))/c^(2/3)+1/3*b*d*Chi(b*(c^(1/3)-(d*x+c)^(1/3)))*sinh(a+b*c^(1/3))/c^(2/3)-1/3*(-1)^(1/3)*b*d*Chi(b*((-1)^(1/3)*c^(1/3)+(d*x+c)^(1/3)))*sinh(a-(-1)^(1/3)*b*c^(1/3))/c^(2/3)
```


$$+1/3*(-1)^{(2/3)}*b*d*Chi(-b*((-1)^{(2/3)}*c^{(1/3)}-(d*x+c)^{(1/3)}))*sinh(a+(-1)^{(2/3)}*b*c^{(1/3)})/c^{(2/3)}$$

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {5473, 5397, 5388, 3384, 3379, 3382}

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx$$

$$= \frac{bd \sinh\left(a + b\sqrt[3]{c}\right) \text{Chi}\left(b\left(\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right)}{3c^{2/3}}$$

$$- \frac{\sqrt[3]{-1}bd \sinh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \text{Chi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c} + \sqrt[3]{c + dx}\right)\right)}{3c^{2/3}}$$

$$+ \frac{(-1)^{2/3}bd \sinh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \text{Chi}\left(-b\left((-1)^{2/3}\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right)}{3c^{2/3}}$$

$$- \frac{bd \cosh\left(a + b\sqrt[3]{c}\right) \text{Shi}\left(b\left(\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right)}{3c^{2/3}}$$

$$- \frac{(-1)^{2/3}bd \cosh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \text{Shi}\left(b\left((-1)^{2/3}\sqrt[3]{c} - \sqrt[3]{c + dx}\right)\right)}{3c^{2/3}}$$

$$- \frac{\sqrt[3]{-1}bd \cosh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \text{Shi}\left(b\left(\sqrt[3]{-1}\sqrt[3]{c} + \sqrt[3]{c + dx}\right)\right)}{3c^{2/3}} - \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x}$$

[In] Int[Cosh[a + b*(c + d*x)^(1/3)]/x^2,x]

[Out] -(Cosh[a + b*(c + d*x)^(1/3)]/x) + (b*d*CoshIntegral[b*(c^(1/3) - (c + d*x)^(1/3))]*Sinh[a + b*c^(1/3)]/(3*c^(2/3)) - ((-1)^(1/3)*b*d*CoshIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]*Sinh[a - (-1)^(1/3)*b*c^(1/3)]/(3*c^(2/3)) + ((-1)^(2/3)*b*d*CoshIntegral[-(b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3)))]*Sinh[a + (-1)^(2/3)*b*c^(1/3)]/(3*c^(2/3)) - (b*d*Cosh[a + b*c^(1/3)]*SinhIntegral[b*(c^(1/3) - (c + d*x)^(1/3))]/(3*c^(2/3)) - ((-1)^(2/3)*b*d*Cosh[a + (-1)^(2/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(2/3)*c^(1/3) - (c + d*x)^(1/3))]/(3*c^(2/3)) - ((-1)^(1/3)*b*d*Cosh[a - (-1)^(1/3)*b*c^(1/3)]*SinhIntegral[b*((-1)^(1/3)*c^(1/3) + (c + d*x)^(1/3))]/(3*c^(2/3)))

Rule 3379

Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] := Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f, fz}, x] && EqQ[d*e - c*f*fz*I, 0]

Rule 3382

```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz}, x]
&& EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

Rule 3384

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol]
:> Dist[Cos[(d*e - c*f)/d], Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x]
+ Dist[Sin[(d*e - c*f)/d], Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /;
FreeQ[{c, d, e, f}, x] && NeQ[d*e - c*f, 0]
```

Rule 5388

```
Int[((a_.) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol]
:> Int[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /;
FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Rule 5397

```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_)^(m_.))*((a_.) + (b_.)*(x_)^(n_))^(p_), x_Symbol]
:> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x]
- Dist[d*(e^m/(b*n*(p + 1))), Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x], x], x] /;
FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1, 0]
&& LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Rule 5473

```
Int[((a_.) + Cosh[(c_.) + (d_.)*(u_)^(n_)])*(b_.))^(p_.)*(x_)^(m_.), x_Symbol]
:> Dist[1/Coefficient[u, x, 1]^(m + 1), Subst[Int[(x - Coefficient[u, x, 0])^m*(a + b*Cosh[c + d*x^n])^p, x], x, u], x] /;
FreeQ[{a, b, c, d, n, p}, x] && LinearQ[u, x] && NeQ[u, x] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned} \text{integral} &= d\text{Subst}\left(\int \frac{\cosh(a + b\sqrt[3]{x})}{(-c + x)^2} dx, x, c + dx\right) \\ &= (3d)\text{Subst}\left(\int \frac{x^2 \cosh(a + bx)}{(c - x^3)^2} dx, x, \sqrt[3]{c + dx}\right) \\ &= -\frac{\cosh(a + b\sqrt[3]{c + dx})}{x} - (bd)\text{Subst}\left(\int \frac{\sinh(a + bx)}{c - x^3} dx, x, \sqrt[3]{c + dx}\right) \end{aligned}$$

$$\begin{aligned}
&= -\frac{\cosh(a + b\sqrt[3]{c + dx})}{x} - (bd)\text{Subst}\left(\int\left(\frac{\sinh(a + bx)}{3c^{2/3}(\sqrt[3]{c} - x)} + \frac{\sinh(a + bx)}{3c^{2/3}(\sqrt[3]{c} + \sqrt[3]{-1}x)}\right.\right. \\
&\quad \left.\left.+ \frac{\sinh(a + bx)}{3c^{2/3}(\sqrt[3]{c} - (-1)^{2/3}x)}\right)dx, x, \sqrt[3]{c + dx}\right) \\
&= -\frac{\cosh(a + b\sqrt[3]{c + dx})}{x} - \frac{(bd)\text{Subst}\left(\int\frac{\sinh(a+bx)}{\sqrt[3]{c-x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad - \frac{(bd)\text{Subst}\left(\int\frac{\sinh(a+bx)}{\sqrt[3]{c+\sqrt[3]{-1}x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad - \frac{(bd)\text{Subst}\left(\int\frac{\sinh(a+bx)}{\sqrt[3]{c-(-1)^{2/3}x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&= -\frac{\cosh(a + b\sqrt[3]{c + dx})}{x} \\
&\quad + \frac{(bd \cosh(a + b\sqrt[3]{c})) \text{Subst}\left(\int\frac{\sinh(b\sqrt[3]{c-bx})}{\sqrt[3]{c-x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad + \frac{(ibd \cosh(a - \sqrt[3]{-1}b\sqrt[3]{c})) \text{Subst}\left(\int\frac{\sin((-1)^{5/6}b\sqrt[3]{c+ibx})}{\sqrt[3]{c-(-1)^{2/3}x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad + \frac{(ibd \cosh(a + (-1)^{2/3}b\sqrt[3]{c})) \text{Subst}\left(\int\frac{\sin(\sqrt[6]{-1}b\sqrt[3]{c+ibx})}{\sqrt[3]{c+\sqrt[3]{-1}x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad - \frac{(bd \sinh(a + b\sqrt[3]{c})) \text{Subst}\left(\int\frac{\cosh(b\sqrt[3]{c-bx})}{\sqrt[3]{c-x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad - \frac{(bd \sinh(a - \sqrt[3]{-1}b\sqrt[3]{c})) \text{Subst}\left(\int\frac{\cos((-1)^{5/6}b\sqrt[3]{c+ibx})}{\sqrt[3]{c-(-1)^{2/3}x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&\quad - \frac{(bd \sinh(a + (-1)^{2/3}b\sqrt[3]{c})) \text{Subst}\left(\int\frac{\cos(\sqrt[6]{-1}b\sqrt[3]{c+ibx})}{\sqrt[3]{c+\sqrt[3]{-1}x}}dx, x, \sqrt[3]{c + dx}\right)}{3c^{2/3}}
\end{aligned}$$

$$\begin{aligned}
&= -\frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x} + \frac{bd\text{Chi}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right) \sinh\left(a + b\sqrt[3]{c}\right)}{3c^{2/3}} \\
&- \frac{\sqrt[3]{-1}bd\text{Chi}\left(\sqrt[3]{-1}b\sqrt[3]{c} + b\sqrt[3]{c + dx}\right) \sinh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right)}{3c^{2/3}} \\
&+ \frac{(-1)^{2/3}bd\text{Chi}\left(-(-1)^{2/3}b\sqrt[3]{c} + b\sqrt[3]{c + dx}\right) \sinh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right)}{3c^{2/3}} \\
&- \frac{bd \cosh\left(a + b\sqrt[3]{c}\right) \text{Shi}\left(b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&- \frac{(-1)^{2/3}bd \cosh\left(a + (-1)^{2/3}b\sqrt[3]{c}\right) \text{Shi}\left((-1)^{2/3}b\sqrt[3]{c} - b\sqrt[3]{c + dx}\right)}{3c^{2/3}} \\
&- \frac{\sqrt[3]{-1}bd \cosh\left(a - \sqrt[3]{-1}b\sqrt[3]{c}\right) \text{Shi}\left(\sqrt[3]{-1}b\sqrt[3]{c} + b\sqrt[3]{c + dx}\right)}{3c^{2/3}}
\end{aligned}$$

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.64

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx$$

$$= \frac{e^{-a} \left(-3e^{-b\sqrt[3]{c+dx}} \left(1 + e^{2(a+b\sqrt[3]{c+dx})} \right) + bdx\text{RootSum} \left[c - \#1^3 \&, \frac{e^{2a+b\#1} \text{ExpIntegralEi} \left(b \left(\sqrt[3]{c + dx} - \#1 \right) \right)}{\#1^2} \right] \& \right)}{6E^a x}$$

[In] Integrate[Cosh[a + b*(c + d*x)^(1/3)]/x^2,x]

[Out] ((-3*(1 + E^(2*(a + b*(c + d*x)^(1/3)))))/E^(b*(c + d*x)^(1/3)) + b*d*x*RootSum[c - #1^3 &, (E^(2*a + b*#1)*ExpIntegralEi[b*((c + d*x)^(1/3) - #1]])/#1^2 &] - b*d*x*RootSum[c - #1^3 &, (Cosh[b*#1]*CoshIntegral[b*((c + d*x)^(1/3) - #1]) - CoshIntegral[b*((c + d*x)^(1/3) - #1])*Sinh[b*#1] - Cosh[b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1]) + Sinh[b*#1]*SinhIntegral[b*((c + d*x)^(1/3) - #1]])/#1^2 &])/(6*E^a*x)

Maple [F]

$$\int \frac{\cosh\left(a + b(dx + c)^{\frac{1}{3}}\right)}{x^2} dx$$

[In] `int(cosh(a+b*(d*x+c)^(1/3))/x^2,x)`

[Out] `int(cosh(a+b*(d*x+c)^(1/3))/x^2,x)`

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(245) = 490$.

Time = 0.29 (sec) , antiderivative size = 706, normalized size of antiderivative = 2.15

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx = \text{Too large to display}$$

[In] `integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="fricas")`

[Out] `-1/12*(2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*cosh(a + (b^3*c)^(1/3)) + 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*cosh(-a + (-b^3*c)^(1/3)) - 2*(b^3*c)^(1/3)*d*x*Ei(-(d*x + c)^(1/3)*b + (b^3*c)^(1/3))*sinh(a + (b^3*c)^(1/3)) - 2*(-b^3*c)^(1/3)*d*x*Ei((d*x + c)^(1/3)*b + (-b^3*c)^(1/3))*sinh(-a + (-b^3*c)^(1/3)) - (b^3*c)^(1/3)*(sqrt(-3))*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) + (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) + (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei(-(d*x + c)^(1/3)*b - 1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) + 1) - a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x + d*x)*Ei((d*x + c)^(1/3)*b - 1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) + 1) + a) - (b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei(-(d*x + c)^(1/3)*b + 1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(b^3*c)^(1/3)*(sqrt(-3) - 1) + a) - (-b^3*c)^(1/3)*(sqrt(-3)*d*x - d*x)*Ei((d*x + c)^(1/3)*b + 1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-b^3*c)^(1/3)*(sqrt(-3) - 1) - a) + 12*c*cosh((d*x + c)^(1/3)*b + a)/(c*x)`

Sympy [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx = \int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx$$

```
[In] integrate(cosh(a+b*(d*x+c)**(1/3))/x**2,x)
```

```
[Out] Integral(cosh(a + b*(c + d*x)**(1/3))/x**2, x)
```

Maxima [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx = \int \frac{\cosh\left(\left(dx + c\right)^{\frac{1}{3}}b + a\right)}{x^2} dx$$

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Giac [F]

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx = \int \frac{\cosh\left(\left(dx + c\right)^{\frac{1}{3}}b + a\right)}{x^2} dx$$

```
[In] integrate(cosh(a+b*(d*x+c)^(1/3))/x^2,x, algorithm="giac")
```

```
[Out] integrate(cosh((d*x + c)^(1/3)*b + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(a + b\sqrt[3]{c + dx}\right)}{x^2} dx = \int \frac{\cosh\left(a + b(c + dx)^{1/3}\right)}{x^2} dx$$

```
[In] int(cosh(a + b*(c + d*x)^(1/3))/x^2,x)
```

```
[Out] int(cosh(a + b*(c + d*x)^(1/3))/x^2, x)
```

CHAPTER 4

APPENDIX

4.1 Listing of Grading functions 367

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7, 2022. add second output which gives reason for the grade *)
(*      Small rewrite of logic in main function to make it*)
(*      match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)

(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
```

```

(*      GradeAntiderivative[result,optimal] returns*)
(* "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(* "C" if result involves higher level functions than necessary*)
(* "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(* "A" if result can be considered optimal*)

GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafCo
  expnResult = ExpnType[result];
  expnOptimal = ExpnType[optimal];
  leafCountResult = LeafCount[result];
  leafCountOptimal = LeafCount[optimal];

  (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
  If[expnResult<=expnOptimal,
    If[Not[FreeQ[result,Complex]], (*result contains complex*)
      If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
        If[leafCountResult<=2*leafCountOptimal,
          finalresult={"A",""}
          ,(*ELSE*)
          finalresult={"B","Both result and optimal contain complex but leaf count is
        ]
      ,(*ELSE*)
      finalresult={"C","Result contains complex when optimal does not."}
    ]
    ,(*ELSE*)(*result does not contains complex*)
    If[leafCountResult<=2*leafCountOptimal,
      finalresult={"A",""}
      ,(*ELSE*)
      finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $"}
    ]
  ]
  ,(*ELSE*)(*expnResult>expnOptimal*)
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    finalresult={"C","Result contains higher order function than in optimal. Order "<}
    ,
    finalresult={"F","Contains unresolved integral."}
  ]
];

finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)

```



```

(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)

```

```

ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
    If[ListQ[expn],
      Max[Map[ExpnType, expn]],
      If[Head[expn]===Power,
        If[IntegerQ[expn[[2]]],
          ExpnType[expn[[1]]],
          If[Head[expn[[2]]]===Rational,
            If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
              1,
              Max[ExpnType[expn[[1]], 2]],
            Max[ExpnType[expn[[1]], ExpnType[expn[[2]], 3]],
          If[Head[expn]===Plus || Head[expn]===Times,
            Max[ExpnType[First[expn]], ExpnType[Rest[expn]]],
          If[ElementaryFunctionQ[Head[expn]],
            Max[3, ExpnType[expn[[1]]],
          If[SpecialFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 4]],
          If[HypergeometricFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 5]],
          If[AppellFunctionQ[Head[expn]],
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 6]],
          If[Head[expn]===RootSum,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 7]],
          If[Head[expn]===Integrate || Head[expn]===Int,
            Apply[Max, Append[Map[ExpnType, Apply[List, expn]], 8]],
          9]]]]]]]]]]

```

```

ElementaryFunctionQ[func_] :=
  MemberQ[{
    Exp, Log,
    Sin, Cos, Tan, Cot, Sec, Csc,
    ArcSin, ArcCos, ArcTan, ArcCot, ArcSec, ArcCsc,

```

```

    Sinh, Cosh, Tanh, Coth, Sech, Csch,
    ArcSinh, ArcCosh, ArcTanh, ArcCoth, ArcSech, ArcCsch
  }, func]

SpecialFunctionQ[func_] :=
  MemberQ[{
    Erf, Erfc, Erfi,
    FresnelS, FresnelC,
    ExpIntegralE, ExpIntegralEi, LogIntegral,
    SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
    Gamma, LogGamma, PolyGamma,
    Zeta, PolyLog, ProductLog,
    EllipticF, EllipticE, EllipticPi
  }, func]

HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1, Hypergeometric2F1, HypergeometricPFQ}, func]

AppellFunctionQ[func_] :=
  MemberQ[{AppellF1}, func]

```

Maple grading function

```

# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017 Use Maple leaf count instead since buildin
#Nasser 03/23/2017 missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017 corrected the check for complex result
#Nasser 10/27/2017 check for leafsize and do not call ExpnType()
#
# if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019 Added debug flag, added 'dilog' to special functions
#
# see problem 156, file Apostol_Problems
#Nasser 4/07/2022 add second output which gives reason for the grade

GradeAntiderivative := proc(result, optimal)
local leaf_count_result,
      leaf_count_optimal,
      ExpnType_result,
      ExpnType_optimal,
      debug:=false;

      leaf_count_result:=leafcount(result);

```

```

#do NOT call ExpnType() if leaf size is too large. Recursion problem
if leaf_count_result > 500000 then
    return "B","result has leaf size over 500,000. Avoiding possible recursion issues
fi;

leaf_count_optimal := leafcount(optimal);
ExpnType_result := ExpnType(result);
ExpnType_optimal := ExpnType(optimal);

if debug then
    print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
fi;

# If result and optimal are mathematical expressions,
# GradeAntiderivative[result,optimal] returns
# "F" if the result fails to integrate an expression that
# is integrable
# "C" if result involves higher level functions than necessary
# "B" if result is more than twice the size of the optimal
# antiderivative
# "A" if result can be considered optimal

#This check below actually is not needed, since I only
#call this grading only for passed integrals. i.e. I check
#for "F" before calling this. But no harm of keeping it here.
#just in case.

if not type(result,freeof('int')) then
    return "F","Result contains unresolved integral";
fi;

if ExpnType_result<=ExpnType_optimal then
    if debug then
        print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
                return "B",cat("Both result and optimal contain complex but leaf count of
                                convert(leaf_count_result,string)," vs. $2 ("
```

```

                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_c
        end if
    else #result contains complex but optimal is not
        if debug then
            print("result contains complex but optimal is not");
        fi;
        return "C","Result contains complex when optimal does not.";
    fi;
else # result do not contain complex
    # this assumes optimal do not as well. No check is needed here.
    if debug then
        print("result do not contain complex, this assumes optimal do not as well");
    fi;
    if leaf_count_result<=2*leaf_count_optimal then
        if debug then
            print("leaf_count_result<=2*leaf_count_optimal");
        fi;
        return "A"," ";
    else
        if debug then
            print("leaf_count_result>2*leaf_count_optimal");
        fi;
        return "B",cat("Leaf count of result is larger than twice the leaf count of opt
                                convert(leaf_count_result,string)," $ vs. $2(",
                                convert(leaf_count_optimal,string)," )=",convert(2*leaf_count
        fi;
    fi;
else #ExpnType(result) > ExpnType(optimal)
    if debug then
        print("ExpnType(result) > ExpnType(optimal)");
    fi;
    return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
    return (has(expression,I));
end proc:

```

```

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'^^') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
      max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'+`) or type(expn,'*`) then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else

```

```

9
end if
end proc:

ElementaryFunctionQ := proc(func)
  member(func, [
    exp, log, ln,
    sin, cos, tan, cot, sec, csc,
    arcsin, arccos, arctan, arccot, arcsec, arccsc,
    sinh, cosh, tanh, coth, sech, csch,
    arcsinh, arccosh, arctanh, arccoth, arcsech, arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func, [
    erf, erfc, erfi,
    FresnelS, FresnelC,
    Ei, Ei, Li, Si, Ci, Shi, Chi,
    GAMMA, lnGAMMA, Psi, Zeta, polylog, dilog, LambertW,
    EllipticF, EllipticE, EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func, [Hypergeometric1F1, hypergeom, HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func, [AppellF1])
end proc:

# u is a sum or product. rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
    op(2,u)
  else
    apply(op(0,u), op(2..nops(u), u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17 Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
  MmaTranslator[Mma][LeafCount](u);
end proc:

```

Sympy grading function

```

#Dec 24, 2019. Nasser M. Abbasi:
#           Port of original Maple grading function by
#           Albert Rich to use with Sympy/Python
#Dec 27, 2019 Nasser. Added `RootSum`. See problem 177, Timofeev file
#           added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
                    asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
                    asinh,acosh,atanh,acoth,asech,acsch
                    ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
                    fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
                    gamma,loggamma,digamma,zeta,polylog,LambertW,
                    elliptic_f,elliptic_e,elliptic_pi,exp_polar
                    ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
            return True
        else:
            return False

```

```

except AttributeError as error:
    return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow): #type(expn,'^')
        if isinstance(expn.args[1],Integer): #type(op(2,expn),'integer')
            return expnType(expn.args[0]) #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational): #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0])) #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnTy
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'+') or type(expn,'*')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func): #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0])) #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1) #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:

```



```

    m1 = max(map(expnType, list(expn.args)))
    return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation = ""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation = ""
                    else:
                        grade = "B"
                        grade_annotation = "Both result and optimal contain complex but leaf count of result is large"
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation = "Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation = ""
                else:
                    grade = "B"
                    grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_count_result)
            else:
                grade = "C"
                grade_annotation = "Result contains higher order function than in optimal. Order "+str(ExpnType_result)

```

```

# print("Before returning. grade=", grade, " grade_annotation=", grade_annotation)

return grade, grade_annotation

```

SageMath grading function

```

# Dec 24, 2019. Nasser: Ported original Maple grading function by
# Albert Rich to use with Sagemath. This is used to
# grade Fricas, Giac and Maxima results.
# Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
# 'arctan2', 'floor', 'abs', 'log_integral'
# June 4, 2022 Made default grade_annotation "none" instead of "" due
# issue later when reading the file.
# July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    # print("Enter tree_size, expr is ", expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow: # isinstance(expr, Pow):
        if expr.operands()[1] == 1/2: # expr.args[1] == Rational(1,2):
            if debug: print("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

```

```

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
        'sin','cos','tan','cot','sec','csc',
        'arcsin','arccos','arctan','arccot','arcsec','arccsc',
        'sinh','cosh','tanh','coth','sech','csch',
        'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
    ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")

    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
        'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral',
        'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
        'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
        'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func , " is special_function")
        else:
            print ("func ", func , " is NOT special_function")

    return m

def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']  #[appellf1] can't find this in sagemath

```

```

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(), "base_ring") and hasattr(expn.parent(), "gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in is_atom")
        print ("caught exception" , type(error).__name__ )
        return False

def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list: #isinstance(expn,list):
        return max(map(expnType, expn)) #apply(max,map(ExpnType,expn))
    elif is_sqrt(expn):
        if type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow: #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer: #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0]) #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational: #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)

```

```

        return 1
    else:
        return max(2,expnType(expn.operands()[0])) #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2) #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()): #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(4,m1) #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(5,m1) #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(6,m1) #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
    #is checked before calling the grading function that is passed.
    #but kept it here.
    m1 = max(map(expnType, expn.operands())) #max(map(expnType, list(expn.args)))
    return max(8,m1) #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):

    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)

    expnType_result = expnType(result)
    expnType_optimal = expnType(optimal)

```

```

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation = "none"
            else:
                grade = "B"
                grade_annotation = "Both result and optimal contain complex but leaf count of result is larger than"
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation = "Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation = "none"
        else:
            grade = "B"
            grade_annotation = "Leaf count of result is larger than twice the leaf count of optimal. " + str(leaf_c

else:
    grade = "C"
    grade_annotation = "Result contains higher order function than in optimal. Order " + str(expnType_result)

print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation

```